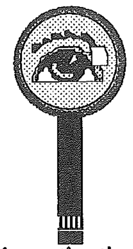
I. PHENOMENA OF INTERFERENCE

Teacher. The term *phenomena of interference* appeared in the 19th century and was applied to some of the phenomena previously called "periodical colors." This term referred to alternating fringes of different colors seen, for instance, in soap bubbles, around narrow obstacles, etc. The new term implied a specific explanation of these phenomena, namely through the principle of interference of light. However, the phenomena themselves had been known long before the discovery of the principle of interference and were explained differently at the time. We are going to examine one of these phenomena, first experimentally and then theoretically.

1. Colors of thin plates

Preliminary part



investigation

Equipment. Two microscopic slides, paper, scissors.

Procedure Put together two microscopic slides, while supporting them with index fingers from below, use your thumbs to move the upper slide over the lower one with so much friction as to produce colors. The colors are seen better against a dark background. If you succeeded in obtaining colors try to change them.

Experiments

Dorothy. I don't see any colors.

Michael. We obtained some, mostly near the edges.

Mary. I saw the colors even in the middle of the plate in the form of irregular circles and rings. They alternate: green, red, green again, red again, and so on.

David. I saw colored rings near my thumb so long as I pressed it to the glass. When I removed the thumb the rings began contracting, until they completely disappeared.

Ruth. I've noticed a change in colors when I looked at the same place at different angles without changing the pressure.

Formulating a problem

What is the cause of the colors created by two glass plates?

Selecting variables

Teacher. Name the factors which affected the appearance of colors.

Mary. Pressure and the angle of reflection. Thus, we have two variables.

Teacher. Since pressure is not an optical concept, its influence must be indirect. We have to find optical factors affected by pressure, and those will be our variables.

John. Perhaps, pressure changes the **shape** of plates: we know that if surfaces are not parallel, the image is distorted and may even be colored.

Main part

a. shape of the plate

Hypothesis

Michael. Let us suppose that pressure bends the plates into a prism.

Test

Ruth. We've bent a single plate (one has to be **careful** not to break the glass), but no colors appeared.

Dorothy. We've also tried it. No luck!

conclusion ➡ The hypothesis is wrong: colors are **not** due to bending of plates. A single plate doesn't create colors.

b. shape of the space between plates

Hypothesis

Mary. Perhaps two plates make an air prism which produces colors.

Test

David. This sounds unlikely, because a prism produces only a **single** spectrum. However, we can test it directly by separating the plates at one end.

Michael. We placed a piece of cardboard between the plates near one edge and pressed the plates together, but we haven't seen any colors, even a single set.

conclusion ➡ The hypothesis is false: the shape of the air gap between the plates is **not** important.

c. substance between plates

Hypothesis

Dorothy. Perhaps the colors are due to the air between the plates

Test

John. Let us remove the air from between the plates. Our group replaced air with water, and observed no change in coloration.

Mary. We did the same and found small bubbles between the plates, which implies a mixture of air and water, but the colors are there nonetheless!

Dorothy. We put some vinegar between the plates and obtained colors , similar to those with water.

conclusion ➡ The hypothesis is false: the air is **not** necessary to produce colors.

d. distance between plates

Hypothesis

Ruth. Let us summarize the facts once more: 1) we need two glasses but not to hold a specific substance between them; and 2) we need pressure but not for bending the plates. We can reconcile the two by assuming that the distance between the plates plays some role. *good point!*

Test

David. We can obtain a variety of distances between the plates in a single trial by inserting something (for instance, a piece of paper or cardboard) between the plates on one side. *good point!*

Dorothy. We did it with cardboard and the result was negative.

Mary. We inserted a paper strip, and it worked. The colors appeared, however, only on the side opposite to the paper strip. No colors appeared near the strip however hard we pressed. This means that the distance necessary to produce colors is much smaller than the thickness of paper.

conclusion ➡ The colors are produced by two glass plates put together only if they have a **very small** distance between them. Pressure is needed to reduce the distance between the plates.

Teacher. We still have the angle of reflection to investigate. Mark a point on the upper plate and watch how the colors will change when you increase the angle of reflection.

e. angle of reflection

Preliminary experiments

Dorothy. When I looked at the plates at a smaller angle I saw a blue ring; when the angle increased the ring became green.

Hypothesis

David. The greater the wavelength the greater the angle of reflection at which we see that color.

Test

Michael. We've seen the red color at a larger angle than the yellow one, and the yellow fringe appeared at a greater angle than the green one.

conclusion ➡ The angle of reflection increases with the wavelength. *important*

Teacher. Very good! You've repeated some of the experiments of Robert Hooke (1635-1703) and Isaac Newton (1642-1727). Let us now see how they explained this phenomenon.

2. Hooke's theory

history

A sickly boy who barely survived childhood, Robert enjoyed making mechanical toys. In 1653, he came to Oxford to become Robert Boyle's assistant, involved in making scientific instruments and performing experiments. In recognition of his outstanding mechanical skills, Hooke became curator of instruments and demonstrator of the newly founded Royal Society of London. Every week he had to prepare several major experiments and demonstrate them at the Society's meeting. This greatly stimulated his interest in scientific research and inventions. On the other hand, it prevented him from realizing in full many of his brilliant ideas and dragged him into priority disputes with Huygens, Newton, and others. Hooke invented an air-pump, a spring balance, which eventually made possible the watch and the maritime chronometer, and many other instruments. He was one of the first to systematically use a microscope for biological observations which he described in his book *Micrographia.* He discovered "Hooke's Law" for elastic deformations, influenced Newton in his work on universal gravitation, gave a theory of the origin of fossils, and distinguished himself as an architect.

In his *Micrographia* Hooke described multiple spectra in thin pieces of mica. He found that the order of these colored rings was inverse to that in the primary rainbow, and that by pressing a particular place he could change the color there. By splitting mica further and further he obtained pieces which exhibited a single color. Hooke supposed that colors depend on the thickness of mica and that this is a property of all thin transparent substances. He confirmed this hypothesis through observations of a very thin glass film, a thin film of air or of a liquid between two glass plates, soap bubbles, and finally of the film that covers tempered steel. Hooke thought that light was a periodical sequence of pulses and he explained the colors of thin plates as follows.

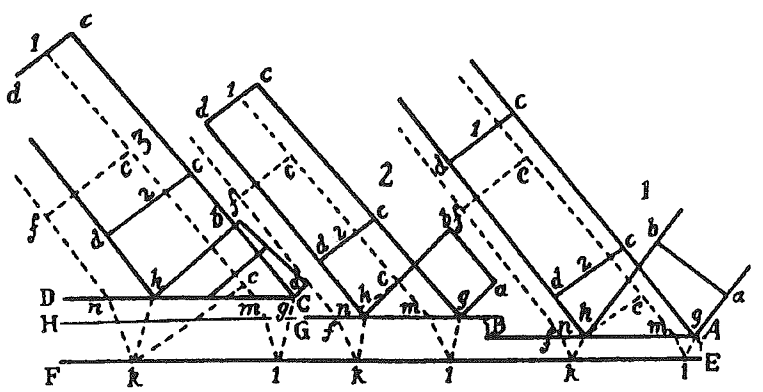


Fig. 10.1. Hooke's theory of thin plates. From Hooke, *Micrographia* (1665)

The pulse **ab** (white light) falls on the font surface of a plate **AB**, which reflects a portion of it as the pulse **cd**. The rest of it is refracted and then reflected by the back surface **EF** as the pulse **ef** (Fig. 10.1). Naturally, the pulse **ef** is weaker than the pulse **cd**. When the combination of two pulses enters the eye it produces a perception of blue color when its weaker part precedes the stronger; and of red color when the stronger part precedes the weaker one. Since the position of the weaker pulse between the two following stronger pulses depends on the thickness of the plate, the latter will determine the resulting color (Hooke, 47-56, 65-7).

3. Newton's theory

history

Newton adopted Hooke's idea that the color of a thin film depends on its thickness. For a film of air between two lenses he was able to find the relation between the radius of a colored ring and the thickness of the film. From his measurements Newton found that the thicknesses that corresponded to the brightest part in consecutive spectra formed an arithmetical progression 1, 3, 5,... while the thicknesses in their darkest parts were 2,4,6,... ... The unit in these progressions was 1/178,000 of an inch, which was the thickness of the film at the brightest part in the spectrum of the first order, which Newton associated with yellow color. He found experimentally the ratio of the thicknesses that corresponded to the red and violet parts of the same spectrum, and using an acoustical-optical analogy, calculated the thickness of the film for every color (*Opticks*, Book II, 193-224). How difficult these quantitative experiments were may be seen from the fact that no one repeated them until the middle of the 19th century. While acknowledging Hooke's experimental results on thin films, Newton rejected his explanation of these colors and offered his own.

In Newton's view, light can periodically change its property of being reflected or refracted (he called this "fits of easy reflection and transmission"). This meant that after entering a refracting medium, at every distance from the surface multiple of 2/178,000 in. light acquired a property to be transmitted further, while in the middle between these distances it could be reflected back. Therefore, light either passes through a film or returns, depending on whether the film's thickness consists of an odd or even number of 1/178,000's of an inch (Fig. 10.2).

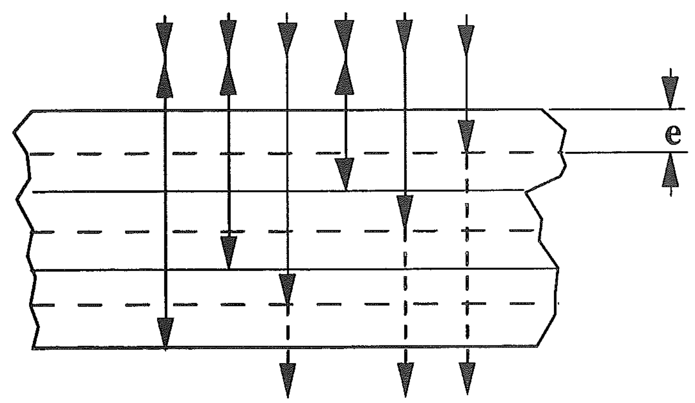


Fig. 10.2. Newton's "fits of easy reflection & transmission".

Newton called the distance between the two consecutive points of easy transmission or easy reflection the *interval of fits*. Its magnitude depended on the color and on the index of refraction of the film. Thus, to some extent the interval of fits resembled the wavelength. Although Newton's concept of periodicity of light differed somewhat from the modern one, he was able to obtain some interesting results with its help. The most important of these was calculating the dimensions of colored rings produced by concave glass mirrors ("colors of thick plates"). Newton's measurements confirmed his theory with great precision (*Opticks*, 278-315).

The first wave explanation of the colors of thin films was provided by Thomas Young.

history

II. PRINCIPLE OF INTERFERENCE OF LIGHT

1. Thomas Young and his discovery

Teacher. Thomas Young was the eldest of ten children in a family of Quakers. His grandfather stimulated his interest in classic literature, and his neighbor, a surveyor, acquainted Thomas with mathematical and physical instruments. Most of his knowledge he obtained by his own efforts. Thomas first distinguished himself in classical languages. At the age of 14, he was more or less versed in Greek, Latin, French, Italian, Hebrew, Persian, and Arabic. While studying botany he decided to make a microscope from a description in a book. For this purpose, he acquired a lathe and for a while abandoned science for learning the art of turning. Then he found in a book some fluxional symbols and did not stop until he mastered an introductory calculus. Classical scholars were greatly impressed with his Greek translations, but Thomas decided to pursue a career in medicine. In 1792, Young began his medical studies in London, continuing them in Edinburgh and finally in Göttingen. He became interested in physics because he believed that a thorough knowledge of that science was important for becoming a good physician. In 1796, while preparing for his medical degree in Göttingen, Young studied the formation of human voice, which led him to acoustics. Upon returning to England in 1797, he found that to practice medicine he had to get a medical degree from an English university. Thus, he enrolled at Cambridge, and since he was already well prepared for the examinations, Young devoted his time to acoustical experiments. While studying beats of sound he discovered in 1799 the principle of interference for sound: two sounds may not only reinforce but also destroy one another. In 1801, he generalized this idea for all sorts of waves, including light. This meant that light added to another light can produce darkness. Young first presented this new concept to the Royal Society of London in November 1801 in his article "On the Theory of Light and Colours." He called it *principle* (or *law*) *of interference.*

discussion

Dorothy. You mean that turning on two lights instead of one can make a room darker?

Teacher. No, the principle of interference is applicable not to every light. Neither two electrical bulbs, nor two candles, nor any other two independent sources of light can produce interference.

John. Can we produce darkness by combining direct light from the sun with sunlight reflected by a mirror?

Teacher. No. Another limitation concerns the angle between interfering rays: it must be close to zero, which does not take place in the experiment you have described.

Mary. How about two mirrors set at almost 180° to one another and reflecting sunlight? Will this work?

Teacher. A similar idea was realized by Augustin Fresnel in 1816. To succeed, this experiment has to fulfill another condition: the two rays must differ very little in the length of their routes. This is possible if the reflection occurs at a very small angle. Besides, for this experiment, the source of light must be very small, and sunlight can be used only after passing through a pinhole.

important

Michael. So many restrictions! It is probably a very rare phenomenon,

which would be difficult to observe or reproduce. If so, what is the point in studying it?

Teacher. Indeed, interference of light occurs much less frequently than reflection or refraction of light. It can be observed only under the following conditions (called *conditions of coherence*):

The interfering rays must:

1. **originate from the same light source;**
2. **have the same wavelength (color);**
3. **have about the same intensity;**
4. **form a very small angle between themselves;**
5. **have a very small path difference; and**
6. **in some cases, come from a luminous body of a small angular size.**

important

When these conditions are not fulfilled our eye cannot detect any interference pattern. We study this phenomenon because of its great theoretical and practical value. As to its rarity and difficulty to observe, you simply don't know that some well known phenomena belong to this category. For instance, the colors of soap bubbles, with which you all have played are produced by interference. The same is true about the multicolored spots in pools of water contaminated by gas or mineral oil.

Mary. Do colors we've observed between glass plates also result from interference?

Teacher. Precisely.

Ruth. How can this be? These colors have another explanation!

