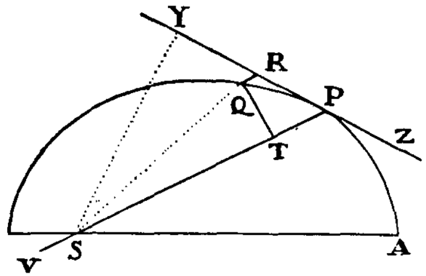
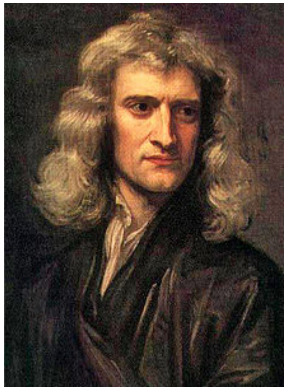
Module 1

Newton's mathematical force

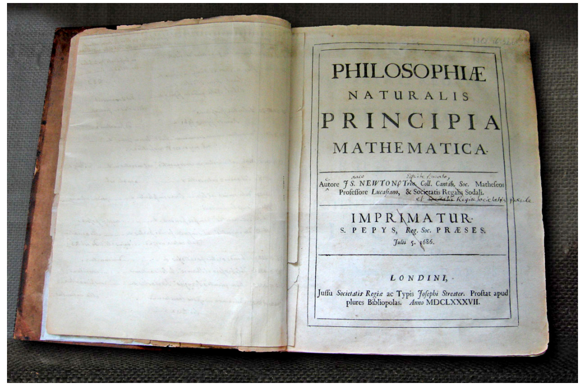


UNIVERSITY OF COPENHAGEN

Philosophiæ Naturalis *Principia* Mathematica (1687)



Newton (1687)



Dec 14, 2016 theguardian

Isaac Newton masterwork becomes most expensive science book sold

First edition of Principia Mathematica, which was published in 1687 and sets out Newton's laws of motion, raises £3m at auction

"The *Principia* is perhaps the greatest intellectual stride that it has ever been granted to any man to make" (Einstein)

"The *Principia* marked the epoch of a great revolution in physics. The method followed by its illustrious author Sir Newton … spread the light of mathematics on a science which up to then had remained in the darkness of conjectures and hypotheses" (Clairaut)

"The *Principia* is one of the most influential works in Western culture, but it is a work more revered than read" (Brackenridge)

Mathematics and Reality?

Divergence	Convergence
- Geometry is geometry only through the **abstract simplicity of its object**. Only that makes it certain and demonstrative. The **object of physics is much vaster**. That is what makes it difficult, uncertain and obscure. But this is essential to it: **one is not a better physicist because one is the best of geometers** (Castel, 1743)	- It is not sufficient for a system to satisfy the phenomena only in a vague and general manner, or to **provide plausible explanations of some of them**: the details and the **precise calculations are the touchstone**; only they can tell if one must adopt, reject, or modify an hypothesis (D'Alembert, 1749)

What did mathematics do to physics? (Gingras, 2001)

Motivation to write the *Principia*

January 1684

Hooke

Wren

Halley

How to derive the laws of planetary motion?

Hooke claims to have derived that an inverse square law leads to an ellipse, but shows no evidence.

August 1684

Months passed and Hooke had yet to produce his evidence. Edmund Halley traveled to Cambridge to find out what Isaac Newton had to say on the matter.

