

Group exercise

May 2, 2023

I collect here some exercises we will discuss together in class.

1 Massless bosonic 1+1D fields

Consider a 1+1D massless bosonic scalar and real field $\theta(x, \tau)$ described by the following action in Euclidean time:

$$S_0 = \frac{1}{2\pi} \int dx d\tau \frac{(\partial_\tau \theta)^2}{u} + u (\partial_x \theta)^2 \quad (1)$$

where u is a velocity (it is invariant under rescaling).

1. What is the scaling dimension of the θ field?
2. Based on its scaling dimension, what would you expect from correlation functions of the kind $\langle \theta(r, \tau) \theta(r', \tau) \rangle$?

These correlation function are actually quite non-trivial, and we will often use them in the next lectures. Therefore, it is worth calculating them in some detail.

1.1 Correlations of the massless bosonic field in 1+1D.

Use your favorite technique to show that the following two-point correlation function can be approximated in the following way:

$$\langle \theta(x, 0) \theta(x', 0) \rangle \approx -\frac{1}{2} \ln |x - x'| + \frac{1}{2} \ln \Lambda_{\min}^{-1} \quad (2)$$

where it is convenient to introduce an infrared cutoff $\Lambda_{\min} = 1/L$ and an ultra-violet cutoff $\Lambda_{\max} = 1/a$ such that $a \ll |x - x'| \ll L$.

Hints: In full generality one can calculate $\langle \theta(x, \tau) \theta(x', \tau') \rangle$. In this case, it may be convenient to define space-time vectors $\vec{p} = (\omega/u, -k)$ and $\vec{r} =$

$(u[\tau - \tau'], x - x')$ such that $r = \sqrt{(x - x')^2 + u^2(\tau - \tau')^2}$ and you can apply the following relations:

$$\int_0^{2\pi} \frac{d\alpha}{2\pi} e^{ipr \cos \alpha} = J_0(pr), \quad (3)$$

$$\int_{\Lambda_{\min}}^{\Lambda_{\max}} dp \frac{J_0(pr)}{2p} \approx -\frac{1}{4} \ln \left(\frac{r^2 + \Lambda_{\max}^{-2}}{\Lambda_{\min}^{-2}} \right) \approx -\frac{1}{2} \ln r + \frac{1}{2} \ln \Lambda_{\min}^{-1}, \quad (4)$$

where the approximation is valid for $a \ll |x - y| \ll L$.

2 About the dynamical scaling exponent z

In the examples we saw on rescaling, we always considered systems whose gapless fixed points display a linear dispersion $\omega = vk$. We now want to check what happens when this is not verified. Consider the Lagrangian:

$$L = \int d^{D-1}x \left[\psi^\dagger (i\partial_t) \psi + a \psi^\dagger \vec{\nabla}^2 \psi \right] \quad (5)$$

1. Check that the dispersion is $\omega = ak^2$. L is indeed Gaussian and describes a gapless system, although the dispersion is no longer linear.
2. Try to apply the (wrong) rescaling $x = bx'$, $t = bt'$. Show that there is no possible rescaling of the fields $\psi = b^{-D_\psi} \psi'$ that fulfills the relation:

$$\int dt d^{D-1}x \left[\psi^\dagger (i\partial_t) \psi + a \psi^\dagger \vec{\nabla}^2 \psi \right] = \int dt' d^{D-1}x' \left[\psi'^\dagger (i\partial_{t'}) \psi' + a \psi'^\dagger \vec{\nabla}'^2 \psi' \right] \quad (6)$$

This equation, however, must be fulfilled for any gapless and Gaussian theory.

3. Apply the rescaling $x = bx'$, $t = b^z t'$. Find z and D_ψ such that Eq. (6) is verified.