Teacher. You mean **two** other explanations. There is nothing wrong with advancing a new explanation of the phenomenon which had already been accounted for. The purpose of any new theory is to give a fuller explanation of the given phenomenon and to extend its application to other phenomena as well.

important

Dorothy. But how can the principle of interference explain colors? When light destroys another light, that means darkness rather than colors.

Teacher. This is correct as applied to monochromatic light. According to Young, if two rays of, say, red light meet after traveling different routes, and their path difference at the meeting point is an integer number of the wavelength for red light, we will see a bright red spot. If at another point their path difference equals an odd number of the half-wavelength, this point will appear dark. When two rays of white light interfere, the red component of the first ray will interfere with the red part of the second ray, the blue light from the first ray will interfere with the blue light from the second ray, etc. Since the ratio of the wavelengths for red and blue light is about 1.5, wherever the path difference is one wavelength for red light, it will be three half-wavelength for blue light. This means that at the same point where red light will be reinforced, blue light will be extinguished. As a result, this point will appear not white but reddish. Let us now see how this theory explains the colors of thin films.

2. Young's theory of thin films.

theory

According to Young, the interfering rays are those which are reflected from two parallel surfaces (Fig. 10.3) of the film. The ray **1** is reflected from the first surface and travels the path **SCO**, while the ray **2** first is refracted and then reflected from the second surface following the route **SAEFO.** is P=SA+AE+EF+FO-(SC+CO). Let us draw **AB** perpendicularly to **SC**, and **CD** perpendicularly to **AE**. If the angle **ASC** is very small,

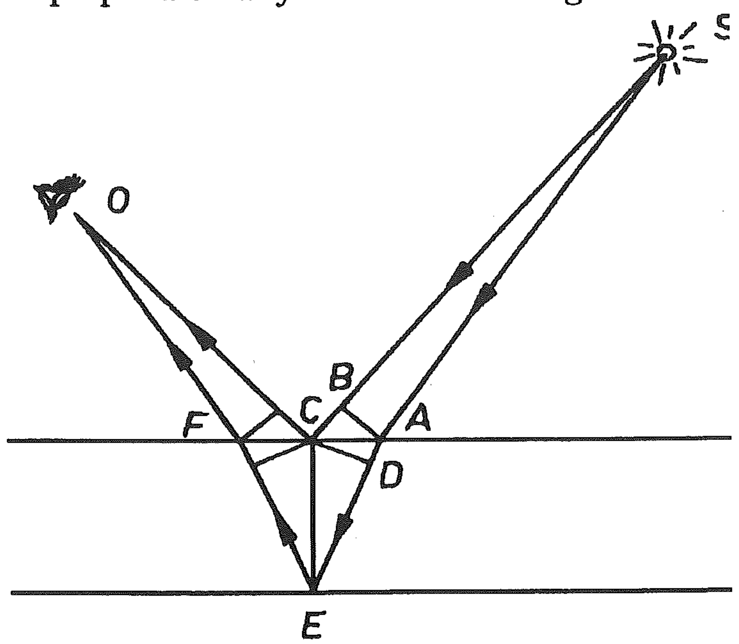


Fig. 10.3. From T. Young ,"Theory of light," *Misc. Works*, v.1, p.169.

SB=SA, and for the time necessary for light to come from **B** to **C** it will also pass from **A** to **D**. Thus, at the points **C** and **E** the ray **2** will be delayed relatively the ray **1** by nDE, where **n** is the film's index of refraction. Similarly, one can find that on the way from points **C** and **E** to the eye **O** the ray **2** will be delayed by the same distance nDE. Thus, the path difference of the two rays is P=2nDE, and since DE=CEcos*r*, we have

$$P=2tn\cos r$$

where **t** is the plate's thickness, and **r** is the angle of refraction. The results calculated with this equation differed from Newton's observations, and to reconcile the two Young supposed that a ray coming from a rarer to a denser medium to be reflected back loses half of a wavelength from its path. Thus, Young's modified equation for the path difference in reflected light is

$$P=2tn\cos r + \lambda/2 \qquad (10.1)$$

The bright fringes will be seen in the directions where P=mλ (λ is the wavelength and **m**=0, 1, 2, ... which is the spectrum's order), or where

$$2tn\cos r_{max} + \lambda/2 = m\lambda \qquad (10.2)$$

math is a friend !

Analogously, the minima of light will appear at

$$2tn\cos r_{min} + \lambda/2 = (2m + 1/2)\,\lambda \qquad (10.3)$$

Thus, if the incident light is white, the rays producing maxima are reflected under different angles for different wavelengths, as the result of which white light is decomposed into fringes of different colors. The spectra of higher orders overlap, and colors mix until they produce white. For this reason, ordinarily, only spectra of the first few orders are seen. Since the number of spectra produced increases with the path difference which, in turn, depends on the film's thickness, only extremely thin films (about 1 micron) produce colored fringes in white light. Thicker plates produce an interference pattern only in more or less monochromatic light, and the thicker the plate, the narrower the range of wavelengths must be.

important

3. Principle of interference and waves of light

discussion

Dorothy. This theory is more complicated than the previous two.
Teacher. Perhaps, but it is better.
John. Why?
Teacher. Hooke's theory showed that a parallel plate can create colors which depend on the plate's thickness. However, it was a qualitative proof, it dealt only with two colors, and it didn't show why only thin plates produce colors. Newton's theory was quantitative, it explained the origin and place of all colors, and it was applicable to two other phenomena. However, it could not explain why thicker plates don't make colors. Young's theory was also quantitative and applicable to all colors. In addition to this, it accounted for the limitation in the thickness of the plates and could be extended to many other phenomena. Thus, we see how each theory improves over its predecessors. Young's theory of thin films was also superseded by others which explained, for instance, the intensity of the interference fringes. However, as far as the location of the fringes is concerned, Young's theory remains a good theory because it provides a precision satisfactory for many purposes by very simple mathematical means.

important

Michael. Perhaps it is a fine theory, but it is very obscure, at least as you presented it. Why did Young compare the path difference to the wavelength? Where did he get this rule from?
Teacher. Young supported the wave hypothesis of light, thus he compared light waves to sound waves. He knew from mechanics that when a point participates in two vibrations the compound amplitude can be either a sum or a difference of amplitudes, depending on

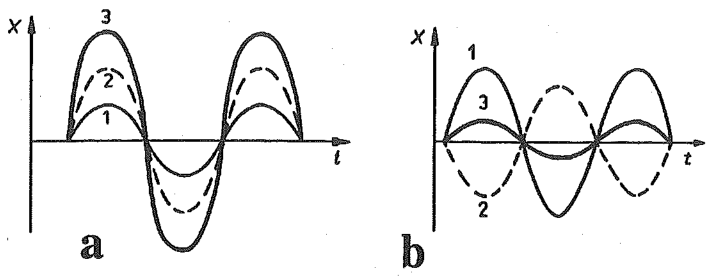


Fig. 10.4. Superposition of waves of the same frequency.

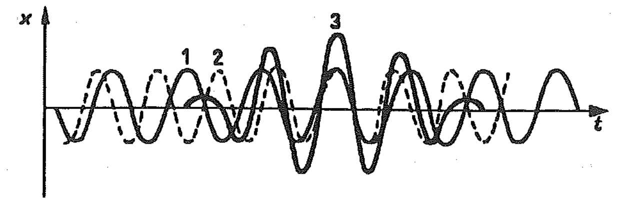


Fig. 10.5. Superposition of waves of different frequency.

whether the vibrations were in the same or the opposite phase. This can be expressed graphically as in Fig. 10.4 (equal frequencies) or in Fig. 10.5 (slightly unequal frequencies). Initially, Young applied such graphical addition to explain beats of sound, or a periodical increase and decrease of loudness (Fig. 10.6) produced by two sounds of slightly different frequencies. He extended this principle to light waves: if two waves bring to to the eye vibrations in the same phase, their amplitudes add; if the phases are opposite, the amplitudes subtract. An addition of amplitudes is supposed to produce a spot brighter than the background, while their subtraction should create a darker spot. The relative phase of the two vibrations depends on the path difference of the two waves. The phases are the same when the path difference is a multiple of the wavelength, and they are opposite when the path difference equals an odd number of the half-wavelength. The concept explaining at which point we should expect an increase or decrease of the intensity of vibrations was also discovered by Young and is called the *principle of superposition of waves*. The principle of superposition is applicable to any two intersecting waves, but the result of such an intersection is not always observable. The principle of interference refers to *observable* phenomena, thus it includes the principle of superposition of waves and the conditions of coherence.

important

David. The idea of adding two waves as expressed in Fig. 10.4 and Fig. 10.5 is so simple that it was probably easy for Young to stumble upon it after he adopted the wave hypothesis. Is that right?

Teacher. Wrong. It took him two years to advance from interference of sound to interference of

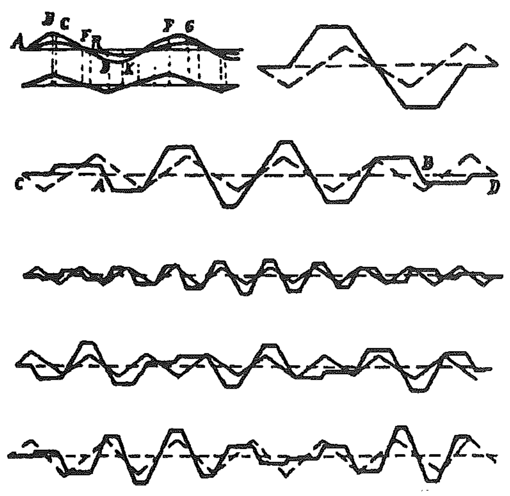


Fig. 10.6. Superposition of waves of unequal frequency. From T. Young, *Misc. Works*, v.1, pl. III.

light. One of the reasons for this can be seen in Fig. 10.6. At the time, mathematicians believed that a sine wave representation of sound (introduced by Newton) was too simplistic and they used more complicated curves. However, such curves are not applicable to light. To produce beats of sound we need two waves of different frequency which produce vibrations of differential frequency changing their intensity in time (it is called *temporal interference*). The eye cannot perceive temporal interference of light because the difference of any two optical frequencies is beyond the range of visible frequencies. The eye can only perceive *spatial interference*, or the one which changes the intensity only in space and not in time. Such interference is produced by waves of identical frequency.

important

Mary. Is that is why you said that red light interferes with red, and blue with blue?

Teacher. Exactly. Of course, you can mix, red light with blue but you won't see any interference pattern.

Dorothy. O.K. So it took Young some time to decide on selecting sine waves to represent light mathematically, but after that everything was obvious, wasn't it?

Teacher. Not at all! For instance, he had to decide whether to use sine waves of finite or infinite length. What would be your choice?

math is a friend!

John. I would probably choose an infinite wave.

Teacher. Wrong. Infinite sinusoidal waves must interfere whatever their path difference, which contradicts experiment. Among other things, Young had yet to discover all the conditions of coherence, and to confirm his theory by a quantitative experiment.

Michael. How was Young's principle of interference received?

Teacher. Most scientists misunderstood it. They were asking (not always explicitly) the same questions you did, however Young didn't bother to answer them in detail. Instead, he relied on analogies, comparing the interference of light with an intersection of waves on the surface of water or with beats of sound. He meant that all these phenomena can be explained by an addition or subtraction of amplitudes of vibrations. His opponents, however, looked at them from an entirely different perspective. In their view, the intersecting water waves did not interfere at all because they preserved their form and velocity; and the beats of sound had a psychological rather than a physical cause. For these reasons, the principle of interference of light was accepted only after 1815, thanks, to a considerable degree, to the efforts of Augustin Fresnel.

interesting!

David. Why didn't Young bring forth new experiments to support his case for interference?

Teacher. He did, and we will repeat some of them in a short while. In 1804, for instance, he offered an experiment (I call it the "screening experiment") which later became very influential in winning support for the principle of interference and the wave theory. The idea was to show that **two** rays of light are necessary for interference, and that blocking one of them destroys the interference pattern. Young placed a wire into the cone of sunlight entering a dark room through a fine

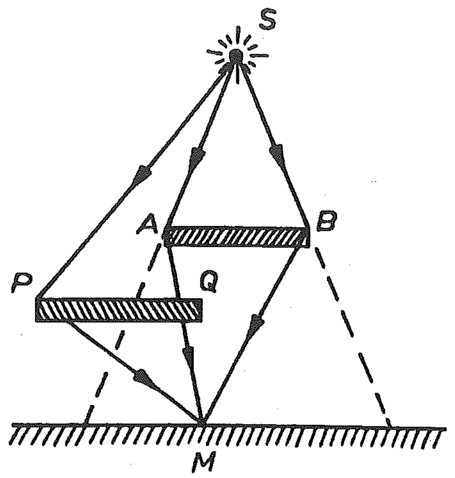


Fig. 10.7. Young's "screening experiment".

interesting!

pinhole and observed parallel fringes inside the shadow. That was a phenomenon of diffraction known since Grimaldi (see Ch. 11). Young supposed that these fringes resulted from the interference of two rays of light **AM** and **BM** (Fig. 10.7) coming to the screen from the opposite edges of the wire **AB** , and his calculations agreed with experiments (Ch.11). When he inserted a small opaque screen **PQ** in the shadow in order to intercept the ray **AM**, he discovered that the fringes disappeared. Young interpreted this outcome as proving that **two** rays of light are necessary for producing the interference pattern. In his view, this refuted Newton's explanation of periodical colors as resulting from periodical changes in a **single** ray.

