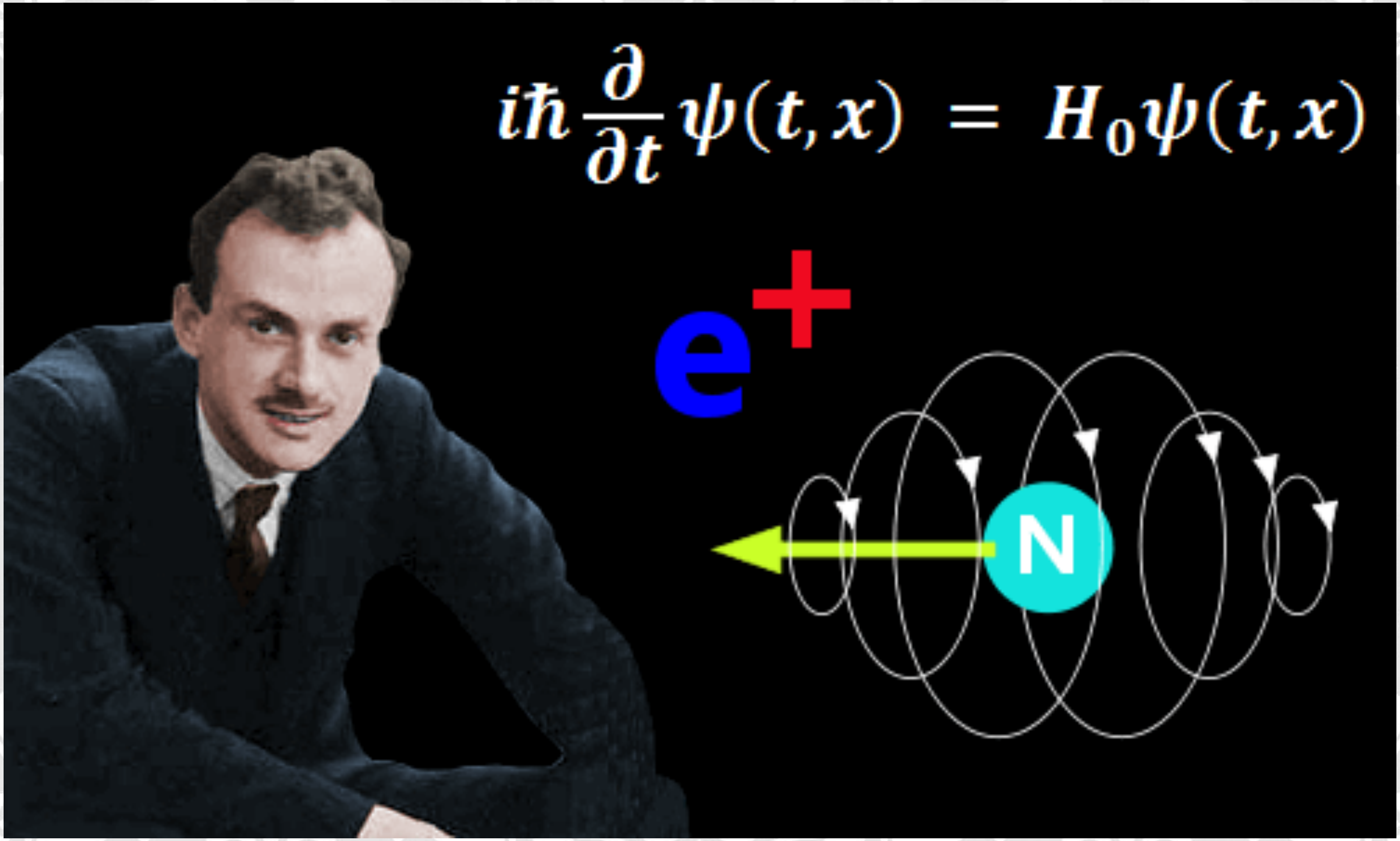
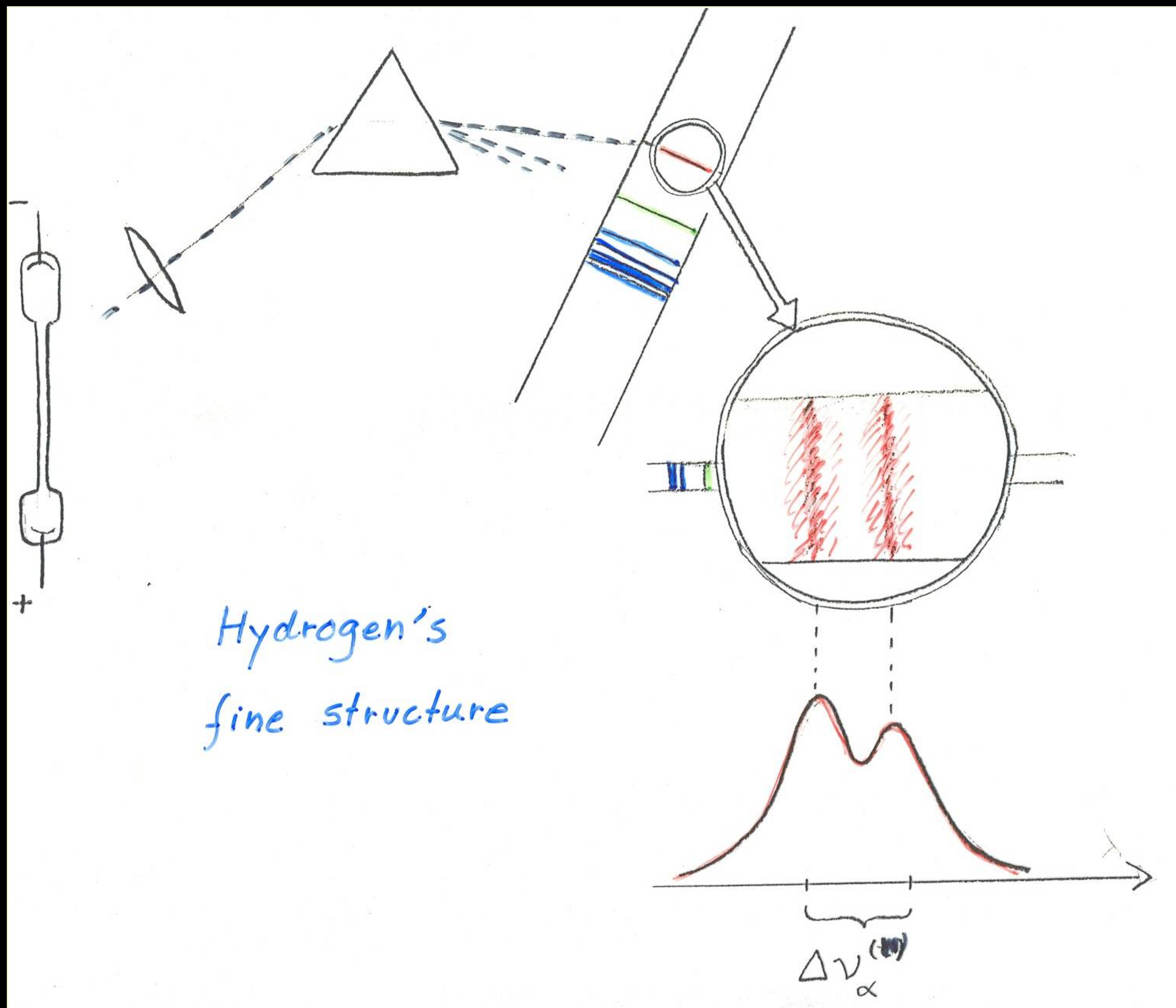
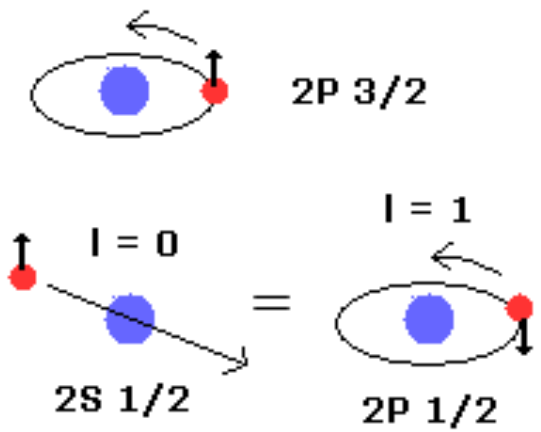
antiparticles & antimatter



The fine structure of the red H line had been known since 1887, and by 1913 the value of the splitting was determined to be $\Delta v^{*} \cong 0.35\ \mathrm{cm}^{-1}$.



