Joseph Fourier's Analytical Theory of Heat: A Legacy to Science and Engineering

F. S. SHOUCAIR, MEMBER, IEEE

Abstract—The movement towards the *mathematization* of the physical sciences by members of the early 19th century French scientific establishment is illustrated by a brief account of J. B. J. Fourier's celebrated "Théorie Analytique de la Chaleur" [1]. The spirit and substance of his unique scientific legacy, and the profound influence it has had on mathematics, physics, and modern engineering methods are highlighted.

INTRODUCTION

THE early part of the 19th century is a particularly interesting period in the history of science because it marks the revolutionary transformation of the "traditional" scientific disciplines into the exact sciences they have become today. The main theme of this revolution can be summarized as the *mathematization* of the sciences in general, and of electricity, magnetism, mechanics, light, and heat, in particular. While the cumulative efforts of many researchers brought about the transformations of the first four of these sciences, Fourier, virtually single handedly, formulated the mathematical theory of heat propagation in solids in use today (including most of his notation). In the course of his researches, Fourier developed a large body of profoundly original mathematical concepts and techniques, whose importance to modern engineering methods is universally acknowledged and properly associated with his name. Another noteworthy revolutionary trend of this period is the emergence of the philosophical approaches of "separatism" (between facts and underlying causes) and of "positivism" (relying on facts and observations to formulate physical laws) in scientific theories. Fourier's "Théorie Analytique de la Chaleur" is an early example of a remarkably successful application of these philosophies to a physical problem of considerable modern relevance. His work is not only a source of many of our most powerful mathematical and engineering tools, but more importantly, a timeless "charter" of our scientific methods as well.

At the beginning of the 19th century, the prevailing view among European (German, in particular) academic physicists paralleled that of 18th century "electricians" in their distinction between the "concrete" (physical) and the "abstract" (mathematical). Indeed, the belief they held was that "mathematics tended to mislead the physicist from his proper field of study" [2]. In France, however, this view increasingly came under challenge because of the influence of a group of exceptional scientists, living, working, and regularly meeting in Paris in the early 1800's. While the first, and most prominent among these, was Simon de Laplace (1749–1827), the list of his peers rapidly grew during his tenure as Governor and Professor at the newly founded Ecole Polytechnique (1795) and soon included such men as Arago, Biot, Ampère, Fourier, Poisson, and Fresnel, most of whom were at some point affiliated with the Ecole as students or teachers. The financial (state) support provided by Napoleon to reform the educational system in France proved an important catalyst to the vibrant contemporary scientific activity because it benefited not only the lycées and institutions of higher learning, but research programs of savants such as Laplace as well.

From the standpoint of the careers of these scientists, the most important research institutions in the physical sciences in Paris were, at the time, the Bureau des Longitudes and the Observatoire. Among the most prestigious academic institutions were the Ecole Polytechnique, the Ecole Normale, the Ecole Centrale, the Faculté des Sciences, and the Collège de France. Because it was common practice for the savants most distinguished in their profession simultaneously to hold teaching, research, and industry positions, a relatively small group of men effectively controlled the highly institutionalized French scientific establishment. Early in the century, for example, Laplace, Arago, Biot, and Poisson, all four members of the Bureau des Longitudes, held between them the most important appointments of the Ecole Polytechnique, the Faculté, and the Collège: the Mathematics, Astronomy and Physics Chairs [3]. It was thus at the instigation of an elite of prominent scientists that mathematics was gradually elevated to an unprecedented status of importance in the physical sciences, and at their hands that physics and engineering, were molded into the shape they largely retain to this day. The principal research workers in physics in Paris between 1820 and 1829, and a breakdown by field of study of their publications are listed in Table I.

This paper's aims are to examine the transformation of physics by early 19th century French scientists by the perspective of J. B. J. Fourier and to discuss the influence of his work. The difficult and unusual circumstances under which he produced his classic work are a compelling

Manuscript received September 5, 1988.
The author is with the Department of Electrical Engineering, Brown University, Providence, RI 02912.
IEEE Log Number 8928919.