III. INTERFEROMETER

1. The idea

In 1816, ***François-Dominique Arago*** (1786-1853), a member of Paris Academy of Sciences, was engaged by the Academy in reviewing Fresnel's early papers. Arago reproduced Fresnel's and Young's experiments on interference of light. While repeating

history

Young's "screening experiment" he happened to have in his hand a glass plate P_1Q_1 (Fig. 10.8) instead of an opaque screen **PQ**. To his surprise, the fringes inside the shadow **CD** of the body **AB** disappeared although the plate did not stop the ray **AM**. Fresnel supposed that the ray **AM** was delayed inside the glass on its way to the meeting point **M**, or, that the path difference of the two rays increased and they met at another point M_1 outside the shadow. There, the background was so bright that the fringe became invisible. To test his hypothesis Fresnel suggested replacing a thick glass plate with an extremely thin one: the fringe reappeared within the shadow at the point M_2. Arago found that the same result could be achieved by means of two thick glass plates P_1Q_1 and P_2Q_2 of a slightly different thickness placed at both sides of the shadow: a small difference in their thickness produced a noticeable shift of fringes. This gave Fresnel and Arago the idea to measure small changes in length or in the index of refraction, because the optical path of a ray depends on both. That is how the first *refractometer-interferometer* was born. Astronomers provided the first task for the new method: to check whether refraction of the air depends on its humidity. That was necessary for calculating the atmospheric

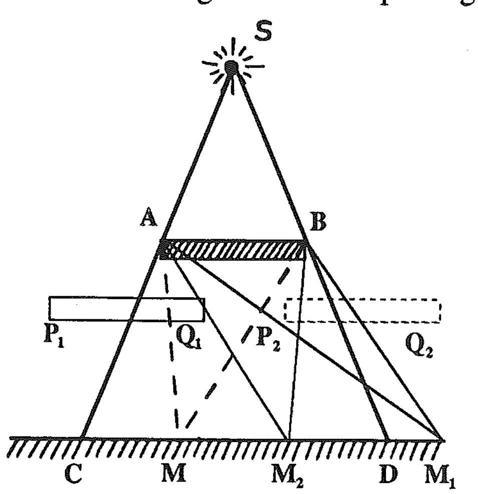


Fig. 10.8. Arago's experiment..

refraction and determining the true positions of heavenly bodies. Interferometers were much improved in the second half of the 19th century, to a considerable extent due to the American physicist Albert Abraham Michelson (1852-1931).

2. Michelson interferometer

The son of a Jewish immigrant from Poland, Albert early displayed courage, inventiveness, and determination. He dreamed of entering the U.S. Naval Academy but was tied with two other applicants, and the place was given to another person. With a letter of support from his senator, Albert intercepted President Grant during his walk and pleaded with him for an extra appointment. He succeeded and eventually became the first American to win the Nobel prize. After graduation he taught at the Academy for a while, and at that time he distinguished himself by measuring the velocity of light (1878). Two years later, he went to Europe for graduate studies, and it was in the laboratory of Helmholtz that he conceived of the idea of a new interferometer. The instrument had to be sensitive enough to discover whether or not the earth moves relative the ether. If "Yes," the "ether wind" must depend on the direction of light relative to the direction of the earth motion, which can be discovered by rotating the interferometer. The result was negative. In 1887, Michelson and Edward Morley repeated the experiment with an improved interferometer, but the result was the same. The negative results of Michelson's experiments played an important role in physics, by preparing the ground for the theory of relativity. Michelson, however, realized that his instrument has also important practical uses. For instance, with the help of his interferometer he found that the Paris standard of meter contained 1,553,163.5 wavelengths of the red line of cadmium. Subsequently, this became the foundation for replacing the metal meter as a standard of length with a much more constant "light meter", based on the wavelength.

The Michelson interferometer consists of two front-surface mirrors **A** and **B** and a beam-splitter **C** (Fig. 10.9). While Michelson used a spectral

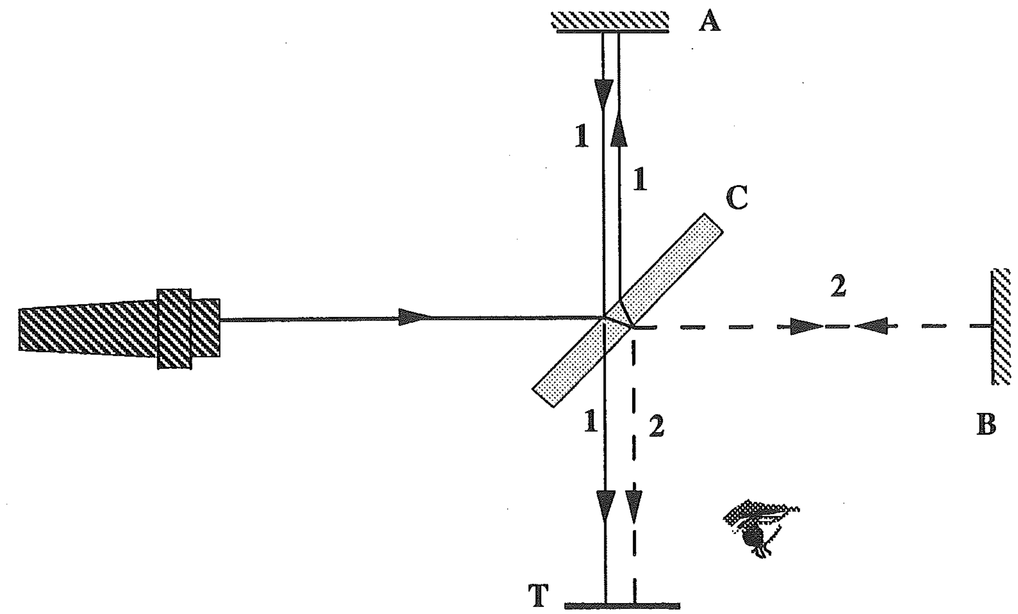


Fig. 10.9. The Michelson interferometer.

tube for a source of light, we will have a laser **L**, which will considerably simplify the procedure. The beam is divided in two parts in the plate **C**. The ray **1** enters the eye after being reflected by the silvered side of the plate **C** and then by the mirror **A**; the ray **2** is transmitted by the plate **C**, reflected by the mirror **B** and again reflected by the silvered side of the plate **C** towards the eye. The two rays interfere in the same way as on a plane-parallel plate (if the mirrors **A** and **B** are perpendicular) and produce a system of concentric bright and dark rings. With white light, one cannot see more than a few rings, but their number increases with the degree of monochromaticity of light. With a laser, one may see many rings, which implies a large path difference and a high monochromaticity of light. When the path difference is very large, it may be difficult to locate the center of the ring system.

The purpose of the interferometer is to measure a **change** in the path difference of two interfering rays, caused by either a movement of a mirror or by a change in the index of refraction of the medium crossed by one or both rays. The index of refraction varies with pressure or temperature. If $\Delta n = n_2 - n_1$ is the change of the index of refraction and **L** is the length that light traverses, the change in the path difference will be $\Delta P = 2L\Delta n$, and, if expressed in the number **N** of wavelengths λ, $\Delta P = N\lambda$. Hence,

$$N = 2L\Delta n / \lambda$$

Thus, when the path difference changes, the fringes move: a shift by N times the width of the fringe corresponds to a path difference change of N wavelengths. The shift is difficult to measure when it is large: one counts how many fringes crossed a chosen mark on the screen. This is easier to do when we see the center of rings, in this case one counts how many new circles appeared from the center (the path difference increased) or how many circles collapsed in the center (the path difference decreased). Since the ray **1** crosses the plate **C** twice (at 45°) while the ray **2** does not cross it at all, the original path difference of the two rays is about 2.8t where **t** is the thickness of the plate **C**. The precision of measurements increases when the initial path difference of the two rays is very small. To achieve this, one has to compensate the extra path of the ray **1** by introducing a proper glass plate (or several plates) into the ray **2**.

IV. EXPERIMENTS

1. Soap bubbles

lab home

Background. The first investigations of soap bubbles were conducted by Boyle, Hooke, and Newton.
Equipment. Plastic cup, detergent, glycerin or corn syrup, cocktail straw.
Procedure. In a plastic cup, prepare a detergent solution, for instance, "Joy" diluted 20 times by volume with a few drops of glycerine. Moisten the walls and the rim with the solution. Make a bubble using a cocktail straw, set it on the rim, and remove the straw.
Investigate.
1. The form of colored fringes and their sequence from the top.
2. Do colors change with time? Watch the same spot on the bubble, for instance, the reflection of a window.
3. Why do colors change? Why do bubbles break? Are the two causes connected?

Conclusion. Can you prove that these colors depend on the thickness of a soap film?
Teacher's note. The soap liquid flows from the top of the bubble to its bottom because of gravity. Thus, the bubble's wall has the same thickness at equal height, and the thickness increases downwards. That is why the colored fringes are horizontal circles. A small bubble has thick walls and does not produce colors.

2. Colors of "mixed plates"

lab home

Background. Young discovered that two glass plates which held between them a mixture of two different substances, water and air, for instance, produced what he called the *colors of mixed plates*. He supposed that these colors resulted from interference of light passing through different media, i.e. water and air, and their path difference was determined by the difference in velocity of light in these two media. After assuming that light travels faster in air than in water, he confirmed this theory by experiment.
Equipment. Glass plates, water, vinegar, corn syrup.
Procedure. Wet two glass plates and press them together.
Investigate.
1. Describe the shape of colored fringes and their sequence when you look at them in reflected light. Repeat the experiment with other liquids.
2. Mark a point on the plate. Watch the color at this point in reflected light, then move the plates without changing their orientation in space so as to see **through** them. Did the color change?
3. Do colors change if you squeeze the plates or look at them at a different angle?

Conclusion. What is the main difference between the colors of thin and mixed plates?
Teacher's note. Those colors are much wider than the colors produced

by the air film. Squeezing the plates should not change these colors.

3. Two-slit interference

lab

Background. This most famous Young's experiment was very briefly described in his *Lectures* (1807). The diagram provided (Fig. 10.10) did not clarify much. Lack of any details about the experimental arrangement led some historians to suggest that it was a thought experiment introduced to explain the principle of interference rather than a real one. Indeed, the hyperbolic lines of maxima and minima in the upper part in Fig. 10.10 remind one of similar lines in another diagram by Young drawn to illustrate the interference of water waves coming from two centers, and which certainly was constructed on the basis of his theory rather than experiment (Fig.10.11) Evidence has been found, however, that Young did perform this experiment, although perhaps not exactly in the way described in physics textbooks, which show a pinhole before the slits and a white screen behind them. He may have used the **direct observation** method, that he had applied in other experiments: holding the slits near the eye and looking through them at a candle flame a few

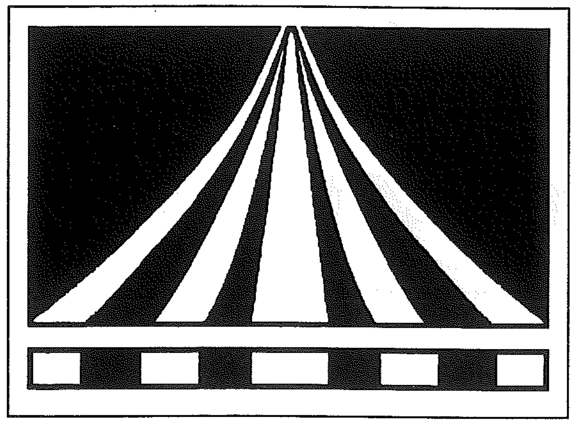


Fig. 10.10. Two-slit interference. From T. Young, *Lectures*, v.1, Fig. 442.

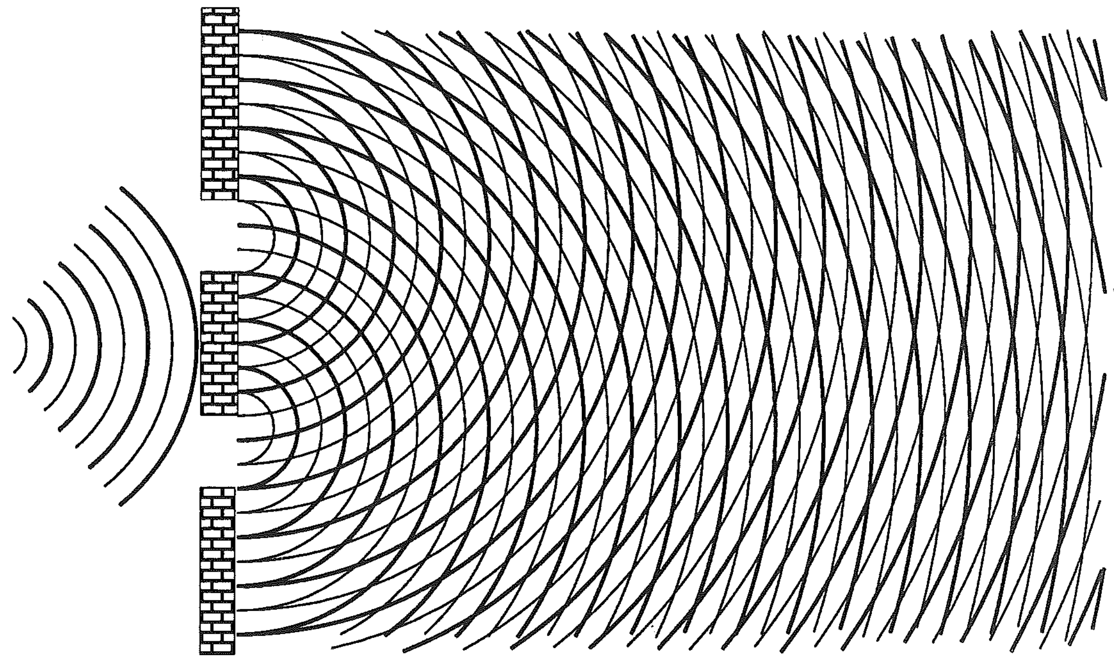


Fig. 10.11. Interference of two water waves , From T. Young, *Lectures*, v.1, Fig. 267.

meters away. It also appears that Young did not realize that the fringes he observed with two slits were very different from those obtained with a single slit (see Ch.11).

Equipment. Two microscopic slides, candle, desklight with a housing, V-slit made out of an index card, pins, razor blades, pieces of wood or cardboard, short rulers, masking tape.

Procedure. To make a V-slit, hold a microscopic slide over a candle flame and deposit soot on the middle part of it. Place the slide **1** (Fig. 10.12) on a table between two supports **2** made of wood or cardboard that are taller than the slide. To prevent the slide from moving while you draw on it, press the supports to it and tape them to the table. Take a fine razor blade and with the aid of a ruler **3** draw a line on the sooty surface. Then, without changing the ruler's position draw another line: a slight change in the position of your hand is sufficient to produce two lines crossing at a very small angle (the intersection point should be visible). At some distance from the two lines draw a single line: you will need it in some experiments where you will compare the phenomena produced by a single slits and by two slits. To make a double slit take two razor blades, separate them with a single layer of masking tape, and tape them together. Using this double razor draw the lines as described above. When a slide with a V-slit or a double slit is ready, wipe the soot at two opposite edges to free a margin about 5 mm wide, and cover it with several layers of masking tape. Place another slide (a clean one) on the top of the first and tape the slides together. This will preserve your slits from destruction. Use a magnifier with a scale to measure the distance between the lines.