The Sommerfeld puzzzle



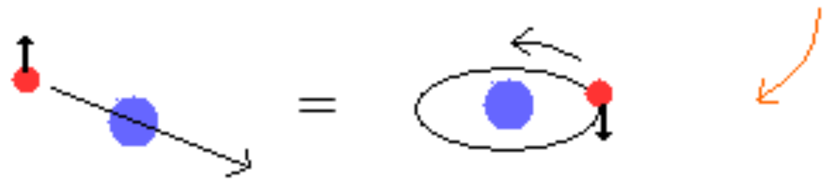
Sommerfeld

$$E_{n_r,n_\varphi} = \frac{mc^2}{\sqrt{1 + \frac{Z^2\alpha^2}{\left(n_r + \sqrt{n_\varphi^2 - Z^2\alpha^2}\right)^2}}}$$

Dirac

$$E_{n,j} = \frac{mc^2}{\sqrt{1 + \frac{Z^2\alpha^2}{\left(n - (j+1/2) + \sqrt{(j+1/2)^2 - Z^2\alpha^2}\right)^2}}}$$

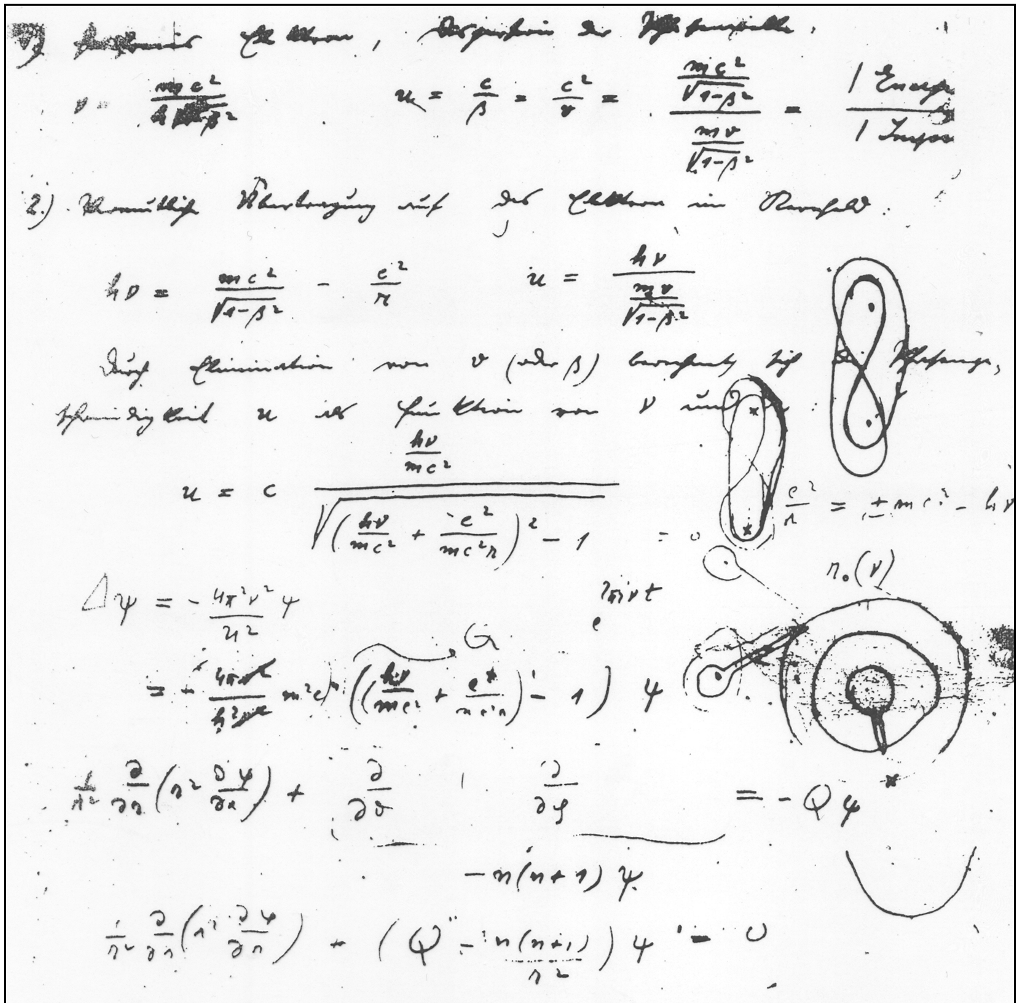
$n_r \rightarrow n - (j + ½)$, $\qquad n_\varphi \rightarrow (j + ½)$



Except for different notations of quantum numbers, the two energy expressions are *exactly* the same. How is this possible!?

How can a patently wrong model result in a fully correct answer?

In the fall of 1925, Schrödinger sought to transform de Broglie's ideas of matters waves into an equation for standing waves, meant to represent particles as wave packets. The equation would have to be relativistic.



Klein-Gordon equation (1926)

$E^2 = \vec{p}^2 c^2 + m^2 c^4$ "translated" into QM by means of $\left\{ \widehat{H} = i\hbar \frac{\partial}{\partial t} \text{ and } \widehat{p} = -i\hbar \frac{\partial}{\partial x} \right.$

yields for a free particle

$$-\frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2} + \nabla^2 \psi = \left(\frac{mc}{\hbar}\right)^2 \psi$$

$$-\Box\psi = m^2\psi$$

d'Alembert operator (4-D Laplace operator)

$$\Box = \partial^\mu \partial_\mu = g^{\mu\nu}\partial_\nu \partial_\mu = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}$$
$$= \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta \ .$$

Particle in EM field

$$E \to E - e\varphi, \quad \vec{p} \to \vec{p} - \frac{e}{c}\vec{A}$$

Oskar Klein, first 5-dimensional theory of QM and GR.
"Klein length":

$$\ell = \frac{h}{4\pi}\sqrt{\pi G} \cong 10^{-32}\ m$$

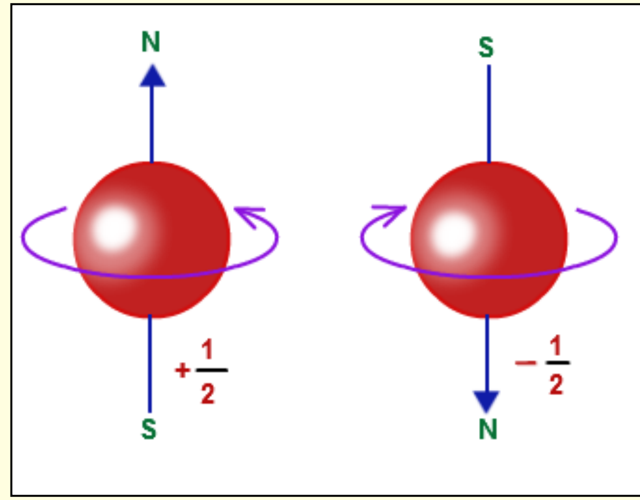
Versions of the Schrödinger equation (without spin), anno 1926-27

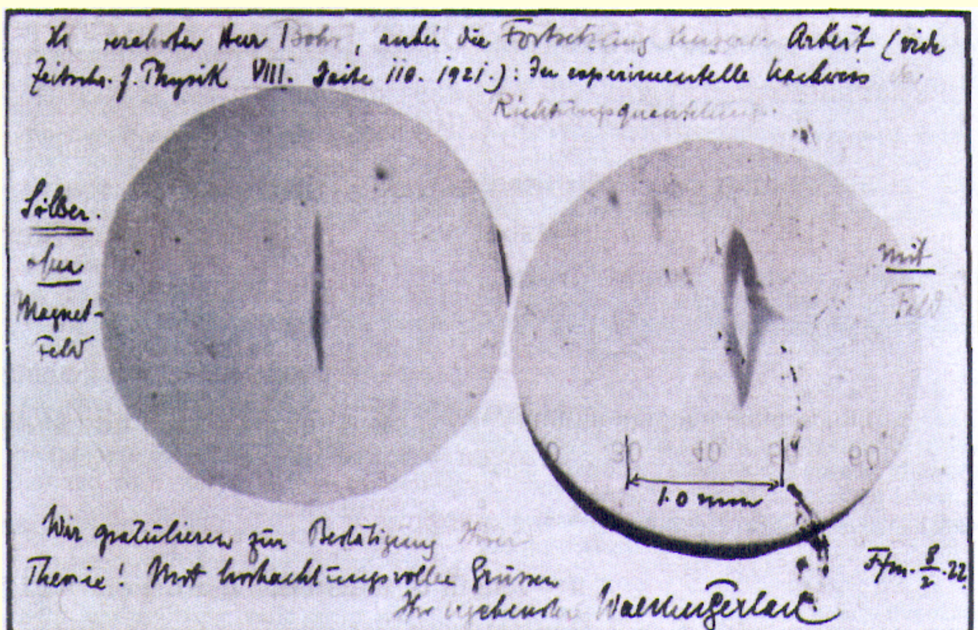
	Eigenvalue eq.	Time-dependent eq.
Non-relativistic ("Schrödinger")	$\Delta\psi + \frac{2m}{\hbar^2}(E - e\phi)\psi = 0$	$\left(-\frac{\hbar^2}{2m}\Delta + e\phi\right)\psi = i\hbar\frac{\partial\psi}{\partial t}$
Relativistic ("Klein-Gordon")	$\hbar^2 c^2 \Delta\psi + [(E - e\phi)^2 - m_0^2 c^4]\psi = 0$	$\hbar^2 c^2 \Delta\psi + \hbar^2 \frac{\partial^2\psi}{\partial t^2} - 2ie\hbar\phi\frac{\partial\psi}{\partial t} + e^2\left(\phi^2 - \frac{m_0^2 c^4}{e^2}\right)\psi = 0$