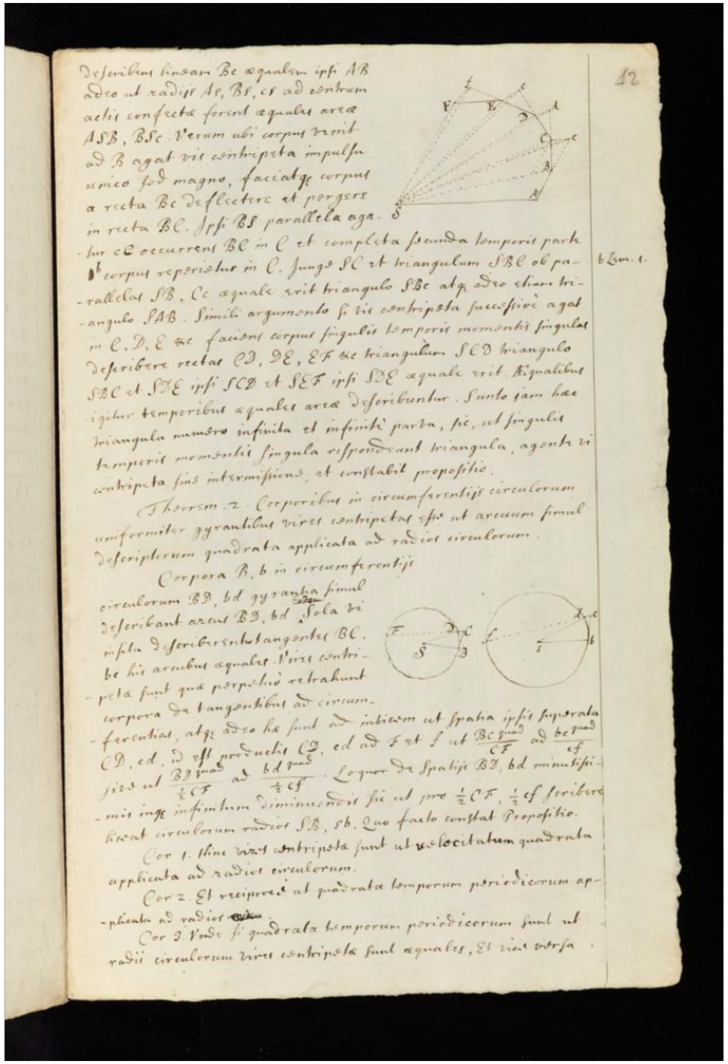
When Halley put the question to Newton, Newton surprised him by saying that he had already made the derivations some time ago; but that he could not find the papers…