0018-9359/89/0800-0359$01.00 © 1989 IEEE

TABLE I
PRINCIPAL RESEARCH WORKERS IN PHYSICS IN PARIS, 1820–1829 AND THEIR PUBLICATIONS [3]

	Light & Wave Motion	Heat	Electricity and Magnetism	Astronomy	Mathematics and Mechanics	Total
Laplace	—	—	—	17	5	22
Fourier	2	5	—	—	16	23
Arago	5	1	5	28	2	41
Biot	5	—	4	7	3	19
Fresnel	18	3	—	1	—	22
Poisson	4	6	6	7	29	52
Ampère	1	—	22	1	6	30
Dulong	—	3	—	1	2	6
Despretz	—	8	1	—	3	12
Péclet	2	—	6	—	3	11
Pouillet	—	2	5	—	1	8
Babinet	1	—	1	1	2	5
Becquerel, A.C.	—	—	32	—	—	32
Cagniard de la Tour	1	2	—	3	6	12
Totals	39	30	82	66	78	295
Percentages[a]	13%	10%	28%	22%	26%	100%

[a]Rounded to the nearest integer.

story of scientific achievement of the highest order against formidable odds and a tribute to the character of a remarkable man. His original contributions to mathematics (pure and applied), to physics, and to modern engineering methods remain ubiquitous and his scientific legacy ranks among the foremost accomplishments of all time. To the modern scholar, Fourier's "Théorie Analytique de la Chaleur" is evidence that a single work by a single man *can* have an epoch-making importance in science. Beyond its originality, Fourier's "Théorie" remains, more than ever, of standard-setting quality because it uniquely exemplifies the spirit and methods of our modern exact sciences.

A portrait of Fourier by Boilly, 1823. (Reproduced from [1], tome second. The original copy is in the possession of the Archives of the Académie des Sciences.)

THE MAN AND HIS ACHIEVEMENT

Jean-Baptiste Joseph Fourier was born on March 21, 1768 in Auxerre (about 100 miles southeast of Paris), France, the ninth of twelve children of Joseph Fourier and Edmie Germaine LeBegue. Although both parents were dead before his tenth birthday, he did receive a good education, as he was enrolled in the local Ecole Royale Militaire, which had been placed under the direction of the Benedictines, a religious order. It was there that he developed a liking for mathematics, which he is said to have studied at night, by the light of candle ends he collected during the day [4]. School records indicate that he won prizes in rhetoric, mathematics, singing, and mechanics [5]. He later took part in local political activities, and was embroiled in the turmoil of revolutionary events. In July 1794, he was arrested and jailed for a period of approximately two months. He moved to Paris the following year, where he entered the newly founded Ecole Normale. Although the school closed within months of its opening (because of renewed political trouble), Fourier had impressed some of the mathematics teachers he met, among whom were the eminent Lagrange, Laplace, and Monge.

Fourier subsequently obtained a special teaching position at the Ecole Centrale des Travaux Publiques, but was arrested again (June–July 1795) after being denounced as a former revolutionary "agitator" by old foes from Auxerre. On September 1, 1795, the Ecole Centrale was renamed the Ecole Polytechnique. It was there that, as a lecturer, Fourier assisted Lagrange and Monge in teaching courses on analysis, differential and integral calculus, descriptive geometry, dynamics, statics, and hydrostatics. He remained at the Polytechnique until 1798, when he was assigned to join a military (30 000 soldier-strong), scientific, and literary expedition under the command of General Napoleon Bonaparte. The destination was Egypt, where Fourier's many administrative duties and studies of antiquities would leave him with "the most agreeable memories" [5, p. 76].

He returned to Grenoble in 1801, after Napoleon appointed him Préfet (the highest governmental position) of the Isère department, on account of the political skills Fourier had demonstrated while in Egypt. Although Fou-

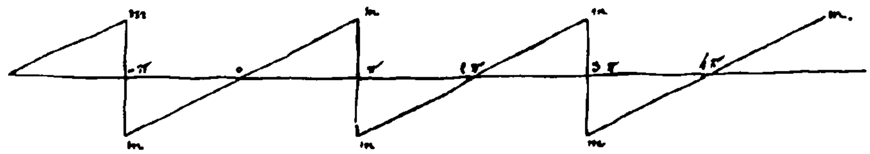

Fig. 1. Fourier's diagram for the function $(1/2)x$. (Reprinted with permission from M.I.T. Press.)

rier's new duties would demand most of his time and considerable administrative skills, he would, throughout his long tenure at Grenoble (1801–1814), long to resume his academic position at the Ecole Polytechnique [5, p. 104]. The title of "Baron" was conferred upon Fourier by Napoleon in 1809.