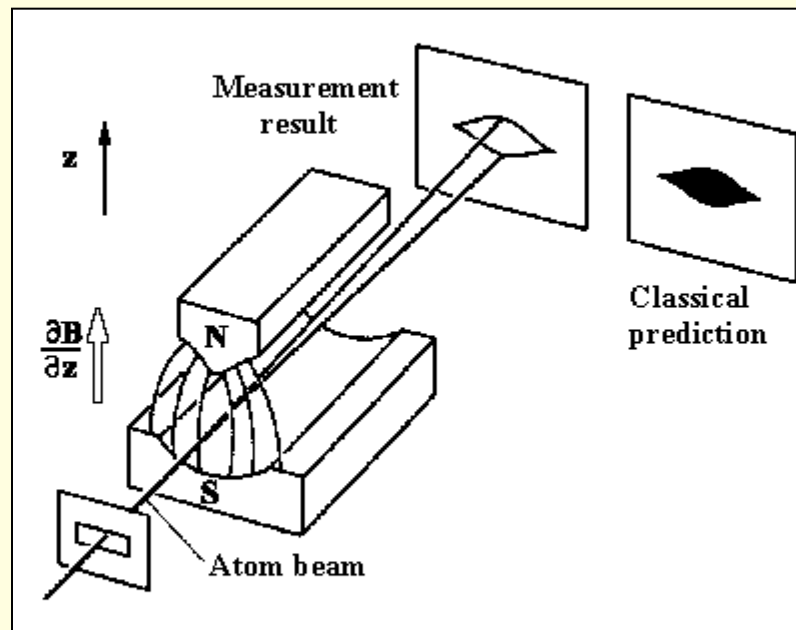
WAVE EQUATIONS, 1926

The electron's spin – an unwelcome discovery

$$r_0 = \frac{e^2}{mc^2}$$







Pauli – Darwin spin theory, 1927

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Spin vektor $\vec{s} = \frac{\hbar}{2} \vec{\sigma}$

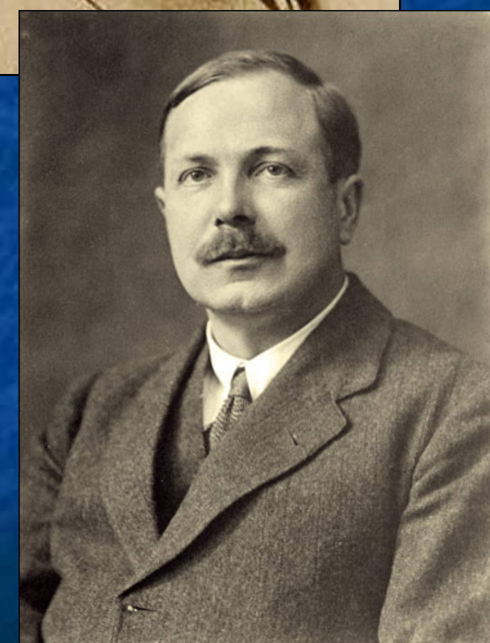
Hamiltonian $H = H_0 + H_1 + H_2$

H_1: spin-orbit

H_2: relativity correction

$$H\psi = E\psi \quad , \quad \psi = (\psi_\alpha, \psi_\beta)$$

$$\begin{cases} \sigma_i \sigma_k + \sigma_k \sigma_i = 0 & (i \neq k;\ i, k = 1, 2, 3) \\ \sigma_i^2 = 1 \end{cases}$$



Dirac's Approach

- Eq. must be Lorentz invariant
- Eq. must be of form $H\psi = i\hbar\, \partial\psi/\partial t$

From KG-eq., free electron

$$i\hbar \frac{\partial\psi}{\partial t} = (\pm)\, c\sqrt{m_0^2c^2 + p_1^2 + p_2^2 + p_3^2}\ \psi$$

Linearization?

$$\sqrt{p_1^2 + p_2^2 + p_3^2 + m_0^2c^2} \stackrel{?}{=} \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3 + \alpha_4 m_0 c$$

Hint (Pauli matrices)

$$\sqrt{p_1^2 + p_2^2 + p_3^2} = \sigma_1 p_1 + \sigma_2 p_2 + \sigma_3 p_3$$

"Playing around with mathematics" leads to 4×4 matrices

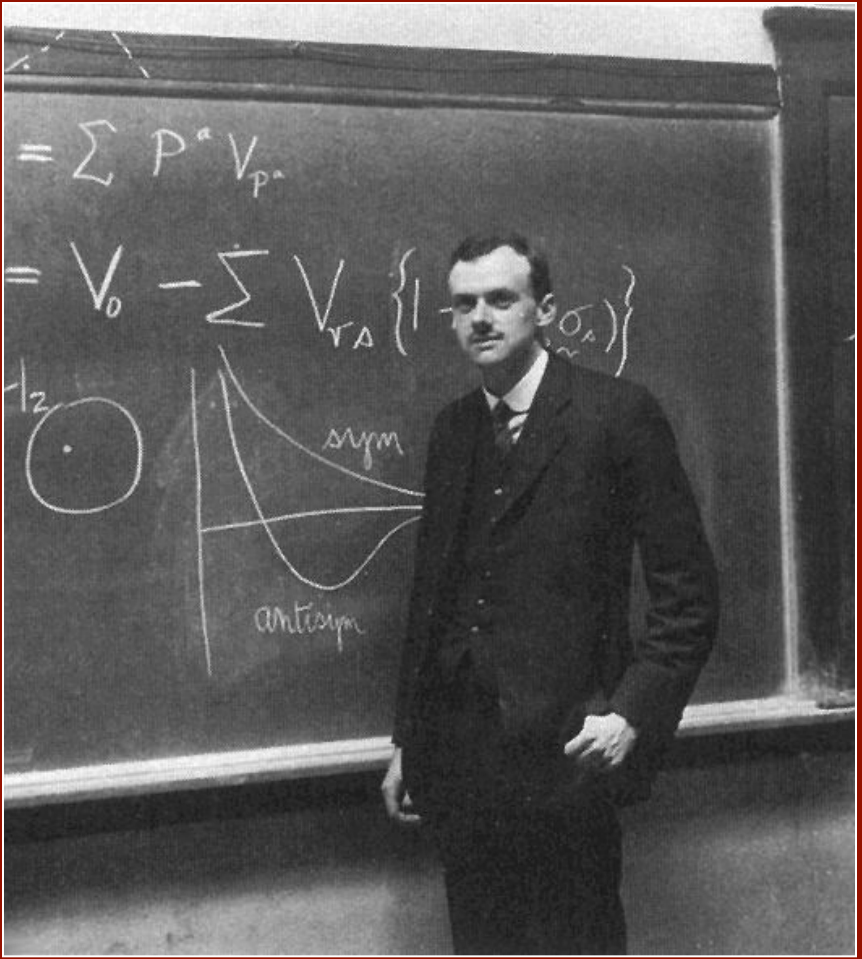
$$\alpha_j = \begin{pmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{pmatrix}, \quad \alpha_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\psi = (\psi_1, \psi_2, \psi_3, \psi_4)$$

$$(\vec{\alpha}\cdot\vec{p} + \alpha_4 m_0 c^2)\psi = -\frac{E}{c}\psi$$

$$[\vec{p} = -i\hbar\nabla, \quad E = i\hbar\, \partial/\partial t]$$

Paul Dirac's route to a linear and relativistically invariant equation for the electron, 1927-28.



The Quantum Theory of the Electron.

By P. A. M. Dirac, St. John's College, Cambridge.

(Communicated by R. H. Fowler, F.R.S.—Received January 2, 1928.)

