XI. *Experimental Researches in Electricity.—Twenty-ninth Series.* By Michael Faraday, *Esq., D.C.L., F.R.S., Fullerian Prof. Chem. Royal Institution, Foreign Associate of the Acad. Sciences, Paris, Ord. Boruss. Pour le Mérite, Eq., Memb. Royal and Imp. Acadd. of Sciences, Petersburgh, Florence, Copenhagen, Berlin, Göttingen, Modena, Stockholm, Munich, Bruxelles, Vienna, Bologna, &c. &c.*

Received December 31, 1851,—Read March 25 and April 1, 1852.

§ 35. *On the employment of the Induced Magneto-electric Current as a test and measure of Magnetic Forces.*

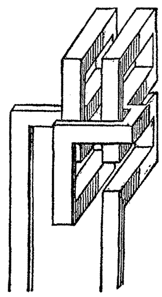
3177. The proposition which I have made to use the induced magneto-electric current as an experimental indication of the presence, direction and amount of magnetic forces (3074.), makes it requisite that I should also clearly demonstrate the principles and develope the practice necessary for such a purpose; and especially that I should prove that the amount of current induced is precisely proportionate to the amount of lines of magnetic force intersected by the moving wire, in which the electric current is generated and appears (3082, 3109.). The proof already given is, I think, sufficient for those who may repeat the experiments; but in order to accumulate evidence, as is indeed but proper in the first announcement of such a proposition, I proceeded to experiment with the magnetic power of the earth, which presents us with a field of action, not rapidly varying in force with the distance, as in the case of small magnets, but one which for a given place may be considered as uniform in power and direction; for if a room be cleared of all common magnets, then the terrestrial lines of magnetic force which pass through it, have one common direction, being that of the dip, as indicated by a free needle or other means, and are in every part in equal proportion or quantity, *i. e.* have equal power. Now the force being the same everywhere, the proportion of it to the current evolved in the moving wire is then perhaps more simply and directly determined, than in the case where, a small magnet being employed, the force rapidly changes in amount with the distance.

¶ i. *Galvanometer.*

3178. For such experimental results as I now propose to give, I must refer to the galvanometer employed and the precautions requisite for its proper use. The instrument has been already described in principle (3123.), and a figure of the conductor which surrounds the needles, given. This conductor may be considered as a square copper bar, 0·2 of an inch in thickness, which passes twice round the plane of vibration of each of the needles forming the astatic combination, and then is continued outwards and terminates in two descending portions, which are intended to dip into cups of mercury. As both the needles are within the convolutions of this bar, an indicating bristle or fine wire of copper is fixed parallel to, and above them upon the same axis, and this, in travelling over the usual graduated circle, shows the place and the extent of vibration or swing of the needles below. The suspension is by cocoon silk, and in other respects the instrument is like a good ordinary galvanometer.

3179. It is highly important that the bar of copper about the needles should be perfectly clean. The vertical zero plane should, according to the construction, be midway between the two vertical coils of the bar, fig. 1; instead of which the needle at first pointed to the one side or the other, being evidently attracted by the upright portions of the bar. I at first feared that the copper was magnetic, but on cleaning the surface carefully with fine sand-paper, I was able to remove this effect, due no doubt to iron communicated by handling or the use of tools, and the needle then stood truly in a plane equidistant from the two coils, when that plane corresponded with the magnetic meridian.

Fig. 1.



3180. The connexions for this galvanometer (3123, 3133.) were all of copper rod or wire 0·2 of an inch in diameter; but even with wires of this thickness the extent of the conductors should not be made more than is necessary; for the increase from 6 to 8, 10 or 12 feet in length, makes a considerable difference at the galvanometer, when electric currents, low in intensity, are to be measured. It is most beautiful to observe in such cases the application of Ohm's law of currents to the effects produced. When the connexions were extended to a distance, straight lengths of wire with dropping ends were provided, and these by dipping into cups of mercury completed the connexion and circuit. The cups consisted of cavities turned in flat pieces of wood. The ends of the connecting rods and of the galvanometer bar were first tinned, and then amalgamated; after which their contact with the mercury was both ready and certain. Even where connexion had to be made by contact of the solid substances, I found it very convenient and certain to tin and amalgamate the ends of the conductors, wiping off the excess of mercury. The surfaces thus prepared are always ready for a good and perfect contact.