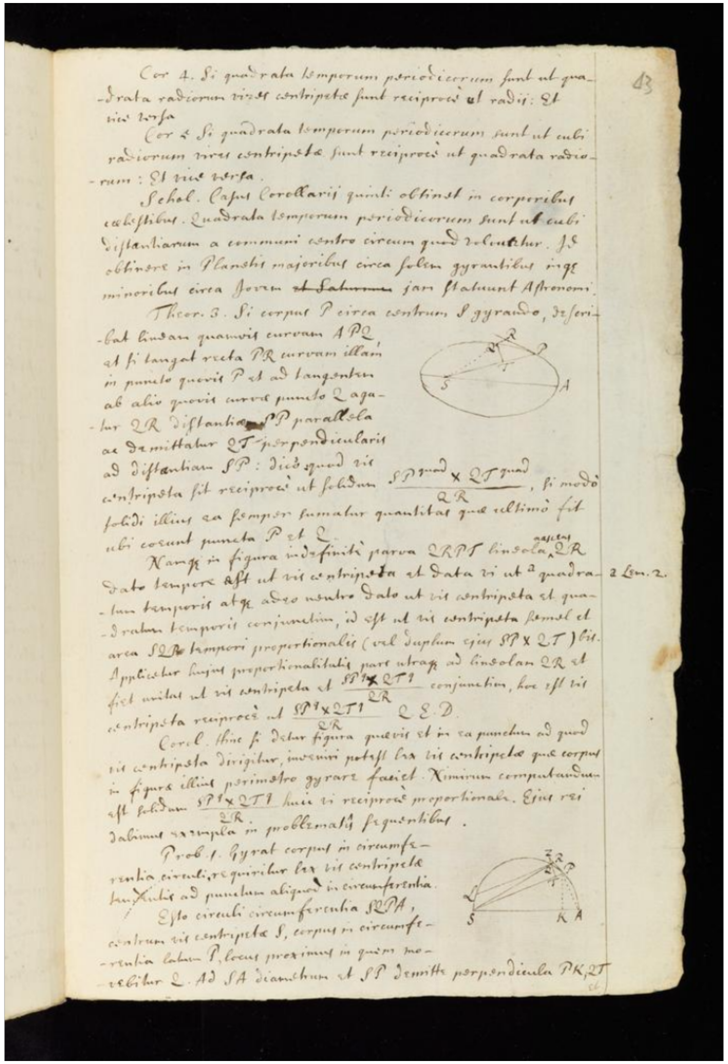
November 1684

Newton sent Halley a nine-page manuscript titled *De Motu Corporum in Gyrum* (On the Motion of Orbiting Bodies).

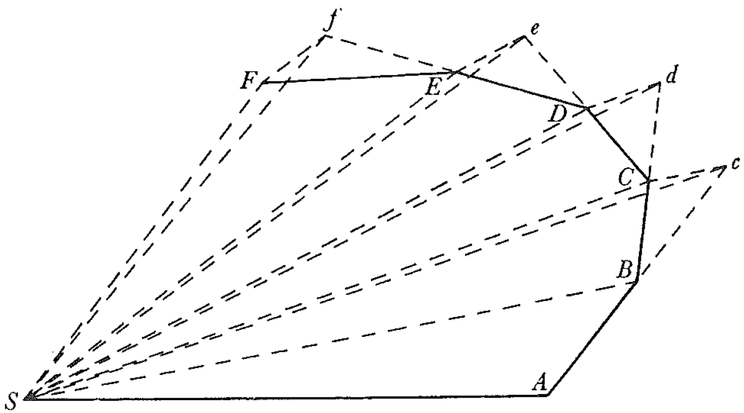
Halley is so fascinated by its content and method that he demands Newton to send more of his work to the Royal Society – which leads to the *Principia* (1687)

De Motu Corporum in Gyrum



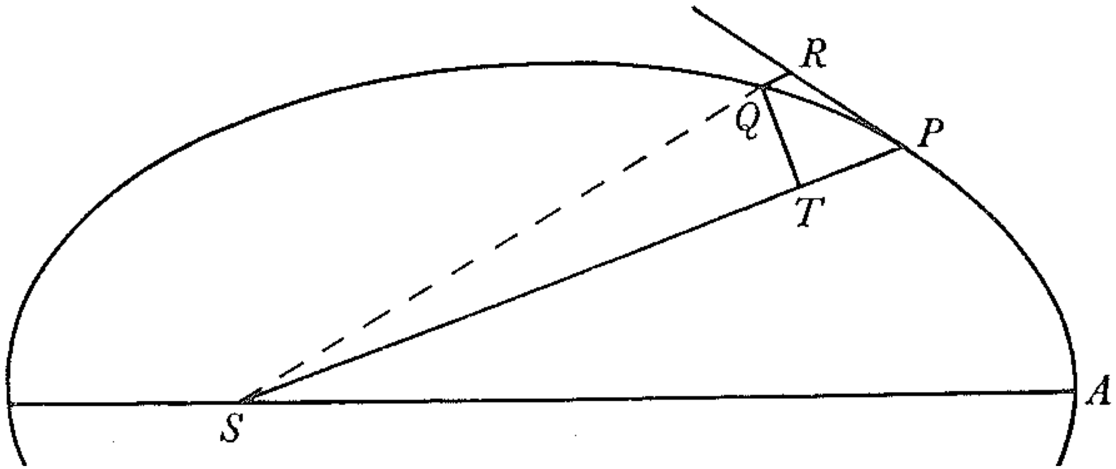


Theorem 1: Central force ➔ Equal areas



Wikipedia: Newtons proof of Keplers second law.gif

Theorem 3: Force proportional to *QR/(SP² x QT²)*



Corollary. Hence if any figure be given and in it a point to which the centripetal force is directed, there can be ascertained the law of centripetal force which shall make a body orbit in the perimeter of that figure: specifically, you must compute (the quantity of) the 'solid' $SP^2 \times QT^2/QR$ reciprocally proportional to this force. Of this procedure we shall give illustrations in following problems.