Fourier's distinguished place in science as an experimental and theoretical physicist and as a mathematician is largely due to a single scientific masterpiece: the manuscript he first submitted to the Institut de France in Paris on December 21, 1807, entitled "Théorie Analytique de la Chaleur" [6]. Although the original 234-page work specifically dealt with the propagation of heat in continuous bodies, it was to have far-reaching consequences for the advancement of pure and applied mathematics. This was due to the sheer volume of original contributions to the theory of functions and their representation as trigonometric (Fourier) series, the mathematical analysis of physical phenomena, and the novel treatment and application of linear differential equations to nontrivial boundary value problems with separable spatial and temporal variables.

The "secrétaire perpétuel" of the Institut de France for mathematical and physical sciences was the astronomer Jean-Baptiste Joseph Delambre (1749–1822), and members of the committee chosen by him to review the Fourier paper were Lagrange, Laplace, Lacroix, and Monge. The initial reaction of this committee to Fourier's manuscript was cool, and the reasons for the unfavorable reception have been the subject of considerable discussion by historians. It is generally agreed, nevertheless, that among the contributing factors were the following.

1) The legitimate concern expressed by Lagrange about the key question of convergence of Fourier's infinite sums of trigonometric functions, such as

$$\tfrac{1}{2}x = \sin(x) - \tfrac{1}{2}\sin(2x) + \tfrac{1}{3}\sin(3x) \cdots$$

which Fourier used to represent a sawtooth function (Fig. 1) [6, p. 170].

Similar functions had arisen in Lagrange's own studies of the vibrating string problem decades earlier, as well as in Euler's work, but no proof of convergence was known in 1807, and Fourier had not provided a rigorous proof valid for arbitrary functions. (Such a general proof has been sought by generations of mathematicians since, and remains unknown to this day.)

2) The scientific rivalries among the panel members, some of whom advocated their own methods and approaches to the heat problem.

3) The fact that Fourier had essentially produced the "Théorie" in total isolation from the Paris scientific circles. He had begun working on it around 1802, upon returning to Grenoble from a long (1798–1801) official assignment in Egypt. Furthermore, as Préfet of the Isère department, he was known to have had heavy political and administrative responsibilities, and little spare time to reflect on other matters, let alone a work of the magnitude of the "Théorie." Moreover, because Fourier was not an active "professional mathematician" in the sense that Laplace and Poisson were during these years, he is unlikely to have benefited from regular intellectual stimulation and scrutiny of peers. This handicap was not considered by at least one scholar who has criticized Fourier for lack of rigor in some of his proofs, and for ignorance about certain contemporary publications [7].

But Fourier was gifted with an uncommon sense of purpose and perseverance which were to prove critical assets in overcoming the opposition his work encountered. Because of Lagrange's and Laplace's reservations (subsequently echoed by Biot and Poisson, about the lack of rigor in Fourier's proof of convergence of infinite sinusoidal series,) the Institut delayed publication of the 1807 work. It was, in fact, not until 1822 that Fourier's "Théorie" was published, at his own instigation, in the form of a book, and not until 1823–1824 that it appeared in the *Annales de l'Académie des Sciences*, after he had replaced Delambre as "Secrétaire Perpétuel" for mathematics of that institution!

The 1811 Prize Paper

The subject for the Institut de France grand prize in mathematics for the year 1811 was announced on January 1, 1810, and pertained to the propagation of heat. The wording of the announcement is given by Darboux in his introduction to the 1888 edition of Fourier's collected works [1, p. VII]:

> . . . The academy having set for the competition, for 1811, the following question: Give the mathematical theory of the laws of the propagation of heat and compare the result of this theory to accurate experiments."