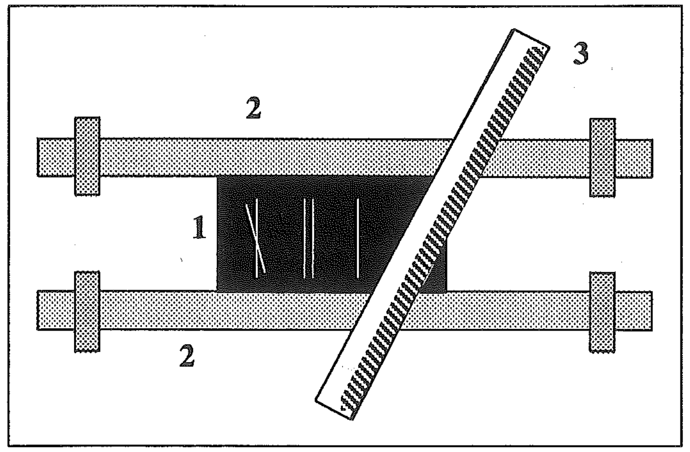


Fig. 10.12. Making double-slits and V-slits (view from above).

Investigate.

1. Look at a distant source of light (a candle flame or a filament of an electric bulb) through a double slit (Fig. 10.13). Describe the fringes.
2. Now look through a single slit. Do you see any difference?
3. Move your eye to and from the vertex and away from it and describe the change. How would you go on establishing the cause of this change?

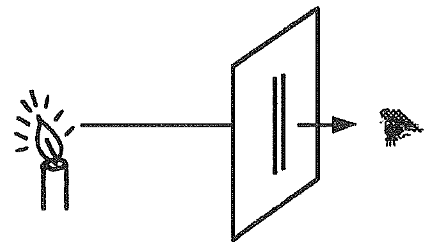


Fig. 10.13. Direct method of observing the two-slit interference.

Conclusion. What is the main difference between the colors produced by a single slit and a double slit?

4. Michelson interferometer

Equipment. Interferometer, laser, air cell, vacuum pump, microscopic slides, ground glass, paper tissue.

lab **Procedure**. Set a laser and the screen **T**, as shown in Fig. 10.9. Direct the laser beam at the middle part of the plate **C** so as to obtain two rows of images on the screen **T**. Using the screws adjust the mirror **B** so that the two rows coincide. At this moment you will see fringes. If you don't see them, repeat the process. It may be helpful to reduce the beam's brightness by placing near the laser's exit window a ground glass or a piece of toilet tissue.

Investigate.
1. Press the base of the instrument with a finger and check whether this affects the fringes. Is there a correlation between the amount of pressure and the shift?
2. Place a microscopic slide perpendicularly to the ray **2** and slowly turn it around the vertical. Describe the change. Suggest a reason for the change and test your hypothesis.
3. Insert several glass plates into the path of the ray **2**. How can you determine whether you increased or reduced the path difference?
4. Measure the index of refraction of the air. Evacuate the cell and place it on the cell support between **C** and **B**. Readjust the mirrors if the fringes disappeared. Mark a dot on the screen, open the valve so as to let the air in **very slowly**. Count the number of rings passing by the dot. Calculate the index of refraction (the length **L** is shown on the cell).

Conclusion. How great a length change can you measure with this instrument?

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Chapter 11

DIFFRACTION of LIGHT

I. EARLY STUDY OF DIFFRACTION

1. Is there light inside shadow?

discussion

Teacher. By definition, the rectilinearity of light means that in a uniform medium the rays of light don't bend. This means that light cannot penetrate shadow. Right? Have we actually tested this?

David. No, we didn't look for light inside a shadow. But isn't this a contradiction in terms? I guess that "shadow" is **defined** as the zone inaccessible to light.

Teacher. You are right, I was not precise. By "shadow" I meant the so-called "geometrical shadow", or the area behind a body limited by the light rays coming from a **point** source and touching the body. Thus, to check whether light bends around corners we must have a point source of light.

Dorothy. Can we use a laser?

Teacher. Yes. I will aim a laser beam at a white screen and place a nail in the path of the beam. Michael, would you please approach the screen and tell us what do you see?

Michael. I see a long shadow of the nail, probably caused by stray light, which appears to be narrowing where the nail crosses the beam with its outlines becoming dim.

Mary. Doesn't this mean that light entered the shadow on both sides of it?

Teacher. It certainly does. Now, I will replace the nail with a pin. What do you see?

John. There is practically no shadow to speak of. Light reaches the middle of the geometrical shadow but it isn't uniform, for there are two narrow dark lines running lengthwise. Where do these lines come from?

Teacher. Forget the dark lines for the moment (we'll discuss them in a short while). Do you still believe that light follows straight lines?

Ruth. Perhaps what we see is an exception? Why don't we try a wider object, such as a pencil?

Teacher. Let's do it.

Dorothy. I moved a pencil all the way from the laser to the screen but there was practically no effect.

Ruth. This may imply that rays bend only around narrow bodies, or, more exactly, a deviation from a straight line is very small and hence it is noticeable only when a body and its shadow are narrow.

good point!

Michael. Sounds interesting. Can this be proven without a laser?

Teacher. Yes, this phenomenon has been known for more than three centuries.

2. Grimaldi

history and discussion

Teacher. That narrowing of the shadow was discovered by Robert Hooke in 1672, when he let sunlight into a dark room and placed in its path a round wooden body. Hooke claimed that it was a new phenomenon rather than the result of reflection or refraction. He interpreted it as the result of bending of light rays inside the shadow, which he called *deflection*. Hooke was unaware that a similar property of light had already been discovered by ***Francesco Maria Grimaldi*** (1618-1663), Professor of Mathematics at a Jesuit Collegium in Bologna. Grimaldi found that when light entered a dark room through a very small opening **AB** (Fig. 11.1) and illuminated a narrow body **EF**, its real shadow **MN** was considerably larger than the geometrical shadow **IL**. On the outside, the shadow was terminated with three colored fringes, red outside and violet inside (Fig. 11.2). These external fringes always followed the body's outlines (Fig.11.3).

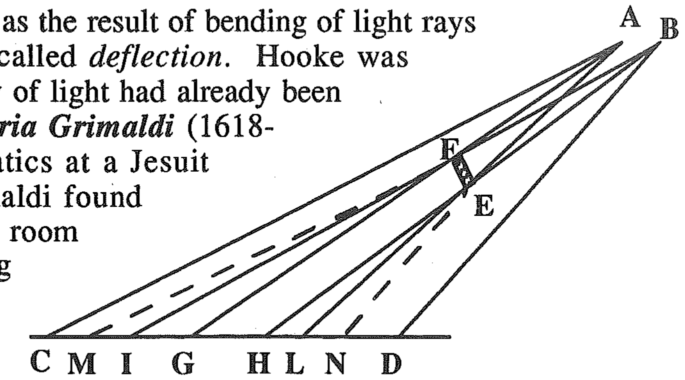

Fig. 11.1. From Grimaldi, *De Lumine* (1665).

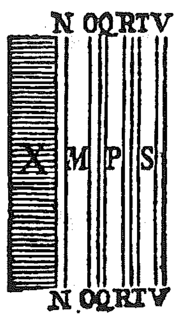

Fig. 11.2. From Grimaldi, *De Lumine* (1665).

He also found dark and light fringes inside the shadow of a narrow body which generally ran lengthwise, except for the corners (Fig.11.4). Grimaldi was also greatly surprised to find that the image of a small opening or of a narrow body produced by a sunbeam appeared to be wider than it was supposed to be for geometrical reasons. Grimaldi stated that he discovered a new phenomenon of bending light around corners, different from reflection or refraction, and called it *diffraction*. To him, diffraction consisted of a splitting of each ray passing near an obstacle into a number of rays, scattered into all directions. Grimaldi's discovery of diffraction was published posthumously in 1665 in the first chapter of a large volume on light and colors. Perhaps the size of the book scared readers off because few people knew of Grimaldi's discovery and still fewer understood its importance. An indifferent review of Grimaldi's book published by Henry Oldenburgh ,the Secretary of the Royal Society, inspired neither Hooke nor Newton to read it. Having failed to repeat Grimaldi's experiments, Huygens even questioned the validity of his discovery.

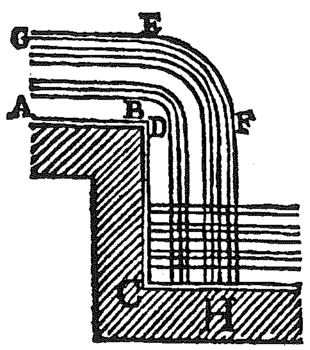

Fig. 11.3. From Grimaldi, *De Lumine* (1665).

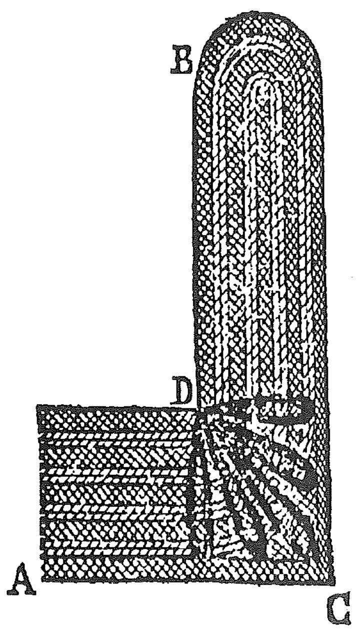

Fig. 11.4. From Grimaldi, *De Lumine* (1665).

Dorothy. How come a scientist of Huygens' stature couldn't observe diffraction?
Teacher. Well, it is not clear whether he learned about diffraction from Grimaldi's book, and not from a secondary source. Moreover, even if he did, he might have missed the some details, such, for instance, as the size of the opening in the shutter. You will see for yourself, how important this size is.
David. How did Newton respond to the discovery? He was probably aware of Hooke's work on diffraction, wasn't he?

3. Newton

history and discussion

Teacher. Yes. Newton had heard Hooke's presentation (a very brief one) in 1675, and he read something about Grimaldi's experiments in a secondary source. He agreed with Hooke that all of Grimaldi's phenomena could be attributed to bending of light rays rather than their splitting. Newton believed, however, that rays bent outward rather than inward, and called this phenomenon *inflection*. Initially, Newton stated that the new phenomenon was actually a refraction produced by a change in the density of the ether adjacent to a body. After 1687, he adopted a new explanation of the bending of light: a body repels light rays passing by, the stronger the closer they approach the body (Fig.11.5).

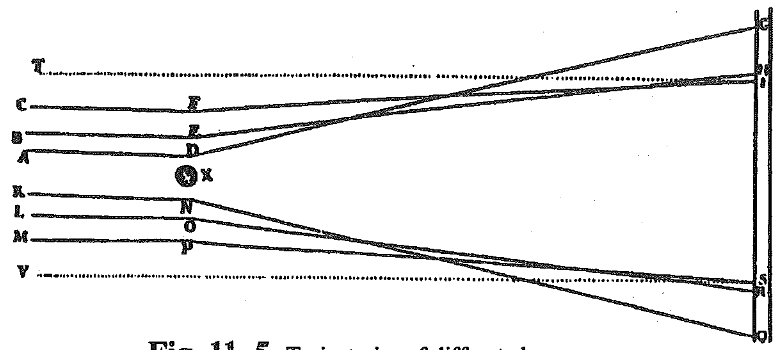

Fig. 11. 5. Trajectories of diffracted rays. From Newton, *Opticks* (1704), Bk.III.

Newton was the first to have performed quantitative diffraction experiments: he measured positions of fringes and tried to express them by a mathematical law. He was also the first to use a monochromatic light to study diffraction. Some of his measurements were very precise and physicists used them as late as the early 19th century for testing new theories. Other observations, however, were erroneous or incomplete. For instance, he never mentioned the internal fringes described by Grimaldi. Actually, since he experimented primarily with a hair, he could not see on the screen multiple internal fringes. However, it is difficult to miss a light band which runs along the hair in the middle of the shadow. It is possible that Newton chose to ignore this phenomenon because it didn't fit into his theory. Newton was the first to have found the following phenomenon: when a glass mirror with a silvered second surface was illuminated by sunlight through a hole in an opaque screen, the light reflected back to the hole displayed colored rings around it. Newton named the rings the *colors of thick plates* and explained them as follows. The rays that fall on the first surface are diffracted in different directions by particles of dust. Some of the diffracted rays travel towards the second surface where they are reflected back and are again diffracted by dust.