Problem 1: Center in the circumference

$F \propto QR/(SP^2 \times QT^2)$

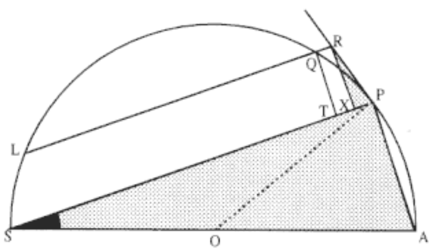


Figure 5.3A
A revised diagram for Problem 1. The perpendicular *RX* and the radius *OP* are added.

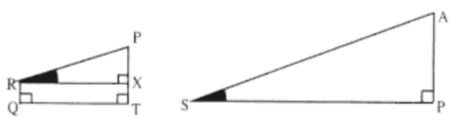


Figure 5.3B
The triangle *RPX* is similar to the triangle *SAP* .

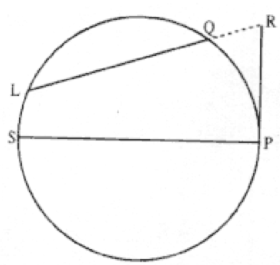
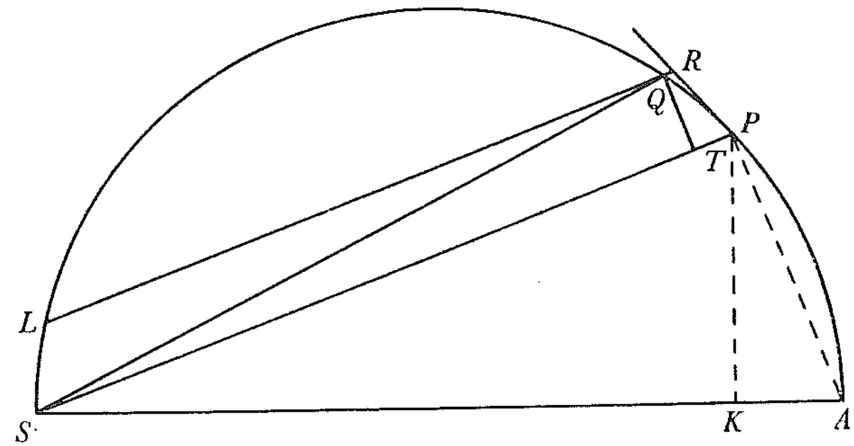


Figure 5.4B
Thus, *RL* / *RP* = *RP* / *QR* or
$RP^2 = QR \times RL$ as required in Problem 1