The deadline for submission was October 1, 1811. Because Fourier resided in Grenoble at the time, his manuscript was hand-carried by his friend Champollion-Figeac (of hieroglyphics deciphering fame) and delivered on September 28. Except for the addition of new sections on the cooling of infinite solids, and on terrestrial and radiant heat, the mathematics and physics was identical to that in the 1807 memoir. In maintaining the original treatment unaltered, Fourier was expressing his disapproval of the objections the examiners had raised about the earlier version. The members of the examining committee were, this time, Lagrange, Laplace, Malus, Haüy, and Legendre. There had been a single other submission, five days earlier, when the Institut received "from Monsieur Antoine Cardon-Michiels, property owner of Bergues in the Nord department, a 21-page document on heat as a symbol of man's return to the fire and on the marriage of ideas in heat for vegetables, plants, and minerals" [6, p. 451].

On January 6, 1812, the commission released its report, which stated that its members were still dissatisfied with Fourier's derivations of the fundamental equations for the propagation of heat, and with his use of trigonometric series in their solutions. It did, nevertheless, concede that the work was of high quality, and that Fourier had won the prestigious prize (along with a 3000 Francs gold medallion) [6, p. 452].

Fourier's subsequent protests to the permanent secretary (J. B. J. Delambre) went unheeded, as he was too far from Paris to defend his case effectively. Lagrange remained unbending with respect to questions of rigor (about proofs of convergence of Fourier's Series), and the Institut did not publish the prize-winning essay. The "standard procedure" for prize-winning papers authored by nonmembers of the Institut de France consisted of archiving until publication in the *Mémoires Présentés par Divers Savants*. The latter journal was not, however, a periodical, and there could be an indefinite delay before the appearance of its next issue. Lagrange's death in 1813 did not alter Fourier's conviction of ill will on the part of the Institut, and he set out to rewrite and extend his studies of heat in yet a third version. This part-time undertaking by the prefect came to a halt, however, with the political turmoil of 1815 when Fourier was (briefly) jailed, released, and reappointed by Napoleon during the Hundred Days. Fourier resigned shortly thereafter, and moved to Paris in pursuit of an academic career [5, p. 118].

When he arrived in Paris, Fourier found, to his dismay, a paper by Poisson in the *Journal de Physique* commenting on Fourier's 1811 prize-winning work! In his paper, Poisson expressed the opinions that Fourier's analysis was "not devoid of difficulties," and that it did not appear to have "all the rigor and generality required by the importance of the question." He furthermore claimed that he had himself found a "more general" method providing a solution in terms of a single arbitrary function, rather than Fourier's trigonometric series, and that he regarded this method as "the true solution to the problem." These comments were so similar to those expressed in the 1812 commission's report that a coincidence seemed unlikely. Fourier's response, which was devastating, marked the beginning of the wide acceptance of his work: he pointed out that the nonhomogeneous heat equation, if subjected to arbitrary initial conditions for all points of the interval, has a unique solution. Hence Poisson's solution, even if it looked different, ***had to be identical*** to Fourier's trigonometric series solution [5, p. 157].

The 1822 Book

The third version of Fourier's analysis appeared in the form of a book which is the famous 1822 *Théorie Analytique de la Chaleur*. The book quickly secured him an international reputation, and a flood of honors.

Format and Style

The book, written in French, contains 433 articles spanning 541 pages. This excludes the "Discours Préliminaire" and the (appended) table of contents. It also forms the first of two volumes of Fourier's collected works, compiled and published by Gaston Darboux in 1888–1890. The second volume ("Tome Second"), which is dated 1890, contains other papers authored by Fourier and published in various journals ("Mémoires publiés dans divers receuils"). The most important of these papers are complementary to the work on heat. The others pertain to a variety of subjects, including the principle of virtual velocities ("vitesses virtuelles"), the roots of transcendental equations, means and experimental errors in measurements, and others.