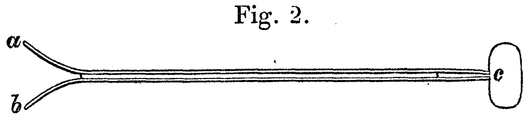
3181. When the needle has taken up its position under the earth's influence, and the copper coil is adjusted to it, the needle ought to stand at true zero, and appears so to do. When that is really the case, equal forces applied in succession on opposite sides of the needle (by two contrary currents through the coil for instance) ought to deflect the needle *equally* on both sides, and they do so. But sometimes, when the needle appears to stand at zero, it may not be truly in the magnetic meridian; for a little torsion in the suspension thread, even though it be only 10° or 15° (for an indifferent needle), and quite insensible to the eye looking at the magnetic needle, does deflect it, and then the force which opposes the swing of the needle, and which stops and returns the needle towards zero (being due both to the torsion and the earth's force), is not equal on the two sides, and the consequence is, that the extent of swing in the two directions is not equal for equal powers, but is greater on one side than the other.

3182. I have not yet seen a galvanometer which has an adjustment for the torsion of the suspending filament. Also, there may be other causes, as the presence about a room, in its walls and other places, of unknown masses of iron, which may render the forces on opposite sides of the instrument zero unequal in a slight degree; for these reasons it is better to make *double observations*. All the phenomena we have to deal with, present effects in two contrary directions. If a loop pass over the pole of a magnet (3133.), it produces a swing in one direction; if it be taken away, the swing is in the other direction; if the rectangles and rings to be described (3192.) be rotated one way, they produce one current; if the contrary way, the other and contrary current is produced. I have therefore always in measuring the power of a pole or the effect of a revolving intersecting wire made many observations in both directions, either alternately or irregularly; have then ascertained the average of those on the one side, and also on the other (which have differed in different cases from $\frac{1}{50}$th to $\frac{1}{300}$th part), and have then taken the mean of these averages as the expression of the power of the induced electric current, or of the magnetic forces inducing it.

3183. Care must be taken as to the position of the instrument and apparatus connected with it, in relation to a fire or sources of different temperatures, that parts which can generate thermo-currents may not become warmed or cooled in different degrees. The instrument is exceedingly sensible to thermo-electric currents; the accidental falling of a sun-beam upon one of two connecting mercury cups for a few moments disturbed the indications and rendered them useless for some time.

3184. In order to ascertain practically, *i. e.* experimentally, the comparative value of degrees in different parts of the scale or graduation of this instrument and so to render it a measurer, the following trials were made. A loop like that before described (3133.), fig. 2, was connected with the galvanometer by communications which removed the loop 9 feet from the instrument, and it was then fixed. A compound bar-magnet consisting of two plates, each 12 inches long, 1 inch broad, and 0·5 in thickness, was selected of such strength as to lift a bunch of clean iron filings, averaging 45 grains at either extremity. Blocks were arranged at the loop, so that this magnet, held in a vertical position, could have one end passed downwards through the loop until the latter coincided with the equator of the magnet (3191.); after which it could be quickly removed and the same operation be repeated at pleasure. When the magnet was thus moved, the loop being unconnected (at one of the mercury cups) with the galvanometer, there was no sensible change of place in the needles; the direct influence of the magnet at this distance of 9 feet being too small for such an effect.

Fig. 2.