Newton's experiments stimulated a considerable interest in diffraction. In 1723,

Giacomo Filippo Maraldi (1665-1729), an astronomer and a member of Paris Academy of Sciences, rediscovered the internal fringes (Fig. 11.6). He found that the interval between them decreases with the thickness of the body, which explains why these fringes cannot be seen in the shadow of a hair. He was the first to use a magni-

Fig. 11.6. **Internal fringes.** From Maraldi, *Mem. Acad. Sci. Paris*, 1723.

fier to observe diffraction fringes on a screen. In 1723, he made a very important discovery by observing a bright spot at the center of the shadow of a small disk or a ball. In 1755, ***Michel Ferdinand d'Albert d'Ailly, Duc de Chaulnes*** (1714-1769), subsequently a general of the French Army and a governor of Picardie, confirmed Newton's explanation of the colors of thick plates and discovered another phenomenon similar to it. He shined a beam of sunlight on a wooden frame with a piece of gauze stretched out on it, with a metal mirror behind it, and observed colored rings

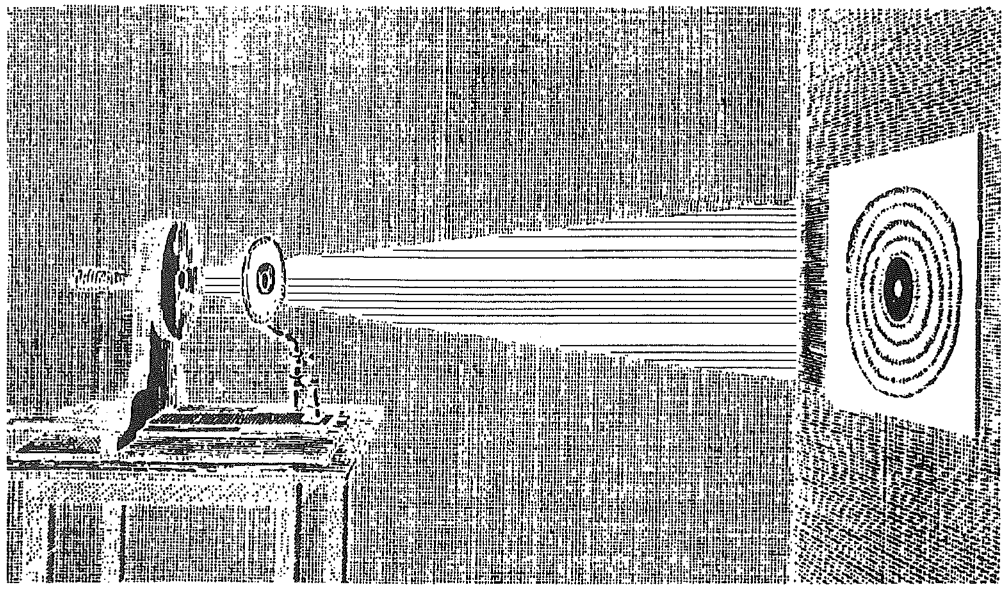

Fig. 11.7. From Duc de Chaulnes, *Mem. Acad. Sci. Paris*, 1755.

(Fig. 11.7). In his view, the fibers of the fabric diffracted light in the same way as particles of dust in Newton's experiments.

David. Why should the results of diffraction on a hair be different from those on nails and other wider objects?

Teacher. If you want a theoretical explanation of the difference, you have to wait until we discuss the wave theory of diffraction. If you doubt the existence of such a difference, see for yourself. We are going to repeat a number of historical experiments, including the experiments with various long and narrow bodies.

4. Experiments with sunlight

lab

Equipment. It is easier to run these experiments when the sun is comparatively low in the sky. Cover the windows with cardboard sheets. Cut with a razor blade several square openings of about 1 cm and cover them with aluminum foil. Make a small hole in the foil with a tip of a fine sewing needle (have one hole per group). Prepare several narrow objects: metal and cardboard strips 3-10 mm wide, nails of about 1-2 mm in diameter, wires 0.1-0.5 mm in diameter, and a V-slit. Use a floor stand with a slider to which you attach the larger objects using masking tape or reusable adhesive ("Tak"). Wires and hairs are to be mounted on a slide frame which is taped to the slider (Fig. 11.8)

Procedure. Bring the object into the sunbeam close to the hole and so as to have the whole object illuminated. Place a white screen about 30 cm from the object and examine its shadow. Move the screen back and forth and watch whether the shadow changes, both inside and outside (remember: the range of distances should be large!). The most difficult part will be to see colored fringes outside the shadow. With a naked eye you should see them at about 3 m from the object. If you cannot move your screen far enough, use a magnifier (x5 - x10), it certainly helps. Make a drawing of what you see.

Purpose. Investigate diffraction on the following objects.

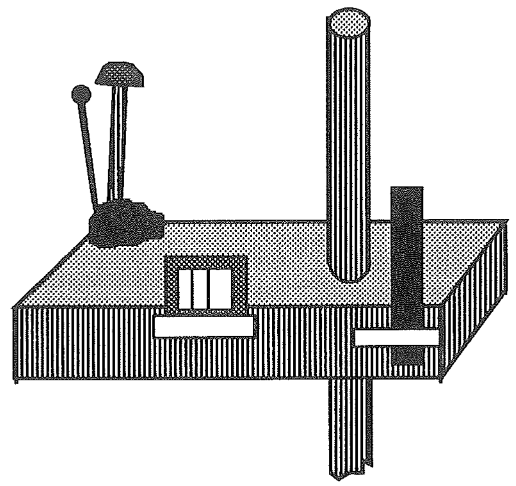

Fig. 11.8. Diffraction experiments with sunlight.

a. narrow body (Grimaldi, Newton)

Equipment. Use the stand with a slider, that carries nails, wires, hair, narrow paper and metal strips as shown in Fig. 11.8.

Procedure. In these experiments your primary concern should be with the coloration of external fringes and the number (or the width) of internal fringes.

Investigate.

1. Under what circumstances the external fringes are colored? Show in a diagram the sequence of colors.
2. How do the internal fringes change with the distance between the object and the screen (what should you keep constant)?
3. Try to establish a qualitative relation between the number of dark inner fringes and the width of the body (what must be constant?).

b. aperture (Newton)

Equipment. A V-slit is made of two pieces of an index card 10 cm long

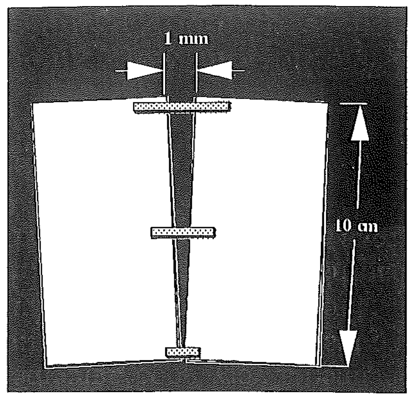

Fig. 11.9. V-slit.

taped together at the ends and in the middle so that on one end they touch one another and on the other end they are 1 mm apart (Fig. 11.9)
Procedure. Place the slit 1 m from the pinhole and shine light on it so as to include the apex.
Investigate.
1. What is the sequence of colors?
2. Is the number of internal fringes the same at different distances from the apex?
3. What is the shape of fringes at different distances from the apex?

c. "thick plate"

1. Newton's experiment.
Equipment. Concave glass mirror, pieces of nylon stocking or some other fabrics. For these experiments you need a larger opening (5-10 mm) in the window covering.
Procedure. Hold the mirror against the window and reflect sunlight back through the opening (Fig.11.10). Move the mirror back and forth until you see colored rings. If you don't see any, try reflecting light by different parts of the mirror and make note about the conditions of the parts which do produce rings.

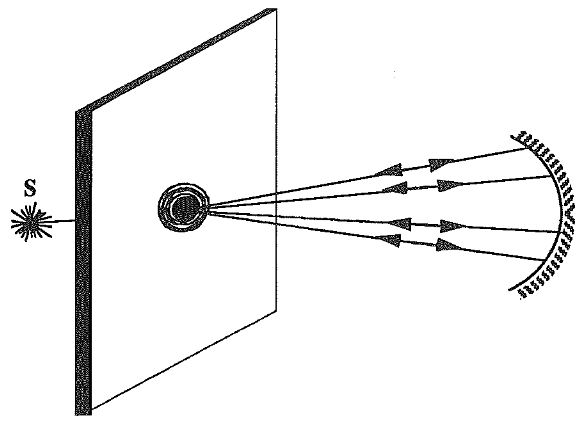

Fig. 11.10. Newton's experiment with colors of "thick plates".

Investigate.
1. Test Newton's hypothesis that particles on the mirror surface are responsible for the phenomenon. Dust the mirror with a chalk powder or other fine powder. Describe the rings observed.
2. Breath on the mirror where you see the sun spot. Why do the rings appear and vanish after a short time?
3. If the rings are produced by particles on the mirror surface, what is essential about these particles: their diameter or the distance between them? How can you test your hypothesis?

2. Duc de Chaulnes' experiment.
Equipment. A concave mirror, pieces of a pantihose and other fabrics stretched over a slide frame.
Procedure. The frame is held close to the mirror. The rest of the experiment is conducted similarly to the previous experiment.

Investigate.
1. Do the colors or the size of the rings depend on the fiber's thickness or on the distances between fibers?

d. ball (Maraldi)

Equipment. Pins with well-rounded heads.
Procedure. Place the pinhead into the sunbeam and watch the center of the shadow. Can you see any light there?
Investigate.
1. Does the shape or brightness of the light spot change if you move the screen to or from the pin?
2. Does the diameter of the pinhead affect the result?

5. Is diffraction a new phenomenon?

investigation

Ruth. O.K. We've completed the experiments and found the claim of Grimaldi and others about light entering the shadow to be correct. But how could they prove it was a new phenomenon?
Teacher. This time, I will let you experiment first and tell about their results afterwards. We are going to resolve the following problem.

Problem

Prove that diffraction is a new phenomenon, different from reflection or refraction.
Michael. What does bending of light have to do with reflection or refraction?
Teacher. Bending of light is an **interpretation** of the phenomenon rather than a fact. What you see is simply light entering a zone supposedly forbidden for it. This can be explained in different ways, and some scientists contended that it was light reflected by the edges of either the aperture or the body, or both. Our purpose is to prove that this is not the case. How would you eliminate reflection?

Hypothesis 1

Mary. A black body reflects much less than a polished one, thus if the material of both the diffracting body and the aperture is black there should be no light inside the shadow.

Test 1

John. If we take a polished nail or a pin and blacken a part of it with ink or paint, we can compare diffraction from both parts.
Dorothy. Our group found no difference between the two.
Michael. So did ours.

Test 2

David. It is difficult to get a sturdy black material in which to make a pinhole. On the other hand, only the edges of the hole are supposed to reflect. Thus, perhaps we can burn a hole in an index card with the tip of a red-hot pin?

Michael. I held a needle over a candle flame with pliers and then pressed it lightly against a card. The holes produced had black edges, and some of them were quite small. The result was the same.

Hypothesis 2

Ruth. The amount of reflected light depends not only on color but also on the size of the reflecting surface. If the diffracting body has a sharp edge, it should not produce fringes.

Test 3

John. Let us try diffraction on a razor blade.

Dorothy. We observed a razor blade producing the same phenomenon as a pencil, although the latter has a much larger reflecting surface.

Teacher. Congratulations! You follow Hooke without knowing it. Now, with reflection out, how can we test the role of refraction? I mean refraction by the surrounding medium.

Hypothesis 3

Ruth. The refraction of air depends on its temperature. Thus, heating a diffracting body may change the diffraction pattern.

Test 4

David. I've used the tip of a soldering gun as a diffracting body and found that the fringes didn't move when it became hot.

general conclusion ➡ **Diffraction is not due to reflection or refraction of light.**

II. WAVE THEORIES OF DIFFRACTION

1. Diffraction and theories of light

history

Diffraction became a major battlefield in the debate on the nature of light. Newton's chief argument against the wave hypothesis was that light does not bend around obstacles as sound does, when we hear it behind a building. In his view, the deviation of light from its rectilinear path observed in diffraction phenomena was too small to support the wave nature of light. Euler, the champion of the wave theory of light, responded to this that hearing a sound behind a building has nothing to do with it bending around corners: sound simply penetrates the walls of the building. Strangely, no one tried to test Euler's statement by direct experiment. While in modern textbooks diffraction is considered to be unquestionably a wave phenomenon, it was not so in the 18th century. Newton's theory of an interaction between light and bodies explained some phenomena of diffraction, and although it was merely a qualitative theory, the "undulationists" had nothing to offer even at the qualitative level. The situation began to change early in the 19th century due to the efforts of Thomas Young and Augustin Fresnel.

2. T. Young

a. diffraction by a hair

history

In 1802, Young discovered a new method of observing diffraction. Instead of projecting the diffraction fringes on a white screen as Grimaldi did, he observed them directly by placing the eye behind a small obstacle and looking "through" it at a small distant source of light (a candle flame). With this method (we will call it "direct observation") he was able to identify without difficulty the internal fringes of a hair missed by Newton. Young explained the colors produced by a hair by an interference of two rays of light diffracted at the opposite sides **A** and **B** of the hair (Fig. 11.11). If **M** is a bright point at a screen, **AM** and **BM** are the interfering rays. If AB«AM, **BC** is perpendicular to both **AM** and **BM**, and θ is a very small angle. Then, the path difference will be P=AM-BM=AC. If b=BN, x=MN, d=AB, then $\theta \approx \sin\theta \approx \tan\theta$, $P = d\sin\theta \approx d\theta$, $x = b\tan\theta \approx b\theta$, and P=xd/b. Since bright fringes correspond to $P=m\lambda$ (m=0,1,2,...), $xd/b=m\lambda$, and the angular radius of the m^{th} bright fringe will be

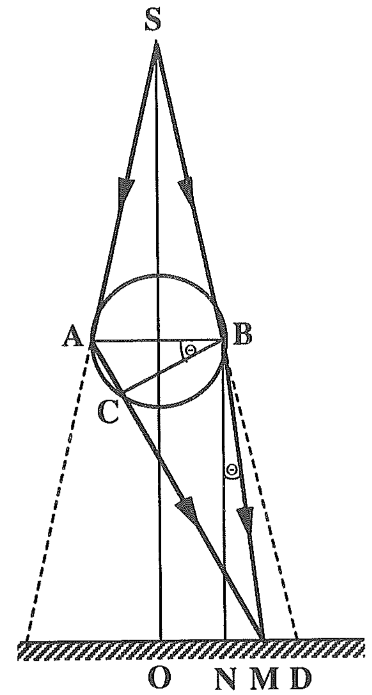

Fig. 11.11. Young's explanation of internal fringes.

$$\theta = m\lambda/d \text{ (rad)} \tag{11.1}$$

Subsequently, Young realized that he could improve the agreement of this equation with his measurements (there were discrepancies for low **m**), by assuming that the inflected ray loses one half of the wavelength, which changed the equation to

$$\theta = (m+1/2)\lambda/d \tag{11.2}$$

Young used this equation to measure the wavelength.

b. diffraction by many particles

One of the best achievements of Young's theory was its explanation of colored rings seen around the sun behind a cloud, or produced by a thin layer of wool illuminated from behind. He supposed that these rings are of the same nature as the fringes produced by a hair: both result from interference of light diffracted by the edges of a single fiber (or water globule). Young asserted that the more equal are the globules or fibers, the sharper and brighter are the rings. He confirmed this theory in experiments with various fibers, globules, and particles using a device he called *eriometer*. Young's eriometer consisted of a piece of a cardboard or metal **A** with a small hole **O** (Fig. 11.12) in the middle. The hole was a center of two circumferences the diameters of which were 1/2 inch and 1 inch, each circumference being pierced through so as to make 8 tiny holes. Young held an object C near his eye E and looked through the hole in the plate A at the candle flame D. He moved the plate A back and forth until a chosen colored ring coincided with