It is difficult not to do injustice to the book by attempting a cursory description of its contents since such a description necessarily involves an element of subjective appreciation. Certain unmistakable features of the text can, nevertheless, be pointed out. Fourier's vast vocabulary and command of the language are evident throughout, as indicated by the unusual clarity of the exposition of his thoughts. The technical subject matter is presented in simple, elegant literary style, and the grammar and punctuation are flawless. At no time is the reader left in doubt about Fourier's thoughts, especially when he presents the reasoning leading to a novel mathematical result. Ambiguities are circumvented by the systematic use of succinct explanations, concise statements, judiciously chosen counterexamples, and intuitively obvious analogies with familiar experiences of daily life.

Fourier's anticipation of the reader's reactions, and his skills as a teacher, are truly compelling. The technical arguments he uses are equally forceful, in that Fourier derives the most important results, such as solutions of boundary value problems for various geometries, in two, and sometimes three different ways. He then shows that his mathematical solutions yield known results for special values of the unknown variable, and finally states that the results are in agreement with experiments! Likewise, all his scientific conclusions were verified by experiments, proofs, or otherwise demonstrated results, and were frequently summarized. This style is maintained throughout the book, which makes for an overpowering classic work of science!

Fourier's approach to the mathematical theory of heat philosophically broke with tradition because he rejected the prevailing notions of imponderable fluid or caloric. He consciously and deliberately chose instead to avoid all questions about the physical nature of heat, thereby adopting a "separatist" philosophical attitude. This attitude, which was held, among others, by Lavoisier (1743–1794) before him, and by Fourier's later colleague Cuvier (1769–1832), the "secrétaire perpétuel" for biology at the Académie des Sciences, essentially consisted of describing facts, while avoiding speculation about their origins. Fourier stated [5, p. 225]

> "To us it seems more important not to give to the principle of communication of heat any hypothetical extension, and we think that this principle suffices to establish the mathematical theory. . . It is always preferable to restrict oneself to the enunciation of the general fact indicated by observation, which is none other than the preceding principle."

He thus formulated differential equations describing (not explaining) known phenomena (temperature distributions in solids) whose derivations were not based on any assumptions about underlying physical causes. In so doing, Fourier treated the problem in the same manner that Newton had mathematically treated gravity: its causes were unknown, but its effects could be observed and quantified.

Because Fourier repeatedly stressed the importance of experimental verification of mathematical results and the paramount importance of facts, the Théorie illustrates his strong "positivist" philosophical attitude. Fourier may thus have influenced the philosopher-mathematician Auguste Comte (1798–1857), the founder of "positivism," who held Fourier's scientific work in high regard [5, pp. 223–228].

By relating the "flux" of heat between two slices of an infinite bar (Fig. 2) to the temperature difference between the slices, and assuming heat was conserved during its flow, Fourier was able to derive the "continuity equation," which was known to hold for the flow of fluids. In one dimension, this equation reads

$$Q = -KA\Delta t \frac{dU}{dx} \tag{1}$$

where U is the unknown temperature function, Q is the quantity of heat flowing across a slice of the bar, K is the thermal conductivity ("conductibilité intérieure"), which depends on the material, A is the cross-sectional area of the bar, Δt is the length of time during which the flow takes place, and x is the spatial coordinate. The minus sign in (1) indicates that Q is positive when (dU/dx) is negative or when the temperature is decreasing in the direction to the right. Equation (1), in turn, led him to the partial differential equation

$$K\frac{d^2U}{dx^2} = \frac{dU}{dt} \tag{2}$$

which is none other than the "diffusion equation." He subsequently derived it in three dimensions and in cylindrical and spherical coordinates, as he formulated the corresponding equations for the annulus, sphere, cylinder, prism, and cube (armille, sphère solide, cylindre solide,