The new quantum mechanics, when applied to the problem of the structure of the atom with point-charge electrons, does not give results in agreement with experiment. The discrepancies consist of "duplexity" phenomena, the observed number of stationary states for an electron in an atom being twice the number given by the theory. To meet the difficulty, Goudsmit and Uhlenbeck have introduced the idea of an electron with a spin angular momentum

scalar potential A_0 and vector potential $\mathbf{A}$, we adopt the usual procedure of substituting $p_0 + e/c \,.\, A_0$ for p_0 and $\mathbf{p} + e/c \,.\, \mathbf{A}$ for $\mathbf{p}$ in the Hamiltonian for no field. From equation (9) we thus obtain

$$\left[p_0 + \frac{e}{c}A_0 + \rho_1\left(\boldsymbol{\sigma}, \mathbf{p} + \frac{e}{c}\mathbf{A}\right) + \rho_3 mc\right]\psi = 0. \qquad (14)$$

This wave equation appears to be sufficient to account for all the duplexity phenomena. On account of the matrices ρ and σ containing four rows and columns, it will have four times as many solutions as the non-relativity wave equation, and twice as many as the previous relativity wave equation (1). Since half the solutions must be rejected as referring to the charge $+e$ on the electron, the correct number will be left to account for duplexity phenomena.

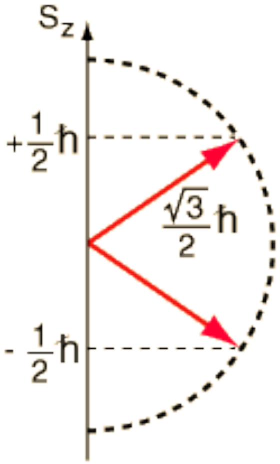
Dirac originally disregarded the spin:

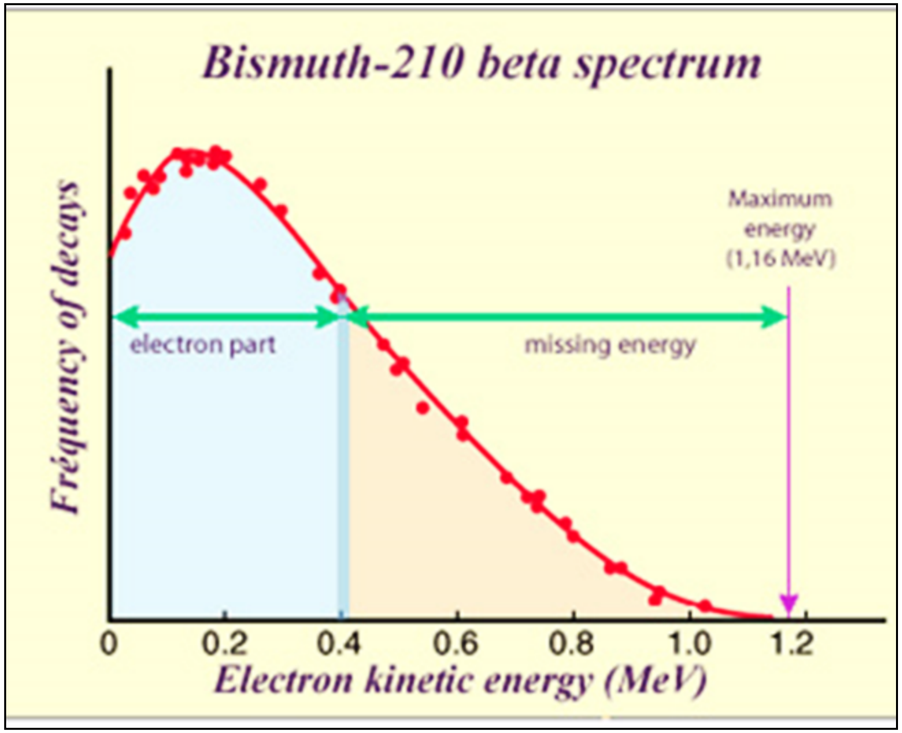
"I was not interested in bringing the spin of the electron into the equation, … The reason was my dominating interest in getting a relativistic theory agreeing with my general physical interpretation and transformation theory. … It was a great surprise for me when I discovered that the simplest case did involve the spin."

He found an additional energy of the electron corresponding to a magnetic moment

I.e., the correct spin derived from quantum principles.

$$\frac{e\hbar}{2mc}\vec{\sigma} = \frac{e}{mc}\vec{s}$$





The mysterious β–spectrum

To explain the β-spectrum Bohr proposed violation of energy conservation in the nucleus.

Pauli suggested, as an alternative, the existence of a "neutron" [:*neutrino*] as a constituent of the atomic nucleus.

> ∾ *This would mean that the idea of energy and its conservation fails in dealing with processes involving the emission or capture of nuclear electrons.* This does not sound so improbable if we remember all that has been said about the peculiar properties of electrons in the nucleus. While we have no relativistic theory and only very few experimental data it is very difficult to give an explanation, but there is no doubt that the question is of fundamental importance and will lead to revolutionary changes in our present picture of the physical world. ∾

(Gamow, *Constitution of Atomic Nuclei and Radioactivity*, 1931)

Pauli to the "radioactive ladies and gentlemen."

The hypothetical neutrino introduced as a massive nuclear constituent, but soon to be expelled from the nucleus (together with the electron).

Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Des. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesats zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen könnte von derselben Grossenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

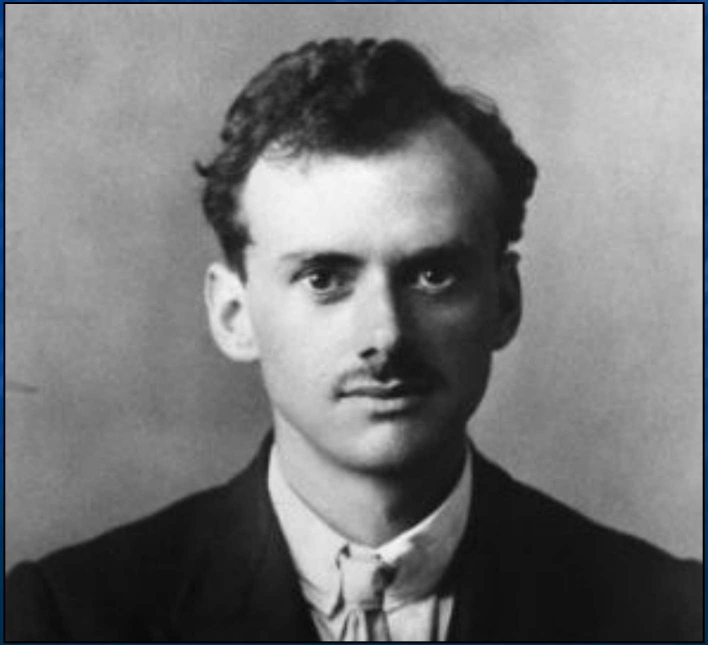
N-14
(14 p, 7 e)
(7 p, 7 n)

Li-6
(6 p, 3 e)
(3 p, 3 n)

From Dirac equation to anti-particles

In a letter to Bohr of 26 November 1929, Dirac introduced his idea of the anti-electron as a ”hole” in a negative-energy world, arguing that the anti-electron is identical to the proton. However, Bohr rejected Dirac’s daring hypothesis.

In 1931, Dirac changed his hypothesis into a theory of positive electrons (positrons), discovered in 1932-33.



The Principle of Plenitude

"From the fact that a thing can exist, I infer readily enough that it does exist." (J. Robinet, 1767)

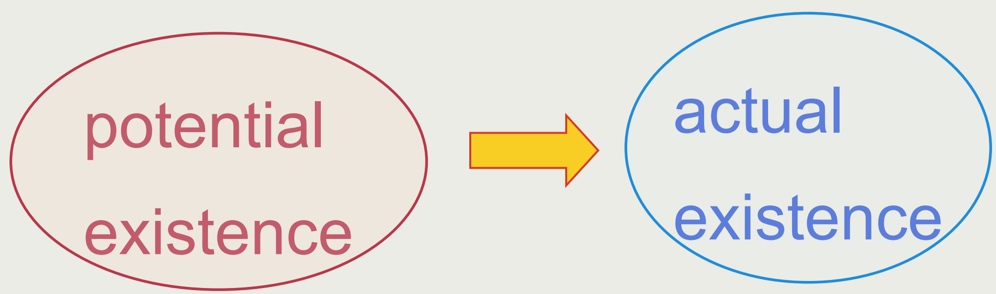
"There appears to be no reason why such processes should not actually occur..." (P. Dirac, 1930)

"Anything which is not probibited is compulsory." (E. Sudarshan, 1972)

The PP has been used to justify numerous entities, e.g.