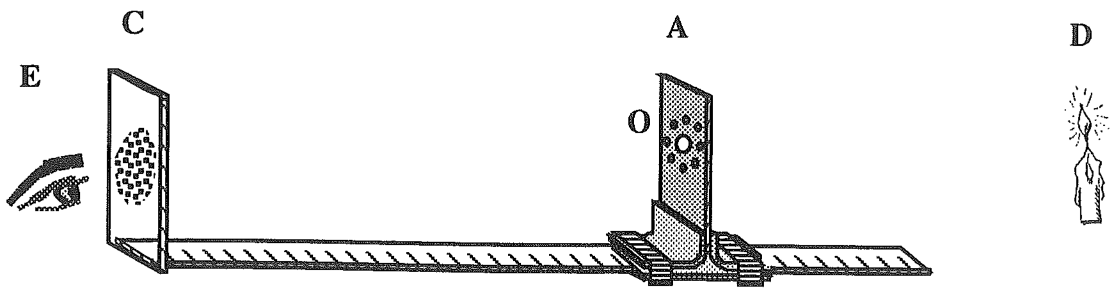

Fig.11.12. Eriometer.

either circle. After measuring the distance **b** between the object **C** and the plate A and the circle's radius **R**, he obtained the angular radius θ of a ring as θ=R/b. By comparing this result with eq.(11.2) he calculated the diameter of particles (or fiber's thickness) **d**:

$$d=(m+1/2)\,\lambda b/R \qquad (11.3)$$

3. Fresnel

history

a. "a man of genius"

Like Young, Augustin Fresnel (1788-1827) entered optics with an ambitious plan to establish the truth of the wave hypothesis of light, and he began with diffraction. He was born in the family of an architect at the North of France. As a child, he was good in math and drawing and hated languages. His health was always poor, and if other kids accepted him to play with them it was because he was very inventive. They called him "a man of genius." One of his discoveries was, for instance, determining the best dimensions of a pipe for a blow-gun. In 1806, he graduated from the *École Polytechnique* (Polytechnical School) in Paris where he received a thorough knowledge of mathematics and chemistry. Then he continued his education at the School of Roads and Bridges and became an engineer. Building roads bored Fresnel, and he was looking for a better application for his brain. He tried his hand in inventing, first in hydraulics then in chemistry, but without much success. In 1814, he turned to optics. His duties did not leave him much time for science until he obtained an unexpected break. In the spring of 1815, Napoleon returned from Elba. Fresnel, who hated Napoleon, went to the South of France to join the royalist army. By the time he reached the army he was so sick that he was turned back. However, when Napoleon seized power, Fresnel was dismissed from his job for disloyalty and exiled to his native village. It was there that he made his first discoveries in diffraction. Lacking any scientific instruments, he made a good use of his inventiveness. For instance, when he needed a short-focus lens, he made it out of a drop of honey covering a small hole in a copper sheet. To measure very small distances he invented a micrometer, which consisted of a thread and a wooden frame.

interesting!

In October 1815, Fresnel rediscovered the principle of interference and applied it to dif-

fraction on a narrow body. Fresnel's theory was the same as Young's, however the response to it was more favorable. In 1817, he entered the contest announced by Paris Academy of Sciences for the best theory of diffraction supported by exact experiments. While working on this, Fresnel discovered a new idea which helped him build a more general and exact wave theory of diffraction than Young's. Fresnel called it the *Huygens Principle*, but later it was renamed the *Huygens-Fresnel Principle* because it differs from the concept introduced by Huygens himself. According to this principle, the wavefront can be divided into small parts **Am**, **mm**$_1$, **m**$_1$**M**, **Mn**, **Nn**$_1$ each of which becomes a source of secondary waves. The intensity of light **I** at the point of observation **P** (Fig. 11.13) is the result of interference of all secondary waves reaching this point. When the number of zones increases to infinity, their width tends to zero, and the compound sum is transformed into a definite integral over the open part of the wave front. Fresnel applied this integral to a number of phenomena of diffraction and obtained a very good agreement with his observations. This impressed his judges so much that, despite their opposition to the wave hypothesis, they awarded the prize to Fresnel. Fresnel's saying was "Nature is not afraid of mathematics." He realized, however, that many people are. Thus, he developed a simplified version of his theory, called the *zone theory*. This is a beautiful theory which is simple enough for high school students.

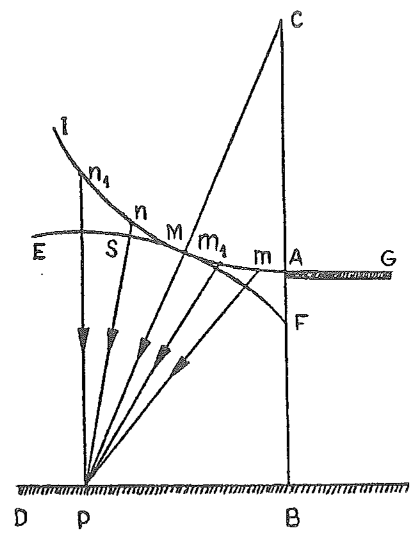

Fig. 11.13. Huygens-Fresnel Principle. After A. Fresnel, *Oeuvres*, v.1, p.174

b. the zone theory

The wave front is divided into *zones*. A zone is a part of the wave front with the origin at S cut off by two spheres with the center at the point of observation **O**, the radii of which differ by a half-wavelength (Fig. 11.14). If the radius of the first wave is b+λ/2, the radius of **m**th wave will be

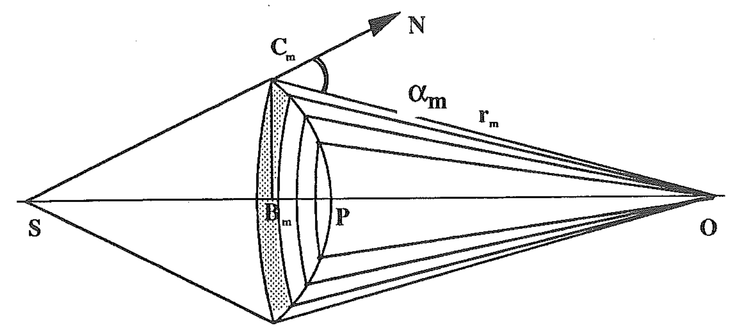

Fig. 11.14. Fresnel's zones.

$$r_m = b + m\lambda/2; \qquad (11.4)$$

math is a friend!

From the triangles **SC**$_m$**B**$_m$ and **OC**$_m$**B**$_m$ one can determine the radius ρ_m of the **m**th zone **C**$_m$**B**$_m$ approximately as

$$\rho_m = \sqrt{\frac{m\lambda ab}{a+b}} \qquad (11.5)$$

where a=SP is the radius of the wave, and b=PO is the minimum distance of the wave from the observation point. Since a path difference of one half-wavelength means an inversion of the phase, the total amplitude of vibrations **A** at the point **O** created by all zones is

$$A=A_1-A_2+A_3-A_4+\ldots; \qquad (11.6)$$

The amplitude of vibrations created at the point of observation **O** by a wave from a given zone diminishes with its area and with an increase of its distance $\mathbf{r_m}$ and the angle α_m. Fresnel studied only cases of small angles of diffraction, for which all zones have approximately the same area. This leaves only $\mathbf{r_m}$ and α_m as variables, both of which increase with the zone number. For this reason, $A_1>A_2>A_3>\ldots$, and equation (11.6) may be rewritten as follows:

$$A=\frac{A_1}{2}+\left(\frac{A_1}{2}-A_2+\frac{A_3}{2}\right)+\left(\frac{A_3}{2}-A_4+\frac{A_5}{2}\right)+\left(\frac{A_5}{2}-\frac{A_6}{2}\right)+\frac{A_6}{2}\ldots; \qquad (11.7)$$

Assuming that $A_k=(A_{k-1}+A_{k+1})/2$, we obtain

$$A=\frac{A_1}{2}\pm\frac{A_m}{2}; \qquad (11.8)$$

where "+" and "-" correspond, respectively, to an odd and even number of open zones. This means that an odd number of zones produces at **O** a maximum of intensity, while an even number of zones creates a minimum.

important

Thus, if only the first zone is open, the total amplitude is A_1, while the whole wave produces only $A_1/2$. When less than the whole first zone is open there will be a bright spot at any distance from the aperture. This leads us to the wave explanation of the rectilinear propagation of light: **almost all energy carried by a light wave is propagated within a very narrow channel connecting the source of light and the eye.** For a century and a half, the rectilinearity of light had been the prime argument against the wave theory. However, by the time Fresnel offered this theory, the debate on the nature of light began to focus on other things.

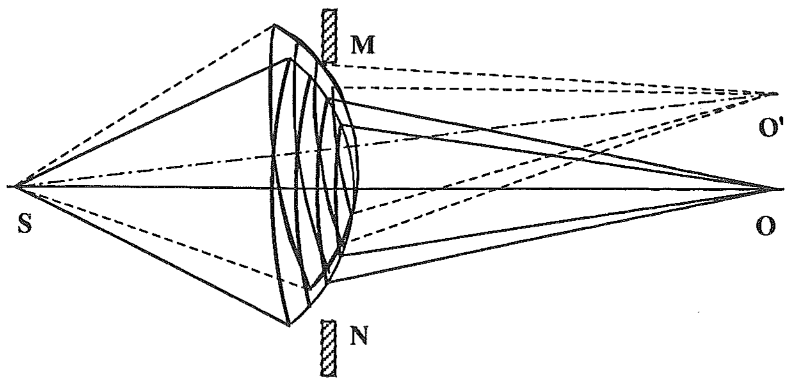

Fig. 11.15. Diffraction on a circular aperture.

Let us now imagine an aperture **MN** of radius ρ (Fig. 11.15). The number of zones **m** seen at this aperture from the point **O** can be determined as

$$m=\rho^2(a+b)/ab\lambda \qquad (11.9)$$

Thus, when the distance from the aperture to the eye increases, the center of the hole will appear alternatively bright and dark. In white light, the central spot is colored, and the colors vary according to the distance **b**. An off-center point **O'** creates another set of zones, in which the number of open half-zones in the upper and lower parts of the aperture is different. For this reason, the intensity of light at **O'** will be different from that at O, but it will be the same at all points equidistant from **O**. As a result,we have alternating bright and dark (or colored) concentric rings. If the aperture is rectangular, we will see parallel dark and bright straight lines (for convenience sake, Fresnel's zones will be made rectangular). In both cases, the number of fringes, as well as whether the central fringe will be dark or bright depends, according to eq. 11.09, on the distances **a** and **b** and the width of the aperture 2ρ.

If instead of a hole we have a a disk (or a ball) covering the first **m** zones (Fig. 11.16), at the shadow's center the total amplitude will depend solely on the amplitude of the first open zone,

$$A=\frac{A_{m+1}}{2} \qquad (11.10)$$

which means that whatever the diameter, the center of the shadow will be bright.

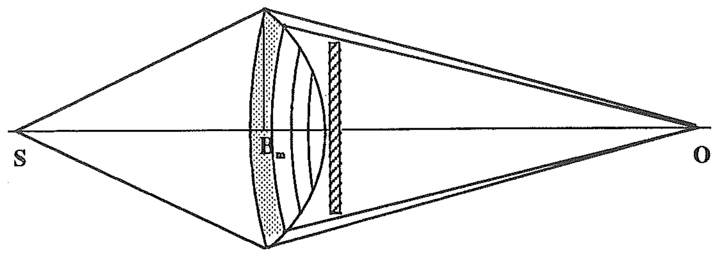

Fig. 11.16. Diffraction on a disk.

math is a friend

In his memoir submitted to the contest on diffraction, Fresnel did not discuss the case of a disk. ***Simon-Denis Poisson*** (1781-1840), a renown mathematician and one of the judges in the contest, applied Fresnel's integral to a disk and found that there must be a bright spot in the center of the shadow. With Maraldi's experiment forgotten, Poisson considered this result to be physically impossible, which was a strong objection to Fresnel's theory. However, François Arago, another judge, performed an experiment together with Fresnel and demonstrated that there was no mistake. It was Poisson's criticism that inspired Fresnel to invent his "zone theory," which made everything crystal clear and easy to grasp.

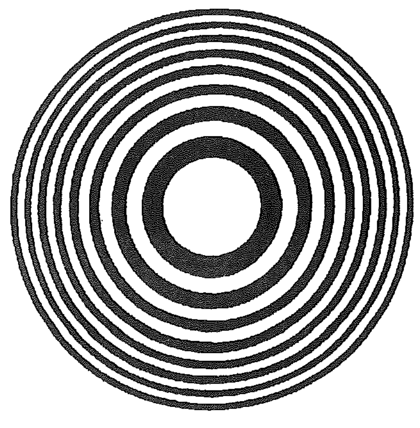

Fig. 11.17. Zone plate.

wow!

We have shown that by covering almost all of the wave front and leaving open only the first zone we actually increase the image's brightness. Therefore, **a small hole plays the role of a converging lens.** It is possible to collect even more light by covering all the even zones, for in this case the total amplitude is

$$A=A_1+A_3+A_5+\ldots. \qquad (11.11)$$

Thus, a plate (*zone plate*) made so as to let through light of the odd zones and absorb that of the even zones, will concentrate light like a lens (Fig. 11.17).

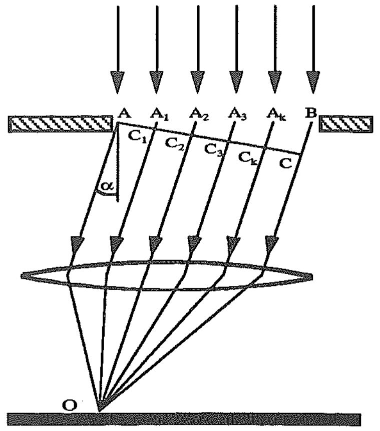

Fig. 11.18. Diffraction on a rectangular slit.