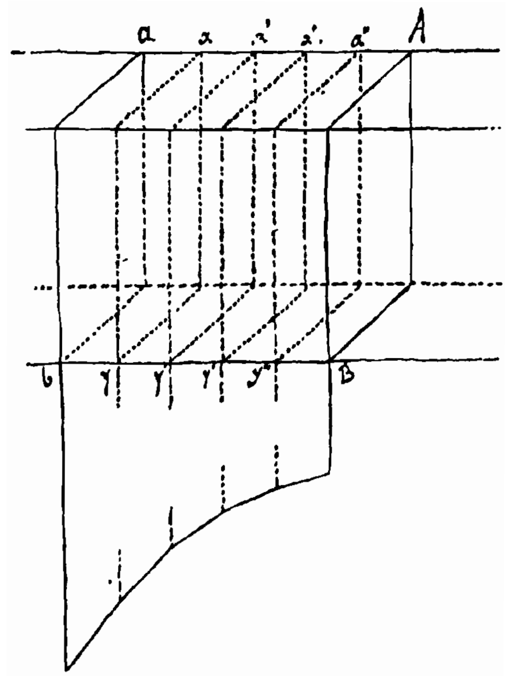

Fig. 2. The sliced infinite bar used by Fourier to derive the "continuity equation." Temperature decreases with increasing x, in the direction from a to A (i.e., towards the right). (Reprinted with permission from M.I.T. press.)

prisme rectangulaire, cube solide) [1, ch. IV, V, VI, VII, VIII, respectively].

Fourier explicitly distinguished between physical behavior at a point interior to a body and that at the surface of the body. This distinction gave rise to separate equations for each. In the process of solving his differential equations, Fourier developed a wide range of novel mathematical techniques. Among these are the method of separation of variables, which allowed him to solve (separately) for the steady-state and time-dependent solutions. The linearity of the differential equations reinforced Fourier's intuition about the physical correctness of superposition of simple solutions. This led him to the formulation of boundary value problems, wherein spatial and temporal initial conditions can unambiguously be "fitted" to determine the unknown constants. In the short article 160 of the book, Fourier stated the fundamental requirement of dimensional correctness and consistency of the two sides of any mathematical equation describing physical phenomena. He was the first to recognize and systematically enforce this principle. Perhaps the most revolutionary contribution to pure mathematics, however, was Fourier's concept of a mathematical function, and his realization that even the most ill-behaved functions (including those containing discontinuities) could be represented by trigonometric functions.

Later Life

Fourier was elected to membership in the Académie des Sciences in 1816, became Secrétaire Perpétuel of the Académie in 1822, and became a foreign member of the Royal

Society of London in 1823. In 1827 he was elected to the Académie Française and the Académie de Médecine, and was made President of the Conseil de Perfectionnement of the Ecole Polytechnique. Fourier enjoyed the respect and loyalty of many mathematicians and physicists in his scientific circle. He is known to have inspired, encouraged, and helped, among others, Sturm, Navier, Oersted, Dirichlet, Sophie Germain, Liouville, and Abel [8].

During the last five years of his life, Fourier was weakened by various illnesses, among which were rheumatism, and possibly malaria he had contracted in Egypt. He also suffered from insomnia. While he continued to work, the quality and legibility of his work declined during this period, on account of his confinement to a box-like chair out of which only his limbs and head protruded. An ailment affecting his thyroid gland caused him the loss of sensitivity to cold, the swelling of his lips and tongue, and impaired his breathing, speech, and memory. On May 4, 1830, he fell on a set of stairs, but continued to work until his death in the afternoon of the 16th. He was 63 years old. He was buried two days later at the side of his long-time mentor and friend Gaspard Monge, in the "Père Lachaise" Cemetery in Paris. The University of Grenoble has, since 1950, published the *Annales de L'Institut Fourier*, and on the occasion of the bicentenary of his birth, in 1968, the secondary school in Auxerre was renamed the "Lycée Fourier."

The Influence of the "Théorie Analytique de la Chaleur"

The most famous mathematical result presented by Fourier in the "Théorie" is the theorem about the representation of an arbitrary function $f(x)$ over a finite interval by an infinite sum of sinusoidal functions. He was first to assert it, and rigorously proved that the expansion is valid for certain simple functions he needed in the problems of the conduction of heat. The theorem has since been refined by many workers, but is essentially valid as originally stated by Fourier.