3185. It must be well understood, that, in all the observations made with this instrument, the *swing* is observed and counted as the effect produced, unless otherwise expressed. A constant current in an instrument will give a constant and continued deflection, but such is not the case here. The currents observed are for short periods, and they give, as it were, a blow or push to the needle, the effect of which, in swinging the needle, continues to increase the extent of the deflection long after the current is over. Nevertheless the extent of the swing is dependent on the electricity which passed in that brief current; and, as the experiments seem to indicate, is simply proportional to it, whether the electricity pass in a longer or a shorter time (3104.), and notwithstanding the comparative variability of the current in strength during the time of its continuance.

3186. The compound bar being introduced once into the loop and left there, the swing at the galvanometer was observed and found to be 16°; the galvanometer needle was then brought to zero, and the bar removed, which gave a reverse current and swing, and this also was 16°. Many alternations, as before described, gave 16° as the mean result, *i. e.* the result of one intersection of the lines of force of this magnet (3102.). In order to comprehend the manner in which the effect of two or more intersections of these lines of force were added together, it should be remembered that a swing of the needle from right to left occupied some time (13 seconds); so that one is able to introduce the magnet into the loop, then break the electric circuit by raising one end of the communicating wire out of the mercury, remove the magnet, which by this motion does nothing, restore the mercury contact, and reintroduce the magnet into the loop, before a tenth part of the time has passed, during which the needles, urged by the first impulse, would swing. In this way two impulses could be added together, and their joint effect on the needle observed; and, indeed, by practice, three and even four impulses could be given within the needful time, *i. e.* within one-half or two-thirds of the time of the full swing; but of course the latter impulses would have less power upon the needles, because these would be more or less oblique to the current in the copper coil at the time when the impulses were given. There can be no doubt, that, as regarded the currents induced in the loop by the magnet, they would be equal on every introduction of the same magnet.

3187. Proceeding in this way I obtained results for one, two, three, and even four introductions with the same magnet.

One introduction	15°
Two introductions	31·25
Three introductions	46·87
Four introductions	58·50

Here the approximation to 1, 2, 3, 4 cannot escape observation*; and I may remark,

* See note to (3189.) $\sin\frac{15}{2} = \sin 7^\circ\,30' = \cdot130526$ $\quad\cdot130526$

$\sin\frac{31\cdot25}{2} = \sin 15\cdot625 = \sin 15\ 37\cdot5 = \cdot269200 \qquad \frac{269200}{2} = \cdot134600$

$\sin\frac{46\cdot87}{2} = \sin 23\cdot435 = \sin 23\ 26\cdot1 = \cdot3976818 \qquad \frac{3976818}{3} = \cdot1328606$

$\sin\frac{58\cdot50}{2} = \sin 29\cdot25 = \sin 27\ 15 = \cdot4886212 \qquad \frac{4886212}{4} = \cdot1221553$

that, whilst observing the place attained at the end of a swing which is retained only for an instant, some degree of error must creep in; and that that error must be greatest, in the first number, where it falls altogether upon the unit of comparison than in the other observations, where only one-half or one-third of it is added to a half or a third of the whole result. Thus, if we halve the arc for two introductions of the pole, it gives 15°·625; if we take the third of that for three introductions, it gives 15°·61;—numbers which are almost identical, so that if the first number was increased by only 0°·6, the proportion would be as 1, 2 and 3. The reason why the fourth, which is 14°·625, is less, may perhaps be referred to the cause already assigned, namely, the declination distance of the needle from the coil when that impulse was given (3186.).

3188. In order to avoid in some degree this case, and to compare the degrees at the beginning of the scale, which are most important for the comparison of future experiments with one another, I took one of the bars of the compound magnet employed above (3184.). The results were as follows:—

One introduction	8°
Two introductions.	15·75
Three introductions	23·87
Four introductions.	31·66

which numbers are very closely as 1, 2, 3 and 4. If we divide as before, we have 8°, 7°·87, 7°·95, 7°·91; so that if only 0·09 be subtracted from the first observation, or 8°, it leaves that simple result*.