Applying Fresnel's zones is simpler for diffraction in parallel rays. For instance, when a plane wave falls on a a parallel slit **AB** (Fig.11.18), light will diffract in all possible directions. Let us select the rays which diffract at an angle α with incident rays and which are collected by a lens **L** at the point **O**. Since refraction in a lens does not change the path difference of diffracted rays, the total path difference of the two rays coming from points A and A_k of the slit **AB** equals A_kC_k, where AC_k. is perpendicular to A_kC_k. If we select the points A_1, A_2, A_3,...so that $A_1C_1=\lambda/2$; $A_2C_2=2\lambda/2$; $A_3C_3=3\lambda/2$..., then A_1A_2, A_2A_3, A_3A_4...can be treated as consecutive Fresnel zones. Whether the point **O** will be dark or bright will depend on the slit **AB** containing an even or an odd number of zones. The total number of zones seen at the angle α can be obtained by dividing the path difference BC=$a\sin\alpha$ of the extreme rays (**a** is the width of the slit) by the half-wavelength. Thus, diffraction minima (dark fringes), will be observed where the number of zones is even, which means

$$a\sin\alpha_{min}=m\lambda; \qquad (11.12)$$

math is a friend !

where m=1, 2, 3... Incidentally, diffraction maxima differ in their intensity. If, for instance, the slit contains three zones, two of them cancel the effect of one another, and the remaining zone produces the amplitude of vibrations equal one third of the maximal amplitude that is produced when there is only a single zone. Similarly, in the case of five zones, four would cancel one another, and the remaining zone would produce only one fifth of the maximal amplitude. The number of zones increases when the point of observation moves away from the center of the slit. For this reason, the central diffraction maximum is the brightest, while the side maxima, left and right, lose in their intensity the more the farther they are from the center.

Two parallel slits will produce a system of dark and bright fringes that depends both on the width of each slit **a** and the distance between them AA_1=**d** (Fig. 11.19). The direction at which a single slit produces dark fringes, according to equation (11.12) will be also the direction of minima for two slits. However, in addition to the diffraction pattern produced by interference of light from different zones of the same slit, we will have interference of light from different slits. The latter can be considered as taking place between the rays originating from such points of the two slits the distance between which is always **d** (the *corresponding rays*). The path difference of such rays will be $A_1C=d\sin\theta_{min}$. Thus there will be additional minima in the directions θ_{min} at which the rays of one slit destroy the corresponding rays of the other slit, such that

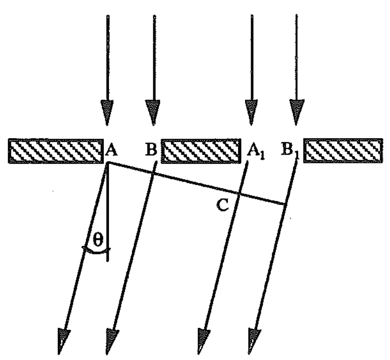

Fig. 11.19. Diffraction on two slits.

$$d\sin\theta_{min}=(m+1/2)\lambda; \qquad (11.13)$$

where m=0,1, 2,... On the other hand, there will be directions θ_{max} at which the corresponding rays from both slits reinforce each other and produce bright fringes, or,

$$d\sin\theta_{max}=m\lambda \qquad (11.14)$$

Since d>a, according to eqs. 11.12-11.14, within the central maximum produced by a single slit (or between the two minima corresponding to m=±1) there will be several maxima produced by interference of light from two slits (Fig. 11.20) The same is true when we have a set of many parallel slits, or a *diffraction grating*, only in this case the maxima are much brighter because in such a direction the resulting amplitude of vibrations equals the amplitude produced by each slit multiplied by their number.

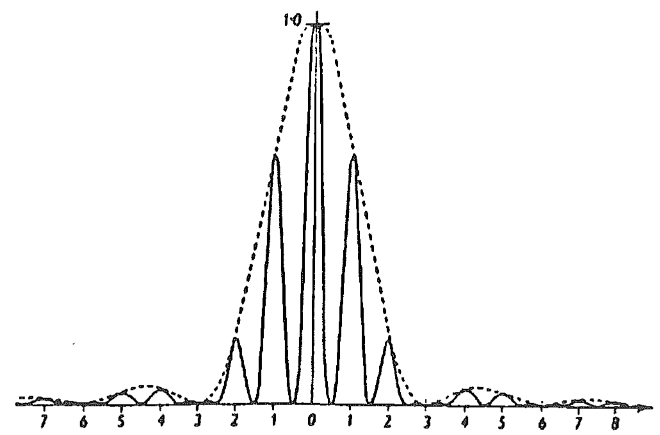

Fig. 11.20. Intensity of light distribution produced by two slits.

III. DIRECT OBSERVATION OF DIFFRACTION

1. Hair (Young)

Equipment. Nails, pins, paper clips from 0.5 mm to 2 mm thick; copper wires of different diameter from 0.5 mm to 48 gauge; hairs, slide frames for mounting hairs and wires, a candle, a tape measure, a meterstick.

lab

Teacher. In the following experiments you will study the external and internal diffraction fringes by looking through an object at a source of light of a small angular size. Investigate how the fringes' width and coloration vary with the size of an object, its shape, material, position, etc. We'll begin with Young's experiment with a hair. Mount a hair on a slide frame and holding it near the eye look at a distant candle flame. What do you see?

Dorothy. Nothing of interest.

John. I see colored fringes on both sides of the flame, several spectra on each side, red and green are visible the best.

Teacher. If some of you cannot see the fringes, move the frame across the eye so as to bring the hair exactly opposite the pupil.

Mary. I've just tried the same experiment with a paper clip instead of a hair. I saw a yellowish narrow band perpendicular to the paper clip but no colors. Why does a hair produce color fringes and a paper clip doesn't?

Michael. Perhaps metals cannot do it. I've tried a nail, and it doesn't work either.

David. I've mounted on a slide frame a few copper wires and found that a thin wire produced colors but the thick ones didn't.

Ruth. I know: it all depends on the object's thickness and not on its material.

John. If this hypothesis is true, a fine thread pulled out of a fabric should produce colored fringes as well. Indeed, it works! Ruth was right!

good point!

Dorothy. If a single thread creates colors, shouldn't the whole piece of the fabric do the same? Let's try it.

Michael. It works, but I saw two perpendicular rows of colored fringes instead of one. Strange, isn't it?

David. Not really. If you look closely at the fabric, you'll see the fibers making a neat rectangular lattice. Since a single fiber produces fringes parallel to it, perpendicular fibers should make perpendicular fringes.

Ruth. But why do many parallel fibers create only a single set of fringes?

John. I mounted on a slide frame a few thin copper wires so that they were all parallel and close to one another. I obtained, however, the same set of colors as with a single wire, only brighter.

Michael. We taped to a glass plate two hairs that were not parallel. What we've found was that each hair produced its own row of fringes which made an angle with one another.

Mary. Probably the spectra produced by parallel wires or hairs coincide, which makes fringes brighter.

Dorothy. Why should they coincide? Can't two parallel wires make two rows of fringes parallel to one another but located at different places?

David. Such two rows of fringes couldn't be spaced vertically because of the whole wire only the part equal to the size of the pupil participates in creating colors. I discovered this by placing behind the wire a small aperture: although only a small part of the wire was visible through the aperture, the fringes were the same as those produced by the whole wire. On the other hand, the two sets of fringes cannot be spaced horizontally (within the same row) either, because, according to equation (11.2), identical wires produce fringes equally distant from the flame.

Teacher. Can you now explain why a hair or a thin wire produce colored fringes while nails, paper clips, and pins don't?

Mary. We see that in the spectrum of each order the red fringe is seen farther away from the object's shadow than the green one, which agrees with equation (11.2). The same equation tells us that the thicker the object, the closer the fringes to the shadow and to one another. Perhaps, when an object is too thick the fringes of different orders overlap, and we see no colors.

Teacher. Can you prove that the same fringe is visible at the same angle whatever the position of an observer?

good point!

John. A given fringe (for instance, the red of the first order) is seen farther from the shadow when you move away from the candle, which hints at the possibility that the ratio of the two distances (x/b) is constant. To test it, we have to measure the distance of the first red fringe to the shadow **x** and the distance between the frame and the light source **b**. For a better precision, we can measure the distance 2x between two red fringes of the first order on different sides of the flame and divide it in two. Two persons are needed to do this: one watches the fringes and directs the other one where to put two white paper riders on a meterstick held near the light. Then the partners exchange their roles, each making two measurements.

Teacher. Is it possible to use Young's experiment to measure the wavelength of light, for instance, of red light?

Ruth. We can use the same equation (11.2). We already know how to measure the angular distance of the fringe from the shadow ($\theta = x/b$). To measure the diameter of the hair **d** we need a microscope with a reticle. To improve the precision we can repeat the experiment with the red fringe of the second order and take the average wavelength.

Teacher. I think you are now ready to do another Young's experiment.

2. Eriometer (Young)

lab

Equipment. An eriometer, a microscope with a reticle, lycopodium powder, a blood smear, a flock of wool, cotton puffs. The eriometer consists of three parts: an object, the measuring plate, and a ruler (or tape). An object is placed between two microscopic slides taped together. To reproduce Young's original measuring plate take an index card, draw two concentric circles, the radii of which are, for instance, 1cm and 2cm; make the central hole 3 mm in diameter and 8 peripheral holes in each circle 1 mm in diameter. Install the object in a holder at the beginning of a ruler with the millimeter scale (or attach it to the ruler with a "Tak"). Place the measuring plate in a sliding holder that can be moved along the ruler.

Teacher Hold the object **C** near the eye (Fig.11.12) and look through it and through the central hole **O** of the plate **A** at a source of light **D** of a small angular size (a candle flame or an electric bulb). You will see colored rings around the light. To measure the angular radii of these rings, move the sliding holder until a chosen color ring coincides with the perforated circle on the plate **A**. Try the red rings of the first and second orders. A candle flame should be sufficiently close to the eye (30-50 cm). To obtain a bright and distinct image, you may have to move sideways both the object (to find a better spot on it) and the plate **A**.

Dorothy. Why do we need two concentric circles at the plate **A**?

Teacher. To cover a large range of fiber thickness without moving the plate **A** too far. Young wanted farmers to use his device to measure the thickness of wool produced by different sheep.

discussion

John. How do we know that Young was right to apply his theory of diffraction on a hair in this experiment. Not only the objects here are different (a flock of wool or a smear of lycopodium, for instance), but even the shape of fringes ?

David. We know that each fiber produces fringes parallel to it, thus a number of identical fibers oriented in all directions will produce arcs say, of red color, of the same radius, which form a red circle.

Teacher. Very good! The key word is "identical." If fibers differ in thickness, they will produce arcs of different colors which overlap and destroy any coloration.

good point!

Mary. We've tried various fabrics instead of yarn and found that they produce straight parallel fringes arranged in two perpendicular rows rather than circles. The only fabric that made circular fringes was nylon stockings. Why is it so?

Ruth As seen under a magnifier, in most fabrics fibers form a rectangular lattice. That is why the result is the same as with a mesh. In nylon stockings, the fibers are curved in different directions which makes them work rather like a yarn than like other fabrics.

Michael. But why do particles produce colored circles like wool fibers?

good point!

Dorothy. Not always! Lycopodium does it, but not a chalk powder or my face powder.

Teacher. We've already noted that when seen through a small hole a fiber creates the same fringe pattern as when seen unobstructed, and we've found that only thin fibers produce colors. Thus, the length of a fiber appears to be of no importance, and cutting it into small pieces should not change the result. But these "small pieces" are nothing else as particles. To produce colored circles the particles or fibers must be: 1) small, 2) identical, and 3) randomly located. Each fiber from the stockings create the same fringe pattern, and when the fringes of the same color coincide the resulting fringe becomes brighter. But if the fibers (or powder particles) differ from one another, they create circles of different radius for light of the same color. As a result, the circles of different colors overlap and produce white. As to the last condition, it can be proven, for instance, by demonstrating that changing distances between fibers doesn't affect the result. Do you know how to do it?

John. I increased the distances by stretching a stocking, but the circles didn't change.

David. I mounted a piece of a stocking on a slide frame. First, I observed circles by holding the frame perpendicularly to the visual line. Then I tilted the frame so as to make a sharp angle with this line. The circles remained the same although in the latter case the distance between the two next fibers, as projected onto the original plane of the frame, certainly decreased.

Ruth It is now clear to me why Young used the equation derived for a single hair in the experiment with a flock of wool: it is a single fiber which is responsible for the angular size of the fringes, the multitude of fibers only improves the fringes' brightness.

Teacher. Very good! Moreover, Young extended this theory to **transparent** particles as well. In his view, colored rings around the sun or the moon hidden behind a cloud are due to diffraction of light on water drops in the cloud.

3. Fresnel's method

lab

Dorothy. Is Young's method of viewing diffraction fringes directly better than Grimaldi's observation of the fringes on a white screen?

Teacher. In certain cases, yes. For instance, you saw more fringes inside a hair's shadow even in a weak candle light, and you didn't have to move the object following the sun's movement. Your were able to perceive a weaker light because this light entered the eye directly without losing much of its intensity while being reflected from a white screen. Fresnel improved the direct method still further by adding a lens before the eye. We will use Fresnel's procedure to solve a practical problem: to recreate classical diffraction experiments when sunlight is not available. Upon the completion of the experiment we will discuss your results.

Equipment.

a. Take as a source of light the "light-house" described above, which consists of a base-mounted light fixture surrounded with a square cardboard house and covered with a cardboard lid. The electrical bulb must be **clear,** of 40 or 60 watts. In one of the walls make a square window of 1x1 cm at the level of the filament. Take a piece of aluminum foil of 3x4 cm, place it on cardboard, punch a tiny hole in it with the tip of a straight pin (if the pinhole is large, you won't see much of diffraction), and tape the foil to a slide frame and the latter to the window. For home observations use any desk lamp with a housing open on one side: cut a cardboard lid slightly larger than the opening, make a window in its center, and tape the lid to the housing.

b. For a narrow object use human hair and wires taped to a slide frame; straight pins with spherical heads, paper clips, nails of 1 to 4 mm in diameter, toothpick, strips of aluminum and of an index card 2 to 4 mm wide, both rectangular and triangular. For an aperture, drill a circular hole of 1 to 2 mm in diameter in a thin aluminum sheet and polish its edges, or punch a hole with a straight pin in an index card. Make aV-slit of two pieces of an index card taped together.

c. To carry objects use a lens-holder with a flat top (Fig. 11.21): tape the strips and slide frames to the top's side and stick pins or nails into a ball of "Tak" (reusable adhesive). Use another lens-holder to support a lens with the focal distances between 2 and 5 cm.