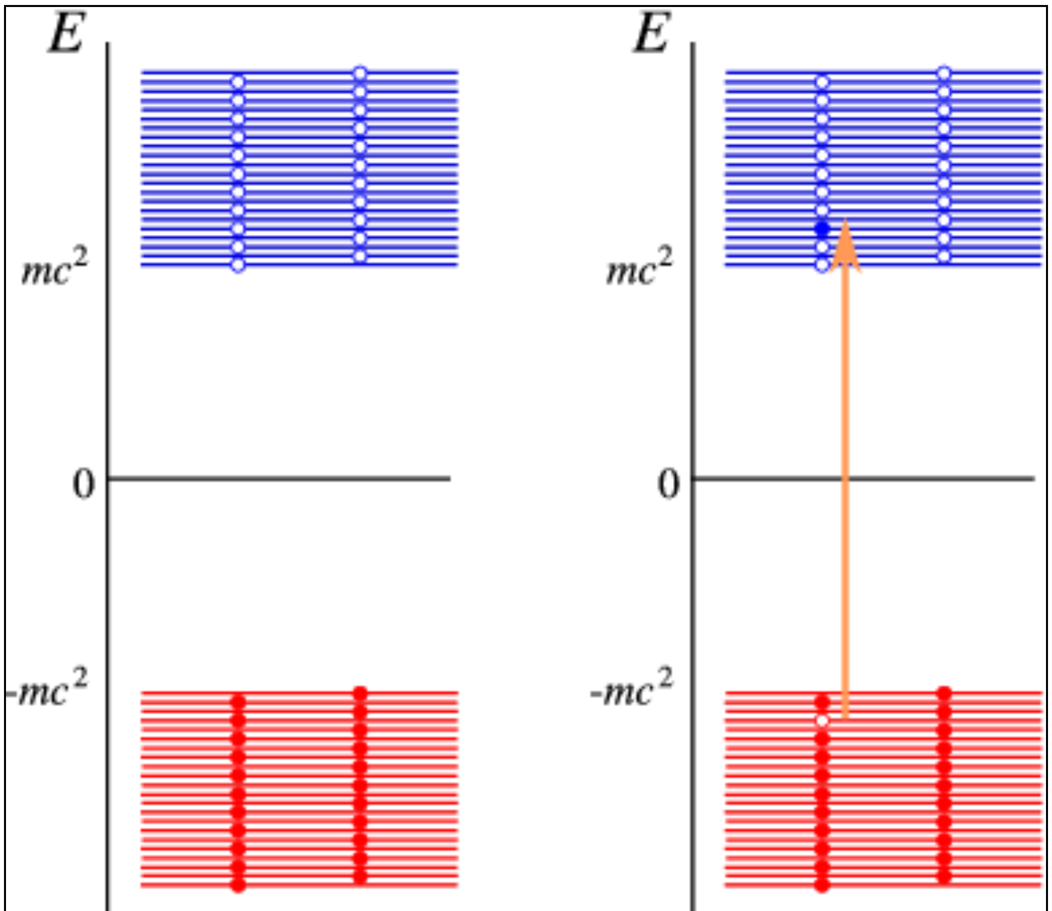
positron, magnetic monopole, tachyon, quarks, black holes, gravitational waves, new elements, supersymmetric particles, synthetic molecules, mermaids (!), ...

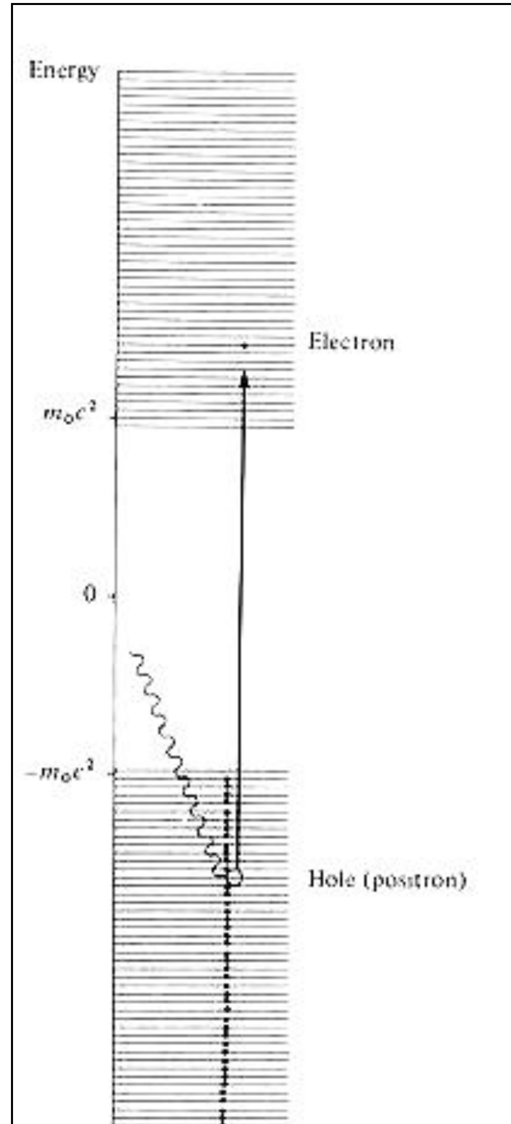
The principle postulates a necessary connection between potential and actual existence.



The Dirac "sea" of negative-energy electrons

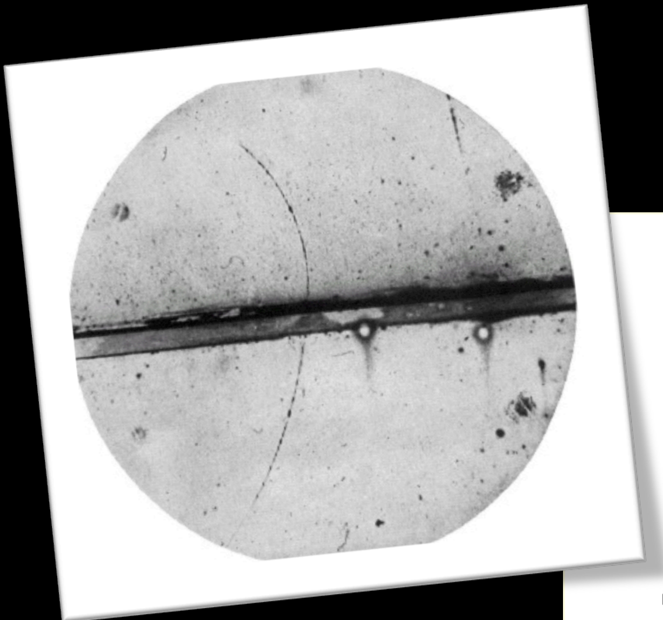
A new picture of the vacuum





(a) Energy levels of spin-1/2 particle

Antiparticles and magnetic monopoles



Quantised Singularities in the Electromagnetic Field.

By P. A. M. Dirac, F.R.S., St. John's College, Cambridge.

(Received May 29, 1931.)

§ 1. *Introduction.*

The steady progress of physics requires for its theoretical formulation a mathematics that gets continually more advanced. This is only natural and to be expected. What, however, was not expected by the scientific workers ... that the line of advancement of

The object of the present paper is to put forward a new idea which is in many respects comparable with this one about negative energies. It will be concerned essentially, not with electrons and protons, but with the reason for the existence of a smallest electric charge. This smallest charge is known to exist experimentally and to have the value e given approximately by*

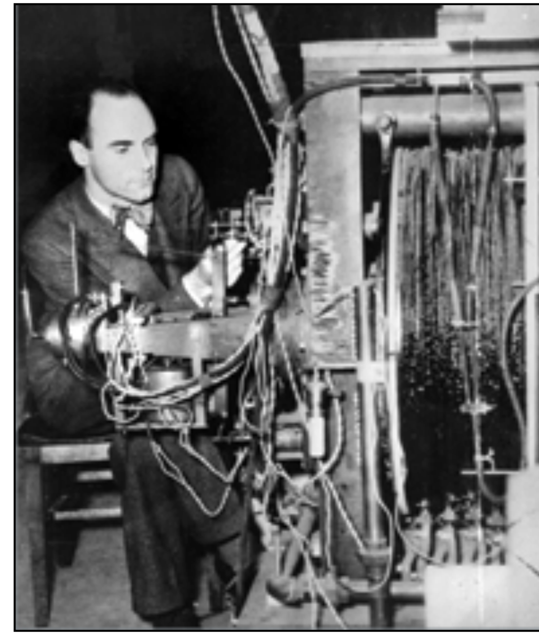
$$hc/e^2 = 137. \quad (1)$$

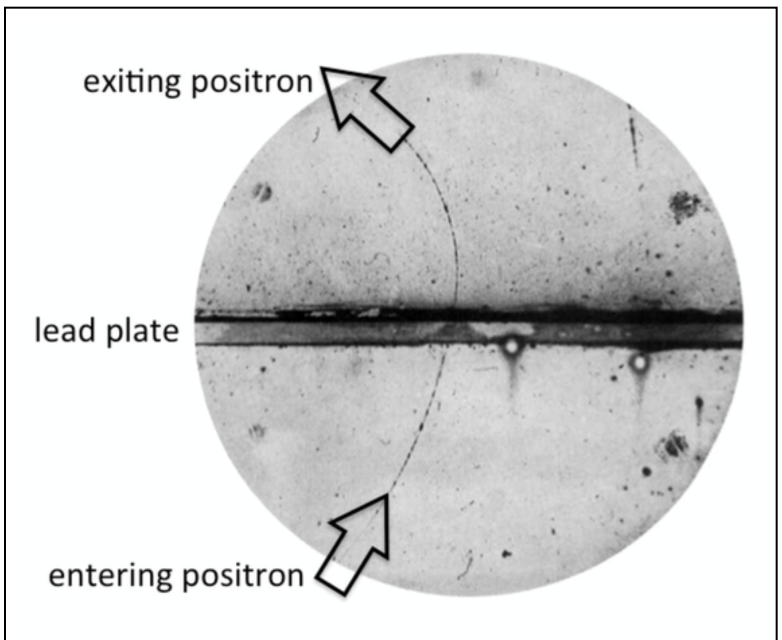
The theory of this paper, while it looks at first as though it will give a theoretical value for e, is found when worked out to give a connection between the smallest electric charge and the smallest magnetic pole. It shows, in fact, a symmetry between electricity and magnetism quite foreign to current views. It does not,