3189. Hence it appears, that in this mode of applying and measuring the magnetic powers, the number of degrees of swing deflection are for small arcs nearly proportional to the magnetic force which has been brought into action on the moving wire†.

* $\sin\frac{8}{2} = \sin 4^\circ = \cdot0697565 \qquad \cdot0697565$

$\sin\frac{15\cdot75}{2} = \sin 7\cdot875 = \sin 7^\circ\,52'\cdot5 = \cdot1370123 \qquad \frac{1370123}{2} = \cdot0685061$

$\sin\frac{23\cdot87}{2} = \sin 11\cdot935 = \sin 11\ 56\cdot1 = \cdot2068019 \qquad \frac{2068019}{3} = \cdot0689340$

$\sin\frac{31\cdot66}{2} = \sin 15\cdot83 = \sin 15\ 49\cdot8 = \cdot2727840 \qquad \frac{2727840}{4} = \cdot0681960$

† Mr. Christie has recalled my attention to a paper in the Philosophical Transactions, 1833, p. 95, in which he has investigated, at p. 111, &c., the effect of what may be called magneto-electric impulses in deflecting the magnetic needle. He found that the velocity of the projection of the needle, which is a measure of the force acting upon it at the instant of its moving, will be proportional to the sine of half the arc of swing. My statement, therefore, would as a general expression be erroneous; but for small arcs the results as given by it are not far from the truth. The error does not interfere with the general reasoning and conclusions of the paper; and as the numbers are the results of experiment, which, though made with a first and therefore rough apparatus, were still made with some care, and are expressed simply as deflections, I prefer their appearance as they are rather than in an altered state. Mr. Christie has been so kind as to give me the true expression of force for many of the cases, and I have inserted the results as foot-notes where the cases occur.—Jan. 26, 1852.

3190. I have found the needles very constant in their strength for days and weeks together. By care, the constancy of their state for a day is easily secured, and that is all that is required in comparative experiments. Those which I have in use weigh with their axis and indicating wire 9 grains; and when out of the copper coil vibrate to and fro once in 26 seconds.

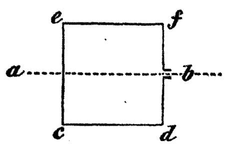
3191. With this instrument thus examined, I repeated most of the experiments with loops formerly described (3133. &c.), with the same results as before. It was also ascertained that the equator of a regular bar-magnet was the place at which the loop should be arrested, to produce the maximum action; and that if it came short of, or passed beyond that place, the final result was less. Employing a magnet 12 inches long, when the loop passed

2·3 inches over the pole the deflection was	5°·91
4·1 inches over the pole the deflection was	7·50
5·1 inches over the pole the deflection was	7·74
6·1 inches over the pole the deflection was	8·16
8·0 inches over the pole the deflection was	7·75
9·0 inches over the pole the deflection was	6·50

¶ ii. *Revolving Rectangles and Rings**.

3192. The form of moving wire which I have adopted for experiments with the magnetic forces of the earth (3177.), is either that of a rectangle or a ring. If a wire rectangle (fig. 3) be placed in a plane, perpendicular to the dip and then turned once round the axis *a b*, the two parts *c d* and *e f* will twice intersect the lines of magnetic force within the area *c e d f*. In the first 180° of revolution the contrary direction in which the two parts *c d* and *e f* intersect those lines, will cause them to conspire in producing one current, tending to run round the rectangle (161) in a given direction; in the following 180° of revolution they will combine in their effect to produce a contrary current; so that if the first current is from *d* by *c e* and *f* to *d* again, the second will be from *d* by *f e* and *c* to *d*. If the rectangle, instead of being closed, be open at *b*, and the ends there produced be connected with a commutator, which changes sides when the rectangle comes into the plane perpendicular to the dip, *i. e.* at every half revolution, then these successive currents can be gathered up and sent on to the galvanometer to be measured. The parts *c e* and *d f* of the

Fig. 3.

