TORIC CODE

$\boxed{GS_s}$
- Symmetries

$\downarrow$

T.O. models symmetries are not necessary

$\boxed{\text{Excitations}}$
- Anyons

$\downarrow$

Mutual statistics

$$H = -J\sum_v A_v - J\sum_p B_p$$

A_v $\quad$ B_p

$[A_v, B_p] = 0 \Rightarrow$ Stabilizers

$A_v|GS\rangle = |GS\rangle \quad \forall v$
$B_p|GS\rangle = |GS\rangle \quad \forall p$

$\Downarrow$

Projectors over the GSs

$P_v = \frac{\mathbb{1} + A_v}{2}$, $P_p = \frac{\mathbb{1} + B_p}{2}$

$P_v^2 = P_v \qquad P_p^2 = P_p \qquad \forall$ GS : $P_v|GS\rangle = |GS\rangle \ \forall v$
$P_p|GS\rangle = |GS\rangle \ \forall p$

EXCITATIONS

- e - excitations : vertices

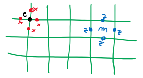
$A_v|\psi\rangle = -|\psi\rangle \Rightarrow$ in the vertex v there is an excitation

$\rightarrow$ Energy $2J$

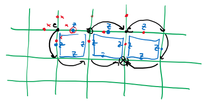
- m - excitations : plaquettes

$B_p|\psi\rangle = -|\psi\rangle \Rightarrow$ in the plaquette p

$\rightarrow$ Energy $2J$



$\sigma_z|GS\rangle$ $\boxed{GS}$

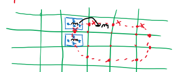


An open string of σ_z creates and moves 2 e-excitations

A closed string creates, moves and annihilates them

Closed strings of z operators commute with A_v, B_p, H

σ_x and plaquette m excitations



- σ_x creates 2 magnetic excitations
- x-strings on the dual lattice creates 2 m excitations
- x strings that are closed in the dual lattice commute with H

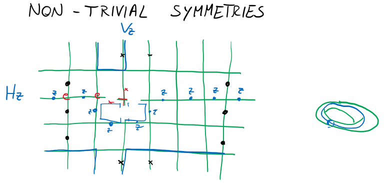
- Any non-trivial single-site operator excites either the plaquettes or the vertices in its neighborhood
- Closed loops are symmetries
- Contractible loops are just products of A's or B's

$\prod_{v\in C} A_v |GS\rangle = \mathbb{1}|GS\rangle$

$\rightarrow$ They do not change the GS [trivial]

- $\boxed{\text{Are there other symmetries?}}$

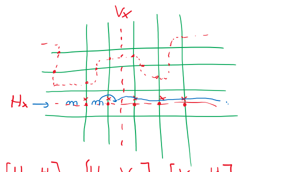
NON-TRIVIAL SYMMETRIES



Non-contractible z strings are not the product of B's

They are non-trivial symmetries

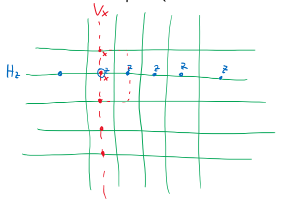
$[H_z, H] = [V_z, H] = [H_z, V_z] = 0$



$[H_x, H] = [H_x, V_x] = [V_x, H]$

4 Non-trivial symmetries

[Non-contractible loops of x and z kind]



$[V_x, H_z] \neq 0$
$\{V_x, H_z\} = 0$
$\{V_z, H_x\} = 0$

We have symmetries that do not commute

$\Rightarrow$ The spectrum is degenerate.

H_z, V_z, H can be simultaneously diagonalized

$|GS\rangle :\Leftrightarrow \forall v \ A_v|GS\rangle = |GS\rangle$
$\forall p \ B_p|GS\rangle = |GS\rangle$

$H_z|GS\rangle = \pm|GS\rangle \qquad H_z^2 = \mathbb{1} \qquad V_z^2 = \mathbb{1}$

$V_z|GS\rangle = \pm|GS\rangle$

$\rightarrow$ I diagonalized H, H_z, V_z together

[There are 4 GSs : from counting]

Maybe $|GS\rangle = \{|\underset{H_z}{\uparrow}\underset{V_z}{\uparrow}\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$: candidates?