$h \rightarrow \hbar = h/2\pi$

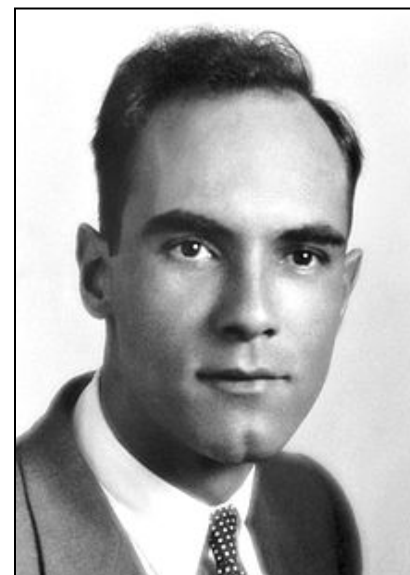
e^+ detection in cosmic rays – but not antielectron

"… secondary particles ejected from atomic nuclei … An incoming primary ray [may] expand the diameter of the proton to the same value as that possesed by the negatron."





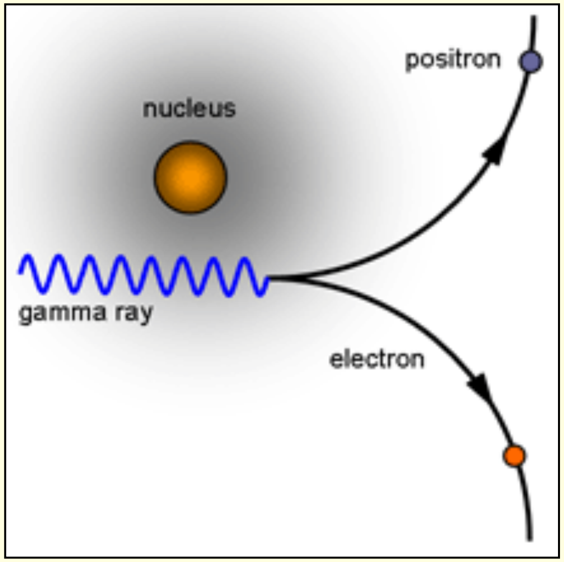
$$r_0 = \frac{e^2}{mc^2}$$



C. D. Anderson, positron, 1932

energy [illegible] mass

mass [illegible] energy



Pair creation

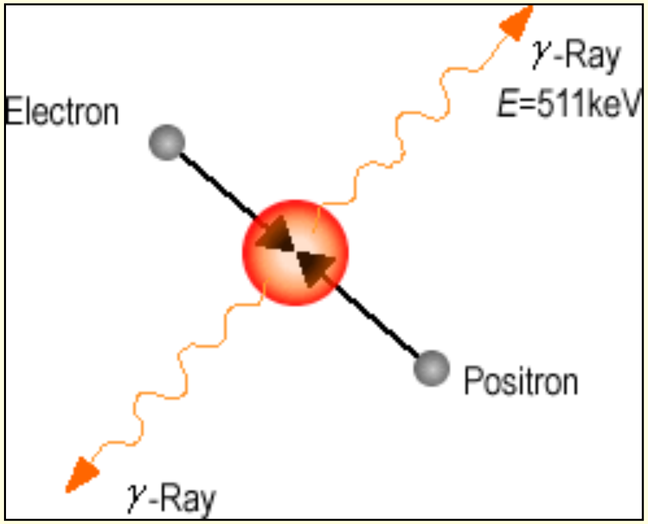
[illegible] [illegible] $e^- + e^+$

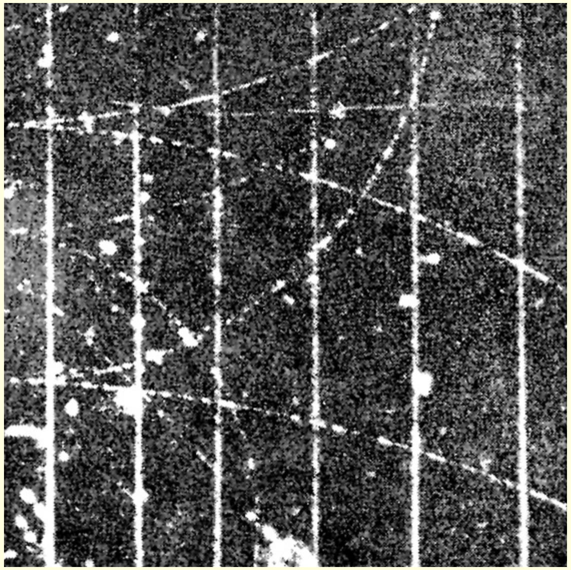
E > 1,02 MeV

m_0c^2 = 0,51 MeV

Annihilation

$e^- + e^+$ [illegible] 2[illegible]

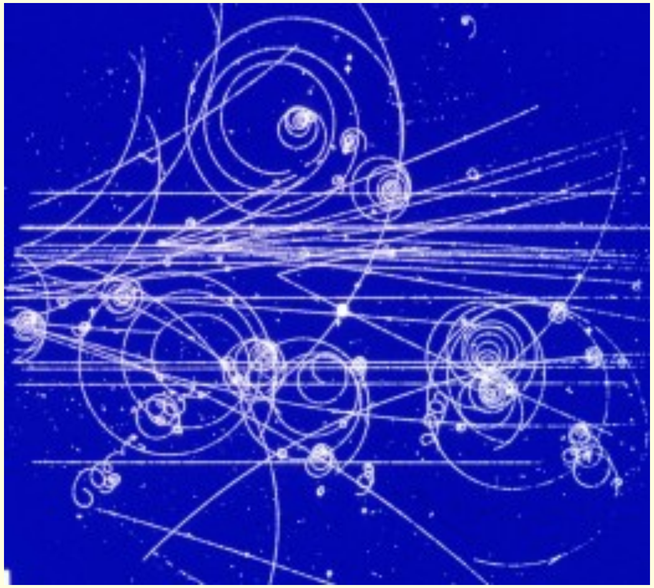




Stellar energy,
ca. 1925-32

$p^+ + e^-$ [illegible] radiation

Eddington, Jeans, a.o.



Artificial (induced) radioactivity, discovered 1934 by F. Joliot and I. Joliot-Curie in the wake of the discovery of the neutron. Nobel Prize 1935.