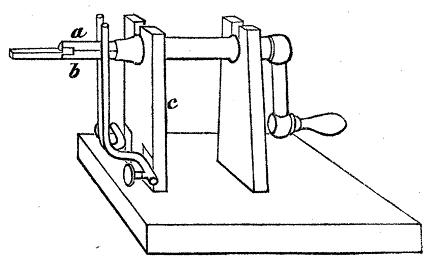


* A friend has pointed out to me that in July 1832, Nobili made experiments with rotating rings or spirals subject to the earth's magnetic influence; they were subsequent to and consequent upon my own experiments upon swinging wires (171, 148.) and revolving globes (160.) of January 1832; but he extended the considerations to the *thickness* of the wire; the *diameter* of the spirals and the *number* of the spirals dependent upon the *length* of the wire. The results (tabulated) will be found in vol. i. page 244, &c. of the Florence edition of his Mémoires.—March 1, 1852.

rectangle may be looked upon simply as conductors; for as they do not in their motion intersect any of the lines of force, so they do not tend to produce any current.

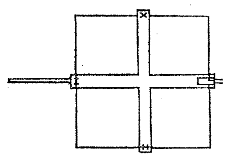
3193. The apparatus which carries these rectangles, and is also the commutator for changing the induced currents, consists of two uprights, fixed on a wooden stand, and carrying above a wooden horizontal axle, one end of which is furnished with a handle, whilst the other projects, and is shaped as in fig. 4. It may there be seen, that two semi-cylindrical plates of copper *a b* are fixed on the axle, forming a cylinder round it, except that they do not touch each other at their edges, which therefore leave two lines of separation on opposite sides of the axle. Two strong copper rods, 0·2 of an inch in diameter, are fixed to the lower part of the upright *c*, terminating there in sockets with screws for the purpose of receiving the ends of the rods proceeding from the galvanometer cups (3180.): in the other direction the rods rise up parallel to each other, and being perfectly straight, press strongly against the curved plates of the commutator on opposite sides: the consequence is, that, whenever in the rotation of the axle, the lines of separation between the commutator plates arrive at and pass the horizontal plane, their contact with these bearing rods is changed, and consequently the direction of the current proceeding from these plates to the rods, and so on to the galvanometer, is changed also. The other or outer ends of the commutator plates are tinned, for the purpose of being connected by soldering to the ends of any rectangle or ring which is to be subjected to experiment.

Fig. 4.



3194. The rectangle itself is tied on to a slight wooden cross (fig. 5), which has a socket on one arm that slides on to and over the part of the wooden axle projecting beyond the commutator plates, so that it shall revolve with the axle. A small copper rod forms a continuation of that part of the frame which occupies the place of axle, and the end of this rod enters into a hole in a separate upright, serving to support and steady the rectangle and its frame. The frames are of two or three sizes, so as to receive rectangles of 12 inches in the side, or even larger, up to 36 inches square. The rectangle is adjusted in its place, so that it shall be in the horizontal plane when the division between the commutator plates is in the same plane, and then its extremities are soldered to the two commutator plates, one to each. It is now evident, that when dealing with the lines of force of the earth, or any other lines, the axle has only to be turned until the upright copper rods touch on each side at the separation of the commutator plates, and then the instrument adjusted in position, so that the plane of the ring or rectangle is perpendicular to the direction of the lines of force which are to be examined, and then any revolution of the commutator and intersecting wire will produce the maximum current which such wire and such magnetic force can produce. The lines of terrestrial magnetic force are inclined at an angle

Fig. 5.



of 69° to the horizontal plane. As, however, only comparative results were required, the instrument was, in all the ensuing experiments, placed in the horizontal plane, with the axis of rotation perpendicular to the plane of the magnetic meridian; under which circumstances no cause of error or variation was introduced into the results. As no extra magnet was employed, the commutator was placed within 3 feet of the galvanometer, so that two pieces of copper wire 3 feet long and 0·2 of an inch in thickness, sufficed to complete the communication. One end of each of these dipped into the galvanometer mercury cups, the other ends were tinned, amalgamated, introduced into the sockets of the commutator rods (3193.), and secured by the pinching screw (fig. 4).