$\boxed{V_x|\uparrow\uparrow\rangle}$: $H_z[V_x|\uparrow\uparrow\rangle] = -V_x[H_z|\uparrow\uparrow\rangle] = -[V_x|\uparrow\uparrow\rangle]$

$V_x|\uparrow\uparrow\rangle = |\downarrow\uparrow\rangle$

$|S_{z_1}, S_{z_2}\rangle$ / $H_z|S_{z_1}, S_{z_2}\rangle = S_{z_1}|S_{z_1}, S_{z_2}\rangle$
$V_z|S_{z_1}, S_{z_2}\rangle = S_{z_2}|S_{z_1}, S_{z_2}\rangle$

$S_{z_i} = \pm 1 = \uparrow, \downarrow$

$\boxed{H_x|\uparrow\uparrow\rangle = |\uparrow\downarrow\rangle}$ $\qquad [\{H_x, V_z\} = 0]$

4 GSs $\Leftrightarrow$ 2 logical qubits

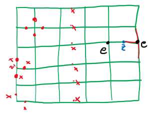
First qubit : $H_z \rightarrow \sigma_{z,1}$
$V_x \rightarrow \sigma_{x,1}$

Second " : $V_z \rightarrow \sigma_{z,2}$
$H_x \rightarrow \sigma_{x,2}$

|| These ground states are characterized and manipulated by non-contractible [non-local] string symmetries

SURFACE CODES

[open boundaries]



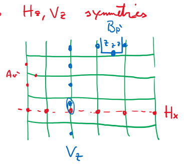
$H = \sum_{v\in bulk} -JA_v - J\sum_{p\in bulk} B_p$

$= -J\sum_{v\in boundary} A'_v$

Trivial choice of the boundary : "smooth" everywhere

$[H_x, V_x] = [H_x, H] = [V_x, H] = 0$

No H_z, V_z symmetries $\Rightarrow$ No degeneracy



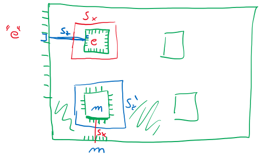
$H = -J\sum_v A_v - J\sum_p B_p$
$-J\sum_{v\in \text{vertical boundary}} A'_v - J\sum_{p'\in \text{Horiz. bound.}} B_{p'}$

$[V_z, H] = [H_x, H] = 0$
$\{V_z, H_x\} = 0$ $\Big\}$ 1 qubit

2 GSs $\qquad V_z|\uparrow\rangle_{GS} = +|\uparrow\rangle_{GS}$
$V_z|\downarrow\rangle_{GS} = -|\downarrow\rangle_{GS}$
$H_x|\uparrow\rangle_{GS} = |\downarrow\rangle_{GS}$

B'_p boundaries are "rough"
A'_v boundaries are "smooth"

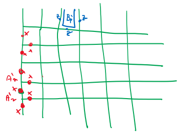
"e"



S_x : x string on the dual lattice

$S_x^2 = \mathbb{1}$: $S_x = \pm 1$

S_z : z string on the lattice



EXCITATIONS of the TORIC CODE

LGT : e and m

e $\downarrow$ electric; m $\downarrow$ magnetic flux exci. $\mathbb{Z}_2$ LGT

$\mathbb{Z}_2$ LGT with $\mathbb{Z}_2$ Higgs matter

$\rightarrow$ unitary gauge

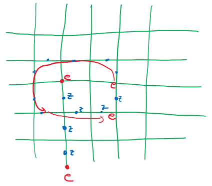
A_v is the mass of the matter charge

B_p is the plaquette magnetic operator

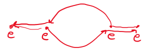
z loops are Wilson loops

H_z, V_z are Polyakov loops $\quad$ move e

H_x, V_x are t'Hooft loops $\quad$ move m



- If I move e around e nothing happens



Nothing happens

- e charges behave like bosons

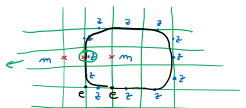
The same is true for m :

if I exchange m_1 with m_2, the GS goes back to itself

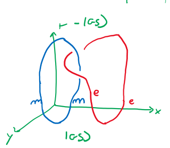
[contractible loops]