$$\text{a} + {}^{27}\text{Al} \rightarrow {}^{30}\text{P} + {}^{1}n$$

$${}^{30}\text{P} \rightarrow {}^{30}\text{Si} + e^{+}$$

$t_{1/2} = 2.5$ min

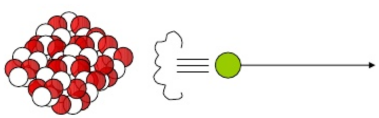
$$n \rightarrow p + e^{-} + \bar{\nu}$$

$$p \rightarrow n + e^{+} + \nu$$

Komperda

Positron Emission

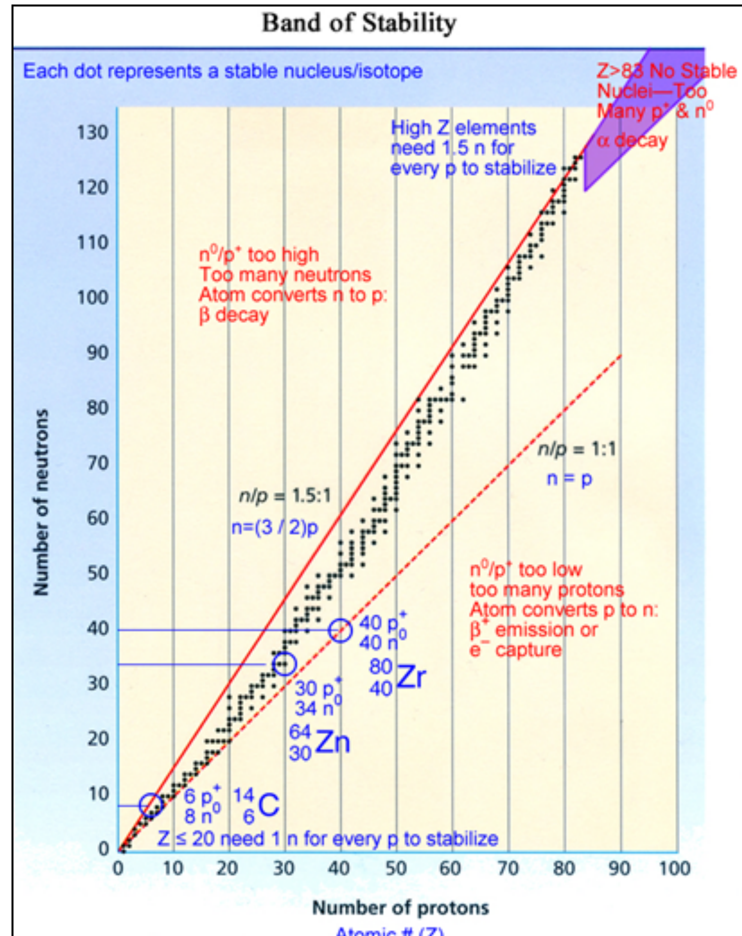
A positron is like an electron but it has a positive charge.

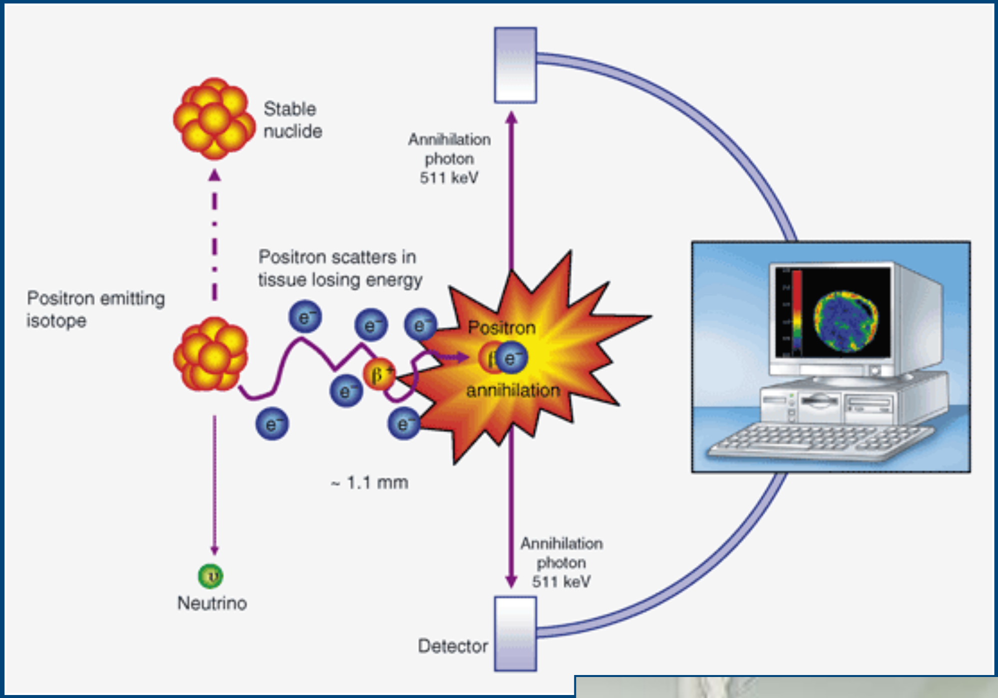


During positron emission a **proton changes into a neutron** and the excess positive charge is emitted.

$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + {}^{0}_{+1}e$$

mass stays in nucleus — positron





PET-method

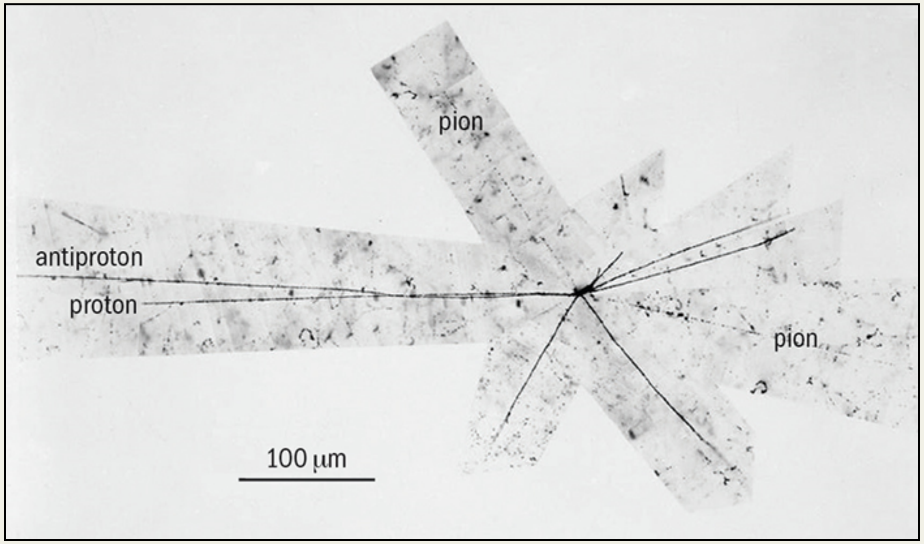
(positron-emission-tomography)

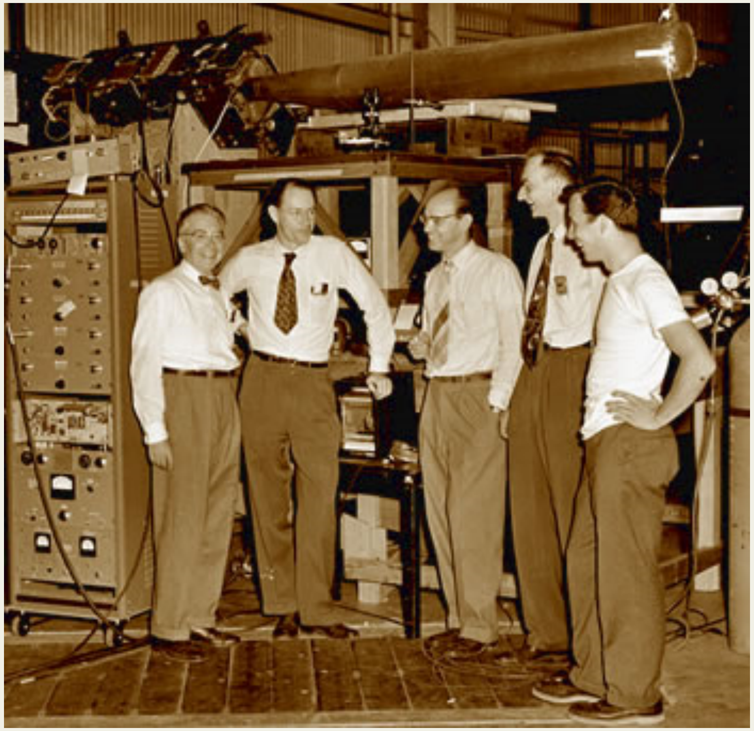
From exotic elementary particle to medicinal practice: diagnosis of cancer, brain research etc.

The radioactive isotope (e^+ emitter) is usually fluor-18, with half-life ca. 110 min.