3195. When a given length of wire is to be disposed of in the form best suited to produce the maximum effect, then the circumstances to be considered are contrary for the case of a loop to be employed with a small magnet (39. 3184.), and a rectangle or other formed loop to be employed with the lines of terrestrial force. In the case of the small magnet, *all* the lines of force belonging to it are inclosed by the loop; and if the wire is so long that it can be formed into a loop of two or more convolutions, and yet pass over the pole, then twice or many times the electricity will be evolved that a single loop can produce (36.). In the case of the earth's force, the contrary result is true; for as in circles, squares, similar rectangles, &c. the areas inclosed are as the squares of the periphery, and the lines of force intersected are as the areas, it is much better to arrange a given wire in one simple circuit than in two or more convolutions. Twelve feet of wire in one square intersects in one revolution the lines of force passing through an area of nine square feet, whilst if arranged in a triple circuit, about a square of one foot area, it will only intersect the lines due to that area; and it is thrice as advantageous to intersect the lines within nine square feet once, as it is to intersect those of one square foot three times.

3196. A square was prepared, containing 4 feet in length of copper wire 0·05 of an inch in diameter; it inclosed one square foot of area, and was mounted on the commutator and connected in the manner already described (3194.). Six revolutions of it produced a swing deflection of 14° or 15°, and twelve quick revolutions were possible within the required time (3104.). The results of *quick* and *slow* revolutions were first compared. Six slow revolutions gave as the average of several experiments 15°·5 swing. Six moderate revolutions gave also an average of 15°·5; six quick revolutions gave an average of 15°·66. At another time twelve moderate revolutions gave an average of 28°·75, and twelve quick revolutions gave an average of 31°·33 swing. As before explained (3186.), the probable reason why the quick revolutions gave a larger result than the moderate or slow revolutions is, that in slow time the later revolutions are performed at a period when the needle is so far from parallel with the copper coil of the galvanometer, that the impulses due to them are less effectually exerted. Hence a small or moderate number of revolutions and a quick motion is best. The difference in the extreme case is less than might have been expected, and shows that there is no practical objection in this respect to the method proposed of experimenting with the lines of magnetic force.

3197. In order to obtain for the present an expression of the power of the earth's magnetic force by this rectangle, observations were made on both sides of zero, as already recommended (3182.). Nine moderately quick direct revolutions (*i. e.* as the hands of the clock) gave as the average of many experiments 23°·87, and nine reverse revolutions gave 23°·37; the mean of these is 23°·62 for the nine revolutions of the rectangle, and therefore 2°·624 per revolution. Now the six quick revolutions (3196.) gave 15°·66, which is 2°·61 per revolution, and the twelve quick revolutions gave 31°·33, which is also 2°·61 per revolution; and these results of 2°·624, 2°·61, and 2°·61, are very much in accordance, and give great confidence in this method of investigating magnetic forces*.

3198. A rectangle was prepared of the same length (4 feet) of the same wire, but the sides were respectively 8 and 16 inches (fig. 6), so that when revolving the intersecting parts should be only 8 inches in length instead of 12. The area of the rectangle was necessarily 128 square inches instead of 144. This rectangle showed the same difference of quick and slow rotations as before (3196.).

Fig. 6.

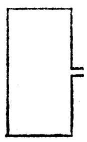


Fig. 7.