1: $\Delta SAP \sim \Delta RPX \therefore (SA/SP)^2 = (RP/QT)^2$

2: $RP^2 = (QR).(LR)$

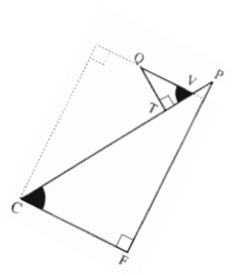
3: R ➔ P LR ➔ SP

$$QR/QT^2 = SA^2/SP^3$$

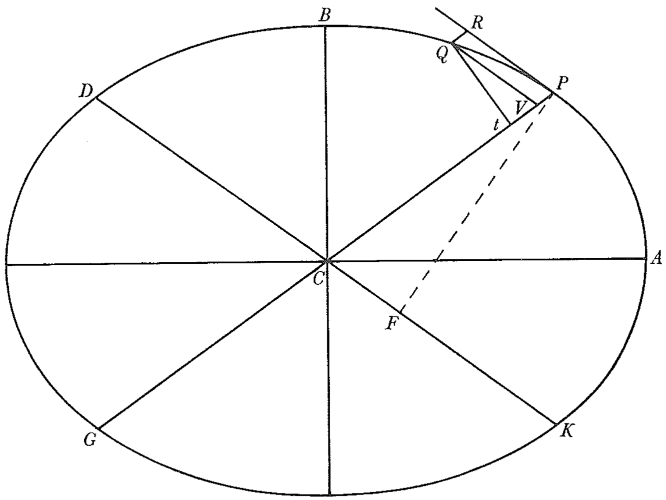
$$F(r) \propto 1/r^5$$

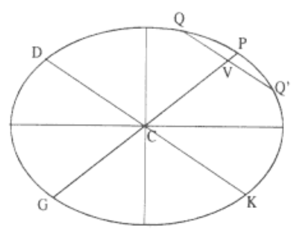
Problem 2: Center in the center of an ellipse

$F \propto QR/(CP^2 \times QT^2)$



1: $\Delta QTV \sim \Delta PFC \therefore (QT/QV)^2 = (PF/PC)^2$





2: $(PV \times VG)/QV^2 = PC^2/CD^2$

Figure 5.12
The diameter *PG* bisects the chords *QQ* ' and *DK* . From Proposition 15 of Book 1 of Apollonius's *Conics* , the ratio of $PV \times VG / QV^2$ is equal to the ratio PC^2 / DC^2 .

1 and 2: $QT^2 = (PV \times VG).(CD^2/PC^2).(PF/PC)^2$

$PV = QR$ $\quad$ $QR/QT^2 = PC^4/(VG \times CD^2 \times PF^2)$

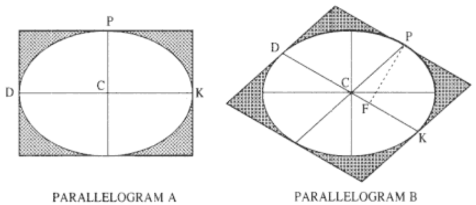


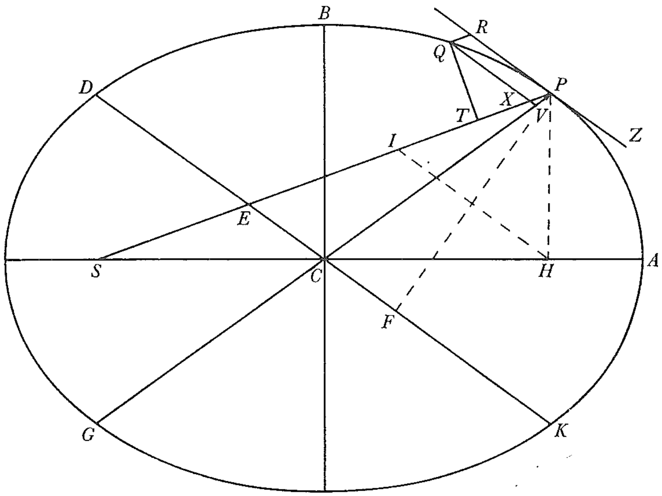
Figure 4.1
The area of parallelogram A is equal to the area of parallelogram B (Proposition 31, Book 7, of the *Conics* of Apollonius of Perga).

3: $BC \times CA = CD \times PF =$ *const.*

4: $R \rightarrow P \quad VG \rightarrow 2PC$

$F(r) \propto r$

Problem 3: Center in the focus of an ellipse (finally!)