PET-center, Aarhus Universitetshospital

Discovery of the antiproton, 1955

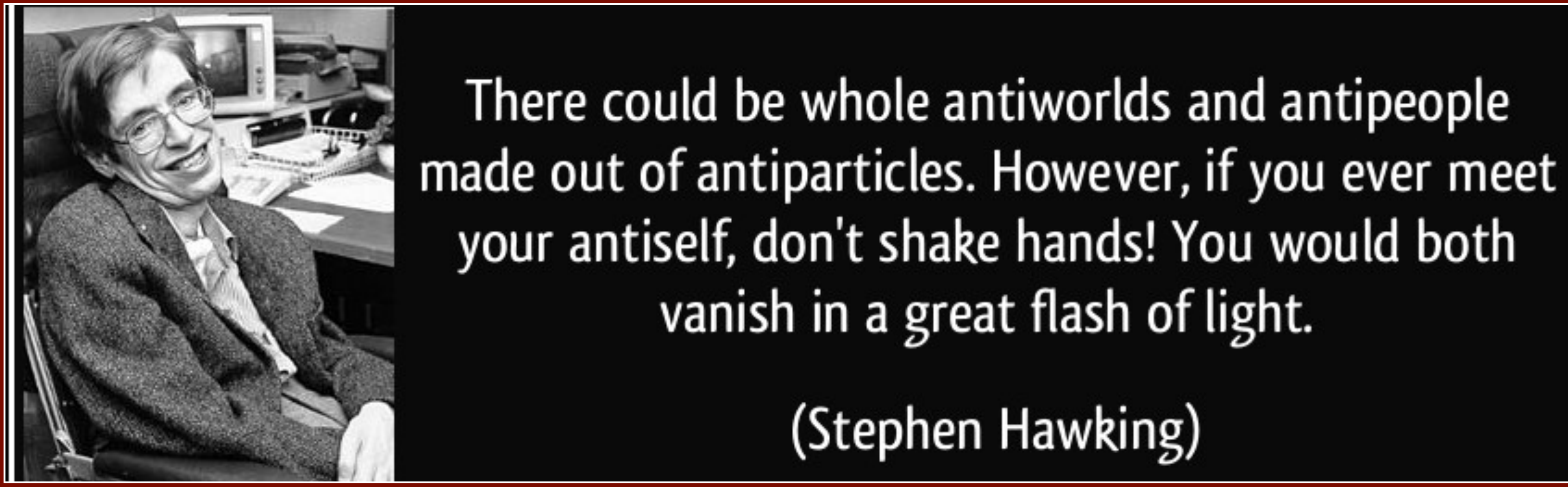


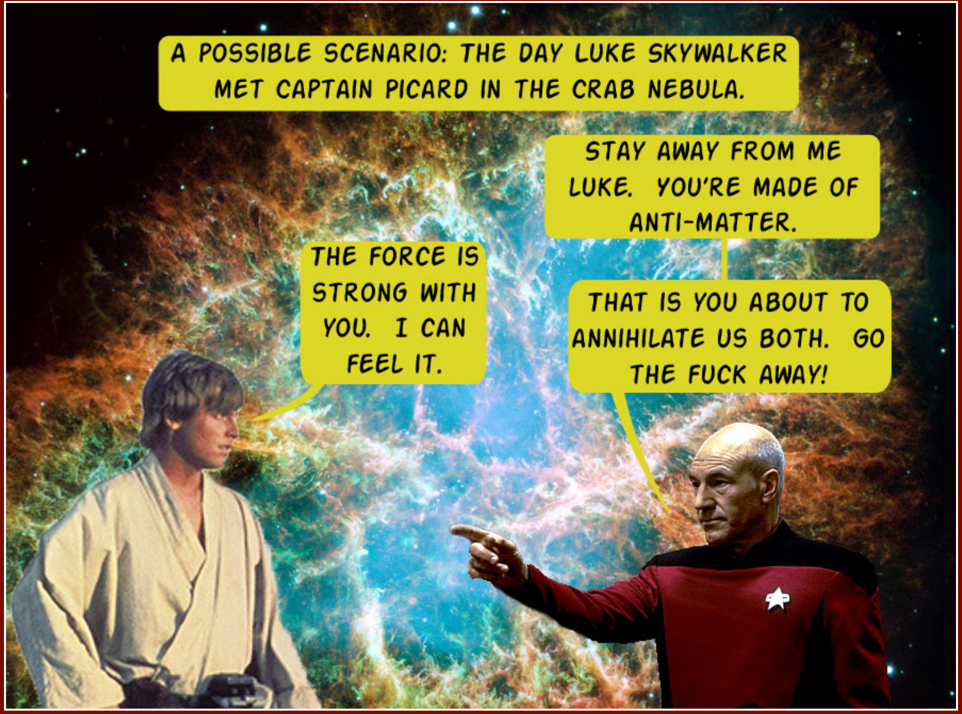


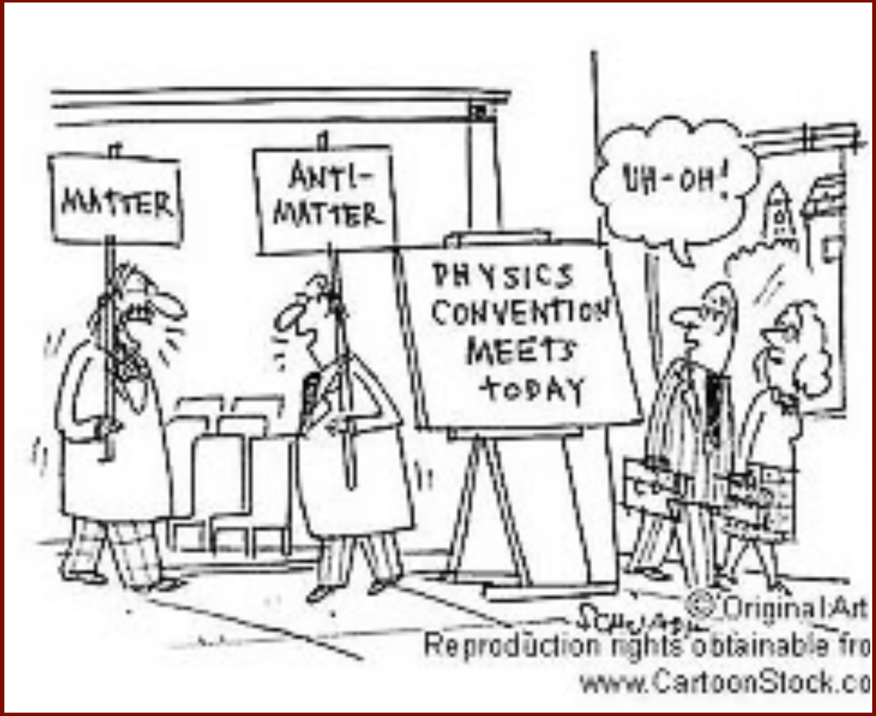
First proof of proton-antiproton annihilation (1956). Antiprotons stopped in photographic emulsion, yielding pions and other particles.

“Natural” antiprotons have been detected in cosmic rays, but they are rare.

Bevatron accelerator (Berkeley), proton energy 6.2 GeV yields antiprotons. NP 1959 to O. Chamberlain and E. Segré. Antineutrons produced in 1957.


There could be whole antiworlds and antipeople made out of antiparticles. However, if you ever meet your antiself, don't shake hands! You would both vanish in a great flash of light.
(Stephen Hawking)


A POSSIBLE SCENARIO: THE DAY LUKE SKYWALKER MET CAPTAIN PICARD IN THE CRAB NEBULA.
THE FORCE IS STRONG WITH YOU. I CAN FEEL IT.
STAY AWAY FROM ME LUKE. YOU'RE MADE OF ANTI-MATTER.
THAT IS YOU ABOUT TO ANNIHILATE US BOTH. GO THE FUCK AWAY!


MATTER
ANTI-MATTER
PHYSICS CONVENTION MEETS TODAY
UH-OH!
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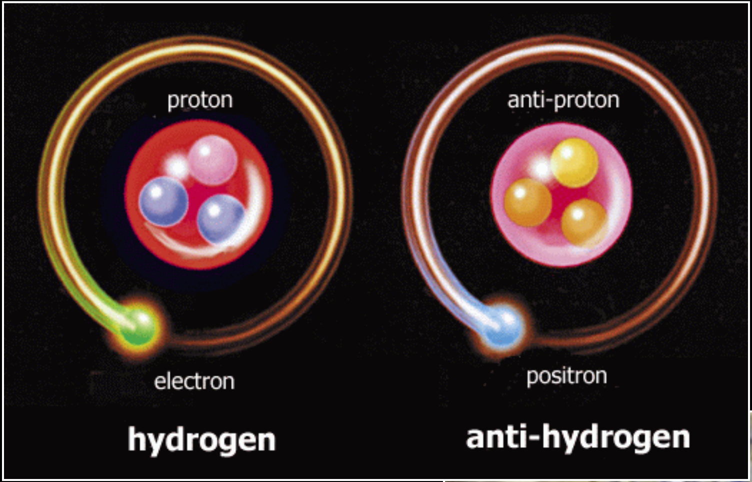
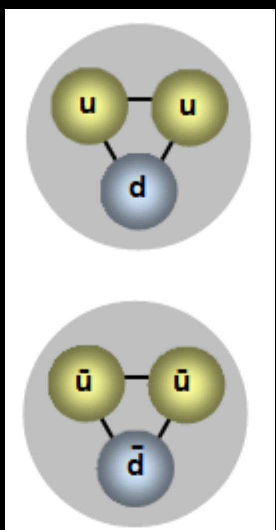


A Danish-led team at CERN have succeeded in manufacturing anti-H, keeping it alive for ca. 20 min and studying its properties.

Is the spectrum of anti-H exactly the same as the one of H?



In 2011, the anti-He-4 nucleus was detected in anti-*p*/anti-*n*