- $\boxed{\text{When I move e around m , I pick a -1 phase}}$

I get a -1

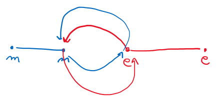


0. I begin from the $|GS\rangle$
1. I nucleate 2 m's
2. I " 2 e's
3. I move 1e around 1m
4. I annihilate the 2e
5. " " the 2m
6. I end in $-|GS\rangle$



Aharonov-Bohm phase

$\oint A\, dl = \pi$



Winding e around m

[counterclockwise]

R^2_{em} : 2 exchanges [c.c.]

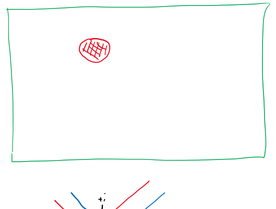
$R^2_{em} = -1$

$\Downarrow$

"$R_{em} = \pm i$"

e and m have a fractional mutual statistics

"anyons"

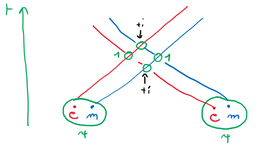


4 topological sectors

- There is no exci [exci are in pairs]

(1) : trivial or vacuum topol sectors

- e
- m
- $e \otimes m = \psi$



$R^2_{mm} = 1 \Rightarrow R_{mm} = +1$

$R_{ee} = 1$

$\boxed{R_{\psi\psi} = R_{em} \cdot R_{me} = -1}$

ψ is a fermion

TORIC CODE

- Gap $\sim 4J$ separates GSs from excitations
- 4-fold degeneracy of the GSs on the torus

$\# GSs = 4^{\text{genus}}$ [closed]

Surface code [open]

$\# GSs = 2^{\text{holes}}$

- $\forall$ each local [single qubit] operator V_α

$\langle \psi_{gs,i}|V_\alpha|\psi_{gs,j}\rangle = \delta_{ij}$

$\Rightarrow$ Topological order

- "Weird" excitations

$\hookrightarrow$ Aharonov-Bohm phase + "fermions"

Abelian anyons

- localized quasiparticles
- gapped [2J]
- When you exchange them $R_{aa} \neq \pm 1$

"Braiding" $\leftarrow R_{aa} = e^{i\theta_a}$

$[\theta_a = \pi \frac{m}{n}]$

R is counterclock

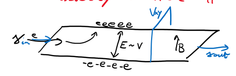
$R^{-1} = R^{\dagger}$ is clockwise

$\boxed{\text{Fractional quantum Hall systems}}$

2D systems : "electron gases"

$\uparrow B \sim 20T$

Classically : Hall effect



$\boxed{R_{xy} = \frac{V_{xy}}{I}}$

$\downarrow$

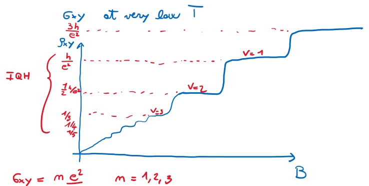
ρ_{xy} : Hall resistivity

σ_{xy} : Hall conductivity

$j = \sigma_{xy} \cdot E$

QUANTUM HALL

σ_{xy} at very low T



Classically $\rho_{xy} \sim B$

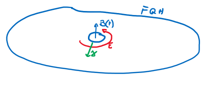
$\sigma_{xy} = m\frac{e^2}{h} \qquad m = 1, 2, 3$

$\sigma_{xy} = \nu\frac{e^2}{h} \qquad \nu = 1/3, 2/3, 4/3, 7/3$

$2/5$

$\frac{1}{2}, 5/2$

$\boxed{j_y = E_x \cdot \sigma_{xy} = E_x \cdot \frac{e^2}{h} \cdot \nu}$



$\phi(t=0) = 0$ + background

$\phi(t=T) = \phi_0$ + background $\qquad \frac{hc}{e} = \phi_0$

Adiabatic process of inserting the flux

$\int dt$