When nine direct revolutions were made, the result was 20°·87 swing. Nine reverse revolutions gave an average of 20°·25 swing; the mean is 20°·56, or 2°·284 per revolution. A third rectangle was prepared of the same length and kind of wire, the sides of which were respectively 8 and 16 inches long (fig. 7), but now so revolved that the intersecting parts were 16 inches, or twice as long as before; the area of the rectangle remained the same, *i. e.* 128 inches. The like effect of slow and quick revolutions appeared as in the former cases (3196. 3198.). Nine direct revolutions gave as the average effect 20°·75; and nine reverse revolutions produced 21°·375; the mean is 21°·06, or 2°·34 per revolution.

3199. Now 2°·34 is so near to 2°·284, that they may in the present state of the investigation be considered the same. The little difference that is evident, was, I suspect, occasioned by centrifugal power throwing out the middle of the longer intersecting parts during the revolution. The coincidence of the numbers shows, that the variation in the arrangement of the rectangle and in the length of the parts of the wires intersecting the lines of magnetic force, have had no influence in altering the result, which, being dependent alone on the number of lines of force intersected, is the same for both; for the area of the rectangles is the same. This is still further shown by comparing the results with those obtained with the square. The area in

$$* \sin\frac{15\cdot66}{2}=\sin 7^{\circ}\cdot83=\sin 7^{\circ}\ 49'\cdot8=\cdot1362343 \qquad \frac{1362343}{6}=\cdot0227057$$

$$\sin\frac{23\cdot62}{2}=\sin 11\cdot81=\sin 11\ 48\cdot6=\cdot2047069 \qquad \frac{2047069}{9}=\cdot0227474$$

$$\sin\frac{31\cdot33}{2}=\sin 15\cdot665=\sin 15\ 40=\cdot2700403 \qquad \frac{2700403}{12}=\cdot0225034$$

that case was 144 square inches, and the effect per revolution 2°·61. With the long rectangles the area is 128 square inches, and the mean of the two results is 2°·312 per revolution. Now 144 square inches is to 128 square inches as 2°·61 is to 2°·32; a result so near to 2°·312 that it may be here considered as the same; proving that the electric current induced is directly as the lines of magnetic force intersected by the moving wire*.

3200. It may also be perceived that no difference is produced when the lines of force are chiefly disposed in the direction of the motion of the wire, or else, chiefly in the direction of the length of the wire; *i. e.* no alterations are occasioned by variations in the *velocity* of the motion, or of the length of the wire, provided the amount of lines of magnetic force intersected remains the same.

3201. Having a square on the frame 12 inches in the side but consisting of copper wire 0·1 of an inch in thickness, I obtained the average result of many observations for one, two, three, four and five revolutions of the wire.

One revolution gave	7°	equal to	7°	per revolution.
Two revolutions gave	13·875	equal to	6·937	per revolution.
Three revolutions gave	21·075	equal to	7·025	per revolution.
Four revolutions gave	28·637	equal to	7·159	per revolution.
Five revolutions gave	37·637	equal to	7·527	per revolution.

These results are exceedingly close upon each other, especially for the first 30°, and confirm several of the conclusions before drawn (3189. 3199.) as to the indications of the instrument, the amount of the curves, &c.†

* Oblong rectangles of 128 square inches area give a mean of 20°·81 (3198.). The rectangle of 144 square inches gave a mean of 23°·62 (3197.).

$$\sin\frac{20\cdot81}{2}=\sin 10^\circ\cdot405=\sin 10^\circ\ 24'\cdot3=\cdot1806049$$

$$\sin\frac{23\cdot62}{2}=\sin 11\cdot81=\sin 11\ 48\cdot6=\cdot2047069$$

$$\frac{128}{144}=\frac{8}{9}\qquad \begin{array}{l}\cdot1806049\times9=1\cdot6254441\\ \cdot2047069\times8=1\cdot6376552\end{array}\qquad \text{Or thus: }\begin{array}{l}\frac{\cdot1806049}{8}=\cdot0225756\\ \frac{\cdot2047069}{9}=\cdot0227452\end{array}$$