Although Fourier did not give a rigorous proof for the general case of arbitrary $f(x)$, the theorem was assumed to hold true for any function from the time his work became known. Poisson examined questions of convergence at points of ordinary discontinuities in a series of papers he published between 1820 and 1835. Cauchy applied his method of residues to similar questions in work published from 1826 onwards. In two papers published in 1829 and 1837, respectively, Dirichlet obtained generalized, limiting values for the sum of the Fourier Series for $f(x)$ in the expansion interval, and showed that the sum remains finite if $f(x)$ has only a finite number of ordinary discontinuities. In a further paper (1837), Dirichlet also showed that the restriction that $f(x)$ must remain finite is not necessary, provided that $\int f(x)\,dx$ over the expansion interval converges absolutely. Riemann (1854) sought to find a necessary and sufficient condition the arbitrary function must satisfy in order that the corresponding Fourier Series converge to $f(x)$ for all points x in the interval. He did not succeed in finding such a condition, however, and Carslaw [9], as late as 1930, stated about Riemann's question " . . . It is quite probable that it is not solvable."

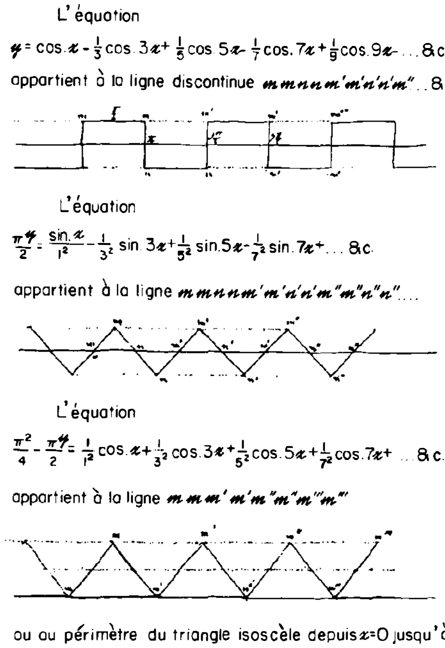

Fig. 3. Examples of periodic functions and their corresponding infinite trigonometric (Fourier) series given by Fourier in the 1807 manuscript (I. Grattan-Guinness, *Joseph Fourier*. Cambridge, MA: M.I.T. Press, 1972, p. 184; reprinted with permission).

The question of uniqueness of a series representing a given function over a given interval was examined by Heine and Cantor from 1870 onwards. This led to Cantor's Theory of Sets of Points. In 1875, du Bois-Reymond proved that if a trigonometrical series converges in $[-\pi, \pi]$ to $f(x)$ where $f(x)$ is integrable, the series must be the Fourier Series for $f(x)$. The concept of uniform convergence was introduced by Stokes (1847) and Seidel (1848) when the nature of the convergence of Fourier's Series became of interest. Lipschitz (1864) and Dini (1880) provided refinements to Dirichlet's sufficient condition, and Jordan (1881) simplified the treatment of Fourier's Series by introducing the concept of functions of bounded variation. In 1893, Parseval obtained the well-known theorem named after him, and in 1902, Lebesgue used the concept of the measure of a set of points to introduce his Lebesgue Integral. This integral has since become a cornerstone of the modern theory of functions of a real variable, and led Riesz and Fischer, in 1907, to the important Converse of Parseval's Theorem.

A debate about the convergence of Fourier series was the object of various letters published by the journal *Nature* between May 1898 and October 1899, and involved, among others, J. W. Gibbs, Henri Poincaré, A. A. Michelson, and A. E. H. Love [10].

In 1970, Hunt stated and proved the Carleson–Hunt theorem: "If $|f|^p$ is integrable on $[-\pi, \pi]$, where $1 < p < \infty$, then the Fourier series of f converges almost everywhere to f," which had been proved by Carleson in 1966 for $p = 2$ [11]. The wording "almost everywhere" is a common terminology in the theory of sets which refers to sets of "measure zero" as the only possible exceptions to "everywhere." Because the latter concept is technically involved, the reader is referred to the literature for further details. In 1930, moreover, Carslaw stated, "the convergence problem for Fourier's Series is still unsolved. There is no property of the arbitrary function $f(x)$ integrable in $[-\pi, \pi]$ which is known to be both necessary and sufficient for the convergence of Fourier's Series. There are simple sufficient conditions, which are known not to be necessary, and the necessary conditions obtained are known not to be sufficient" [9, p. 18]. To this author's knowledge, the statement remains true at the time of this writing (1988).