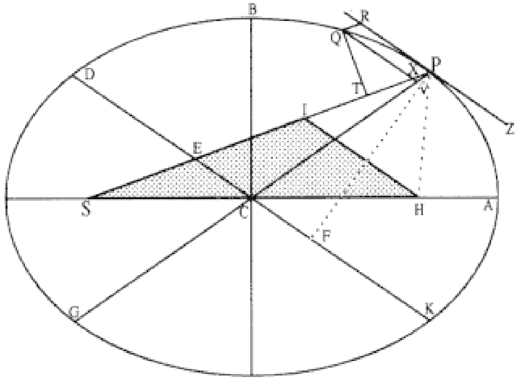


$F \propto QR/(SP^2 \times QT^2)$

Find QR

1: ΔPXV ~ ΔPEC ∴ (PE/PC) = (PX/PV)

Since PX = QR ∴ QR = PV (PE/PC)



2: PE = AC Why? “clearly”

ΔSIH ~ ΔSEC CS = CH (foci)

SE = EI PE = (PS + PI)/2 = (PS + PH)/2

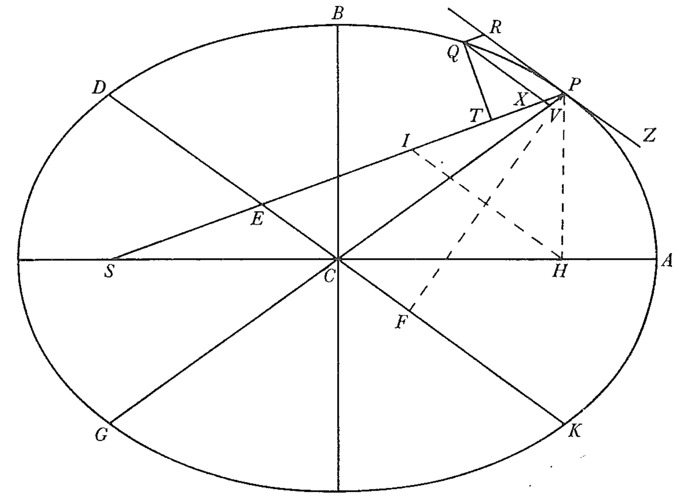
3: $(PV \times VG)/QV^2 = PC^2/CD^2$

Sub (2) and (3) in (1)

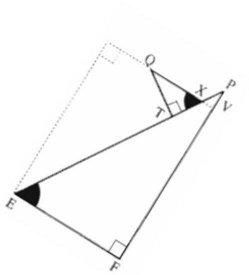
$QR = (QV^2/VG).(PC^2/CD^2).(AC/PC)$

Problem 3: Center in the focus of an ellipse (finally!)

$F \propto QR/(SP^2 \times QT^2)$



Find QT^2



1: $\Delta PEF \sim \Delta QTX \therefore QT/QX = PF/PE$

$QT^2 = QX^2.(PF^2/PE^2)$

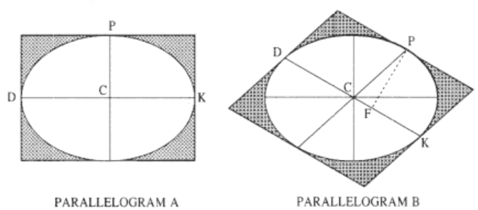


Figure 4.1
The area of parallelogram A is equal to the area of parallelogram B (Proposition 31, Book 7, of the *Conics* of Apollonius of Perga).

2: $AC \times BC = DC \times PF$

Since $PE = AC$

$QT^2 = QX^2.(BC^2/DC^2)$

$QR/QT^2 = (QV^2/QX^2).(VG/PC)(AC/BC^2)$

$R \rightarrow P \qquad QV \rightarrow QX \qquad VG \rightarrow 2PC$

From the definition of the constant *latus rectum* $L = 2BC^2/AC$

$QR/QT^2 = (1/L)$ which is a constant!