† Differences.

$$\sin\frac{7}{2}=\sin 3^\circ\cdot30=\cdot0610485 \qquad\qquad \cdot0610485$$

$$\cdot0597381$$

$$\sin\frac{13\cdot875}{2}=\sin 6\cdot9375=\sin 6^\circ\ 56'\cdot25=\cdot1207866 \qquad \frac{\cdot1207866}{2}=\cdot0603933$$

$$\cdot0620924$$

$$\sin\frac{21\cdot075}{2}=\sin 10\cdot5375=\sin 10\ 32\cdot25=\cdot1828790 \qquad \frac{\cdot1828790}{3}=\cdot0609596$$

$$\cdot0644329$$

$$\sin\frac{28\cdot637}{2}=\sin 14\cdot3185=\sin 14\ 19\cdot11=\cdot2473119 \qquad \frac{\cdot2473119}{4}=\cdot0618279$$

$$\cdot0752595$$

$$\sin\frac{37\cdot637}{2}=\sin 18\cdot8185=\sin 18\ 49\cdot11=\cdot3225714 \qquad \frac{\cdot3225714}{5}=\cdot0645142$$

3202. At another time I compared the effect of equable revolutions with other revolutions very irregular in their rates, the motion being sometimes even backwards and continually differing in degree by fits and starts, yet always so that within the proper time a certain number of revolutions should have been completed. The rectangle was of wire 0·2 of an inch thick; the mean of many experiments, which were closely alike in their results, gave for two smooth, equable revolutions 17°·5, and also for two irregular uncertain revolutions the same amount of 17°·5.

3203. The relation of the current produced to the mass of the wire was then examined; a relation, which has been investigated on a former occasion by loops and small magnets (3133.)*. For the present purpose two other equal squares were prepared, each a foot in the side, but the copper wire of which they consisted was respectively 0·1 and 0·2 of an inch in diameter; so that with the former rectangle they formed a series of three, having the same size, shape and area, but the masses of the moving wire increasing in the proportion of one, four and sixteen. When the rectangle of 0·1 wire was employed, six direct revolutions gave an average result of 41°·75, and six to the left gave 46°·25; the mean of the two is 44°, and this divided by 6 gives 7°·33 as the deflection per revolution. Again, three direct revolutions gave 20°·12, and three reverse revolutions 23°·1; the mean being 21°·61, and the deflection per revolution 7°·20. This is very close to the former result with six revolutions, namely 7°·33, and is a large increase upon the effect of the rectangle of wire 0·05 in diameter, namely 2°·61; nevertheless, it is not as 4 : 1; nor could such a result be expected, inasmuch as the mass of the chief conductor remained the same (3137.). When the results are compared with those made with like wires in the form of loops, they are found to be exceedingly close; in that case the results were as 16° to 44°·4 (3136.), which would accord with a ratio in the present case of 2°·61 to 7°·26; and it is as 2°·61 to 7°·242, almost identical.

3204. The average of the direct and reverse revolutions is seen above to differ considerably, *i. e.* up to 4° and 5° in the higher case. This does not indicate any error in principle, but results simply from the circumstance, that when the needles were quiescent in the galvanometer, they stood a little on one side of zero (3182.). I did not wish to adjust the instrument at the time, as I was watching for spontaneous alterations of the zero place, and prefer giving the numbers as they came out in the graduation, to any pen-and-ink correction of the notes.

3205. The third square of 0·2 wire gave such large swings, that I employed only a small number of revolutions. Three direct revolutions gave an average of 25°·58; three reverse revolutions gave 28°·5; the mean is 27°·04, and the amount per revolution 9°·01. Again, two direct revolutions gave 17°·5; two reverse revolutions gave 18°; the mean is 17°·75, and the amount per revolution 8°·87; the mean of the two final

* See a corresponding investigation by Christie. Philosophical Transactions, 1833, p. 120.