Ohm's Law, which states that the current I flowing through a conductor is proportional to the voltage V applied across it, $I = V/R$, was determined experimentally and theoretically by Georg Simon Ohm in 1826. To obtain this law, Ohm used Fourier's results (which he acknowledged) to draw an analogy between charge in conductors and heat in solids. The analogy used by Ohm is only partially correct, however, because while a thermal conductor stores heat (when it conducts heat), an electrical conductor does *not* store charge when conducting charge [12].

William Thomson (Lord Kelvin) drew an analogy between thermal conduction and electrostatic conduction in his 1842 work on the theory of electrostatics. The mathematical techniques he used in this theory were analogous to those Fourier developed for his mathematical theory of heat. Kelvin successfully applied the continuity equation to the flow of current, thereby underscoring the power of the method [13].

The periodicity of the sine and cosine functions in Fourier's Series has since found applications in the analysis of a wide range of physical phenomena, such as the tides, earthquakes, phases of the moon, the occurrence of sunspots, and many others. These complicated, but periodic phenomena can often be approximated by superposition of simple periodic functions whose resultant corresponds to the original event. The study of the complicated phenomena is thus simplified by analyzing the individual periodic functions. The analysis of a musical sound into its fundamental and higher order harmonics is analogous to the mathematical decomposition of the sound signal into its Fourier Series component functions. In general, the superposition of the "fundamental" and only a few harmonics is sufficient to produce sound of a quality comparable to the "ideal" sound that would result if an infinity of harmonics were present.

In engineering, electrical in particular, a number of functions are important which are not exponential or sinusoidal. These include square or rectangular waves, triangular and square pulses, and impulses. The responses of circuits to such signals are obtained by using the Fourier Series representations of these functions. The sinusoidal steady-state analysis method can then be applied. The concept of the Fourier Transform applies to functions which are *not periodic*, but defined over an infinite interval. Such functions cannot be represented by a Fourier Series, but are treated as if they were periodic, with an infinite period. A *Fourier Transform pair* of functions results from these considerations.

The modern notation $\int_a^b f(x)\,dx$ for the definite integral of a function $f(x)$ between the limits a and b is owed to Fourier. It appeared for the first time in a brief summary of the "Théorie" published in 1816 [6, p. 241] and again in article 224 of the 1822 book in the form $\int_0^\pi \varnothing(x)\,dx$. Because of the usefulness of this notation, it was quickly adopted and used in the curriculum at the Ecole Polytechnique where Cauchy introduced it in his calculus courses starting in 1823 [2, p. 192].

Acknowledgment

The author is indebted to the following people who have contributed valuable criticism about early versions of this paper: Prof. J. Richards of the Department of History, Brown University, Dr. H. R. Sadeghi of the Naval Research Laboratory, Washington, DC, and S. S. Shoucair of Olivetti, Paris, France. The suggestions of the reviewers, which have undoubtedly improved the quality of the paper, are very gratefully acknowledged.

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Additional Bibliography

A considerable literature exists which pertains more or less directly to the influence of Fourier's work on science in general, and on mathematics and engineering in particular. The interested reader may wish to consult the following small sampling thereof.

[14], [15]: For succinct, popularized biographical sketches of Fourier and his work.

[16], [17], [18], [19], [20]: For historical, scholarly, perspectives on 19th century science.

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F. S. Shoucair (S'79-M'83) received the B.S., M.S., and Ph.D. degrees in electrical engineering from Columbia University, New York, NY, in 1979, 1981, and 1983, respectively.

He was an Engineer at the Schlumberger-Doll Research Center, Ridgefield, CT, during the summers of 1979, 1981, and 1982. From 1983 to 1985 he was a member of the VLSI Group, Schlumberger Well Services, Fairchild Research Center, Palo Alto, CA. From 1985 to 1987 he was with the Department of Electrical Engineering, Yale University, New Haven, CT, and concurrently, with the Department of Electrical Engineering, Columbia University, New York, as Adjunct Assistant Professor, during the spring semester, 1987. Since July 1987 he has been with the Division of Engineering, Brown University, Providence, RI. His current research interests include VLSI analog integrated circuits, solid-state device modeling, and the history of science and engineering.