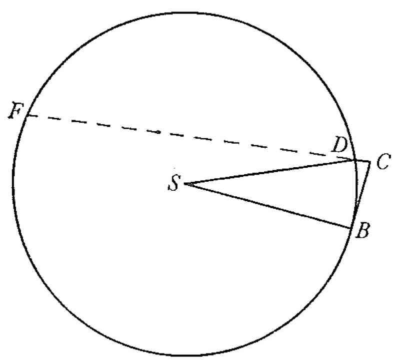
$QR/QT^2 = (QV^2/QX^2).(PC/VG)(2/L)$

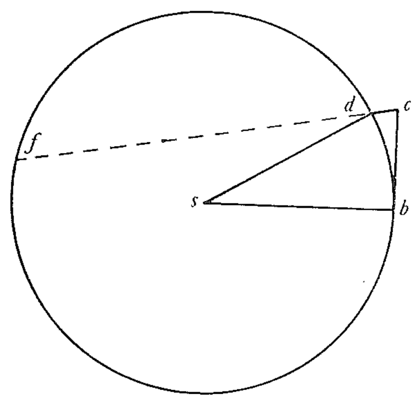
$F(r) \propto 1/r^2$

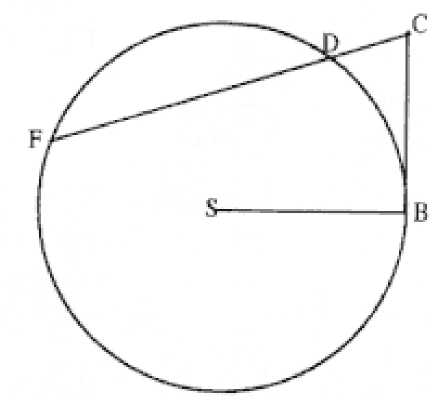
Questions for discussion

- In theorem 2, I do not understand why it is square? I cannot see why BC^2/CF makes more sense then BC/CF. And I cannot see why BC^2/CF can be written as $BD^2/\frac{1}{2}CF$.

Theorem 2: Centripetal force $\propto$ arc^2/R







CD / BC = BC / CF

C1: $F_1/F_2 = (v_1^2/r_1)/(v_2^2/r_2)$

C2: $F_1/F_2 = (T_2^2/r_2)/(T_1^2/r_1)$

C3: If $T_2^2/r_2 = T_1^2/r_1$ then $F_1 = F_2$

C4: If $T_2^2/r_2^2 = T_1^2/r_1^2$ then $F_1/F_2 = r_2/r_1$

C5: If $T_2^2/r_2^3 = T_1^2/r_1^3$ then $F_1/F_2 = r_2^2/r_1^2$

Questions for discussion

- In theorem 2, I do not understand why it is square? I cannot see why BC^2/CF makes more sense then BC/CF. And I cannot see why BC^2/CF can be written as BD^2/½CF.

- It seems that many of Newton's formulations emerge from the idea that we can take motion, divide into small time steps, sum it all up (integrate) and voilá we have a model describing something continuous. Was this widely accepted as a method of derivation in the 1680's?

- I understand that this piece of work is prior to Newton's Principia, but I can't help to question why Newton uses lines and points to derive his theorems. Why is a vector (force vector) not used for these tasks? The force vector does not exist at this time?

- I would like to know if this paper was written before the term force defined or not? Or is this paper the origin of the definition of force (F=m*a)?

Questions for discussion

- Why are the algebraic statements in his theorems and hypotheses not written in algebraic notation?
- Was he only interested in the proportionality relations or were the actual constants, equations and units also a priority for him as well?
- How much of physics is at Newton's time in history is actually mathematical?
- His *scholium* in the paragraph after theorem 4. I do not understand it... Is he trying to say that we need loads of empiric data to be able to say something quantitative about some system? (I mean; if we only take a snapshot of all the heavenly bodies in our solar system, we have no idea which orbit they have. We need to have some historical data (some knowledge of its circle arc) in order to be able to predict the planetary motion.)

End of module feedback

- Please go to b.socrative.com (student login)
- Enter the HISPHYSKU room
- Fill out the short (anonymous) survey
- Tak skal du have!

P.S.: Remember to think about the topic for your seminar!