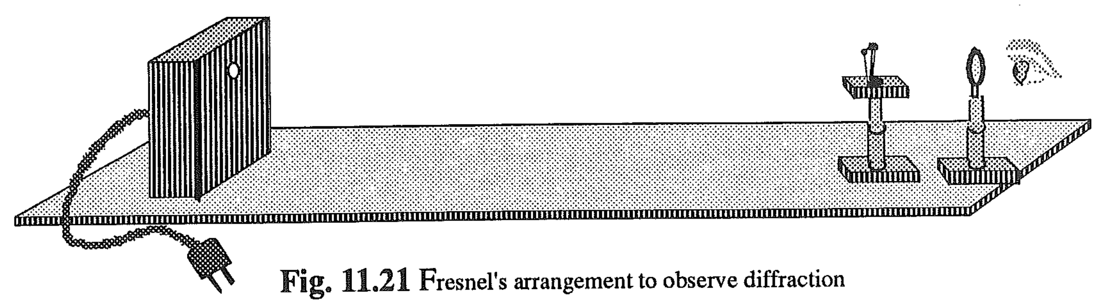

Fig. 11.21 Fresnel's arrangement to observe diffraction

lab

Procedure.
Position yourself at the narrow side of the table and place the "light house" 1.5-2 m from the eye. Move your eye and turn the "light house" until you see the pinhole very bright. Place the lens very close to the eye and adjust it and the eye so as to see a lighted **gray** (not yellow) circle covered with dots, specks, and lines: this is the image of the inside of your eye. Set the object-holder about 30 cm from the lens and adjust it so as to obtain a clear shadow of a pin crossing the circle: if you see parallel lines running lengthwise inside the shadow, your apparatus works fine. In each experiment draw the picture of what you see (use colors if necessary) and compare your diagrams to the original ones. We will check whether such *variables* as the **width** of a body and its **distance** from the lens, so important in the Grimaldi's procedure, retain their significance when using Fresnel's method.

a. narrow body (Grimaldi, Newton, Maraldi)

Watch the fringes produced by long and narrow bodies, such as paper or metal strips, nails, hairs, wires, paper clips. How does the distance between the fringes depend on the body's width and its distance from the lens? Can you make the fringes colored? **Tip**:a nail should be moved much farther from the lens (increase the distance between the light source and the lens), and if this doesn't help, try a smaller pinhole.

b. V-slit (Grimaldi, Newton)

Attach a V-slit to the holder's top vertically and move the eye before the lens up and down so as to look through different parts of the slit. Does the number of fringes depend on the width of that part of the slit you are looking through? Does this number change when you move the slit to and from the eye? Can you produce colored fringes?

c. ball (Maraldi)

This is one of the most impressive optical phenomena illustrating the limitations of the everyday experience. Move the holder carrying a pin with a well rounded head away from the lens and watch the center of the head's shadow. Can you see there a small bright spot? Does the spot change when you move the pin to or from to he lens? Does the spot vary with the size of the head or its shape?

d. small aperture (Fresnel & Arago)

This experiment deserves special attention because of its historical importance for the wave theory. Observe colored rings inside the hole. Investigate how their number varies with the distance between the hole and the lens. Can you obtain at the center a dark spot ? a colored spot? Does this color change with the distance? Do the rings change with the diameter of the aperture?

Teacher. Let us discuss the results starting with a narrow body.

Mary. When investigating the *distance* as a variable, we found that the number of fringes inside a pin's shadow decreased to two when the distance increased, and we adopted this result as a *hypothesis*.

Michael. We *tested* this hypothesis with nails and paper clips and confirmed it. Big nails didn't show any inner fringes, only outer fringes.

discussion

Dorothy. We've tried various paper strips: no success.

John. Apparently, the *conclusion* is that the number of visible inner fringes diminishes as the distance between the body and the lens increases. This agrees with the results of similar experiments with sunlight. What I cannot understand, however, is why no fringes can be seen at small distances?

David. I guess I know: at small distances the intervals between the fringes become two small to see them separately. Can this be explained in the zone theory?

good point!

Teacher. Yes, but before answering this question, let us summarize your findings about the second variable, the body's *width*.

Dorothy. It was clear from the previous experiments that only very narrow bodies deserve attention. Thus, we used a straight pin and a small nail. However, the nail didn't show any fringes.

Ruth. When we ran into a similar problem, we recalled that the fringe's width increases with the distance and moved the holder 1 m from the lens: there we observed the fringes in both the pin and the nail. In the pin's shadow, the fringes were wider than in the nail's, thus we supposed that the thinner the obstacle the wider the inner fringes. We tested this hypothesis with a paper clip and a bigger nail: the results were the same.

John. For our test, we've selected a triangular paper strip of 2 mm at the base and 5 cm tall. At its tip, the inner fringes were wide but towards the base they became more and more condensed until coincided. Thus, the hypothesis is correct.

good point!

Teacher. According to our theory, the central fringe must be always bright. The first dark fringe will be where the first open zone of one part of the wave (say, to the left from the body) is odd, while on the right side the first open zone is even, next to the odd one. It means that the path difference of rays from the edges of the body at the first dark fringe will be $\lambda/2$. Using considerations similar to those on p. 217, one can show that the width of an inner fringe is proportional to the distance from the lens (the light source is far away) and inversely proportional to the body's width.

math is a friend!

Michael. With a V-slit, the result was different from that with a triangular strip: apparently the same fringes run along the slit becoming wider where the slit is wider. The central fringe could be bright or white depending on the distance from the lens. How come that a dark fringe appears against the center of the slit?

Teacher. The two halves of the slit act identically, thus if one half contains an even number of zones, they will destroy light of each other. To calculate the number of zones, take the path difference for the rays coming from the slit's center and one edge and divide it by $\lambda/2$.

Dorothy. What if the path difference is less than $\lambda/2$?
David. This may happen only at a large distance, and the result will be a bright fringe. Incidentally, if we move the slit farther away, the path difference becomes even smaller. This means that at large distances there is only a single zone within the slit, and the central fringe will stay bright whatever the distance.
Teacher. Very good! How about the shadow of a round pinhead?
John. Maraldi was right: we did observe a tiny light spot in the center of the shadow. However, a larger head did not produce this effect. Why?

good point!

Ruth. According to the theory, the intensity of light at the center depends on the first open zone. However, zones of higher orders contribute less light than the zones of low orders. A large head covers many zones, and the light coming from the rest is too weak to excite the eye.
Teacher. Excellent! Did a circular aperture act similarly to a sphere?
Mary. Not at all! The central spot was either dark or colored, red and green were especially clear. There were also several concentric rings, dark in the proximity of the lens, and colored, at larger distances.

good point!

David. Well, we've seen colored fringes with other objects too, although they appeared only at large distances and were fainter.
Michael. Is it correct to say that at small distances fringes of different colors fully overlap and produce an impression of gray?
Teacher. Yes. And who knows why was the center differently colored at different distances?
Ruth. The number of zones depends on both the wavelength and the distance (eq. 11.9).
Teacher. Good! As you see, the zone theory is a powerful tool for explaining diffraction phenomena.
Dorothy. How about regular reflection? You had promised us to explain in the wave theory why an uneven surface can reflect as a mirror.
Teacher. The ideal surface reflects in a single direction because in all other directions the rays reflected by different particles of the surface destroy one another through interference. If the height of the "hills" is less than $\lambda/4$, even at normal incidence the rays reflected by neighboring particles will have the path difference less than $\lambda/2$, and no light reflected at the angle of incidence will be destroyed.

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Chapter 12

POLARIZATION of LIGHT

I. DOUBLE REFRACTION

1. Polarizing sunglasses

Preliminary observations

investigation

Dorothy. Michael, your sunglasses are rather light, do they work well?
Michael. They surely do: these are polarizing sunglasses.
Dorothy. But how it be that your light sunglasses act the same as my dark ones?
John. They don't act the same. I have here both types, and you can try and see that polarizing glasses cut a lot of light from the blue sky and from a water surface but they don't make the ground as dark as the ordinary ones.
Mary. Indeed. How can this be? I know that ordinary sunglasses absorb much of the incident light, which reduces the amount of reflected light (because of the conservation law). That is why such glasses appear dark to an outside viewer. But polarizing glasses must work in a different way because they are not as dark.
David. Maybe they aren't as dark as some absorption glasses but they certainly absorb some light. The main difference must be not in absorption.
Ruth. Some glasses transmit less because they reflect a great deal: they are semi-transparent mirrors. However, polarizing sunglasses don't look like mirrors from outside. But if they neither absorb nor reflect an excess of light, how do they cut the glare?
Teacher. To get a clue to this problem let us do the following experiment. Michael, put your polarizing glasses on! John, hold your polarizing glasses before your eye and look at Michael's eyes! What do you see?
John. I see his eyes, not as well, of course, as without glasses.
Teacher. Now, John, rotate your glasses by 90° in their plane. What do you now see?
John. Wow! Now, I don't see his eyes. How can this be?
Dorothy. I tried the same trick with ordinary glasses but it didn't work: I saw Michael's eyes equally dim in both positions of my glasses.

Formulating a problem

How do polarizing glasses work?

Hypothesis.

David. Apparently, the orientation of polarizing glasses in the plane perpendicular to the line of vision is important in cutting the glare.

Test.

Michael. Let us watch various shiny objects through polarizing glasses while rotating them around the visual line.
Teacher. Good idea! Since only a few of you have polarizing glasses, I'll give you pieces of the same type of a polarizing film as applied in

your glasses, that is placed between two plates of glass or plastic. We'll call such a device *polaroid.*

Mary. We've found that by rotating a polaroid we made the sky appear brighter and darker. The pavement didn't change its brightness, however.

John. The images in the pool changed their brightness twice every full revolution.

Dorothy. We've observed the same with car windows but not with the car fenders.

conclusion There is a specific orientation of a polarizer relative to the visual line at which it transmits more light reflected from certain objects than in the perpendicular orientation.

Michael. Well, we've found that there is a material whose capacity of transmitting light depends on its orientation in space. Are there other materials of this sort?

David. It's not just the material that produces the effect! Have you noticed that neither the pavement nor some parts of car bodies changed their appearances? There should be something different about the light reflected, for instance, by car windows or car bodies.

Teacher. You are right: both the material and the properties of reflected light are essential for this phenomenon. That some materials refract light differently from others was first discovered in a transparent calcite known as *Iceland spar.*

2. Iceland spar

a. Bartholin's experiments

lab

Teacher. In 1669, ***Erasmus Bartholin*** (1625-1698), Professor of Mathematics at the University of Copenhagen, found that an Iceland spar made all objects seen through it double. Let us repeat some of his experiments. Take an Iceland crystal and view different objects through it; if you see them double, investigate the factors affecting the degree of their separation.

Dorothy. I see only the edges of bodies double. Letters, however, do appear double, but it is difficult to see how much they are separated.

David. Among the objects we've tried pencil dots were the best, because the distance between them is larger than the dots themselves. The distance between the images increased with the thickness of the crystal.

Ruth. While rotating the crystal relative to the vertical axis we've seen that one image stood still and the other moved around it. Besides, one image was brighter than the other.

John. When I looked along a diagonal the two images almost coincided, perhaps even exactly.

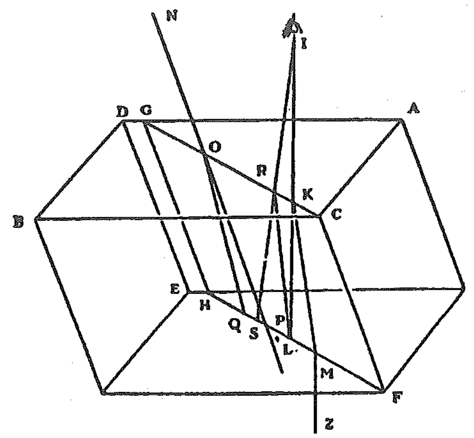


Fig.12.1. From Huygens, *On Light*

Teacher. You have obtained Bartholin's principal results. The direction in which the images coincide is called the *optic axis* (CG in Fig. 12.1). Bartholin concluded that the two images resulted from a different refraction of light. He called the ray that forms the immobile image *ordinary*, because it obeys the law of refraction, and the one that produces the moving image *extraordinary*, because the index of refraction of this ray depends on the direction of incident light.

history and discussion

b. Huygens' theory

In 1678, Huygens presented a detailed account of the properties of Iceland spar, which he published in 1690 (*On Light*, ch. V). He found a specific plane **CFHG** in the crystal, called *principal section*, which retained the incident ray and the two refracted rays for different angles of incident. This plane is parallel to the plane formed by the edge **CF** and the line **CG** bisecting the obtuse angle **ACB**. In the principal section, the incident ray **IK** that is perpendicular to **CG** produces the ordinary ray **KL** and the extraordinary ray **KM**, which form the angle 6°40' between them. Huygens suggested that the extraordinary ray behaves so strangely because its velocity depends on the direction of its propagation in the Iceland spar, while the velocity of the ordinary ray is constant. Since the index of refraction of a medium depends on the velocity of light in this medium, the index of refraction of the extraordinary ray must depend on its direction. According to Huygens, such a phenomenon can be explained in the wave theory by assuming the existence of two types of light waves, spherical and spheroidal. He represented the ordinary light by a spherical wave and the extraordinary light by a spheroidal wave. In the plane of the drawing, the ordinary wave front is represented by a circle and the extraordinary one by an ellipse (Fig. 12.2). The radius drawn from the center C to a point at a wavefront shows the velocity of light in this direction. We see thus that the ordinary wave has a constant velocity, while the velocity of the extraordinary wave depends on the direction of propagation. The two wave fronts touch in points B and S , which means they have the same velocity in the direction BS, or, in other words, **BS** is the direction of the optical axis. To describe the propagation of the extraordinary wave Huygens uses the same principle as for the ordinary wave, with that difference, though, that the secondary waves are semi-spheroids **SVT** rather than semi-

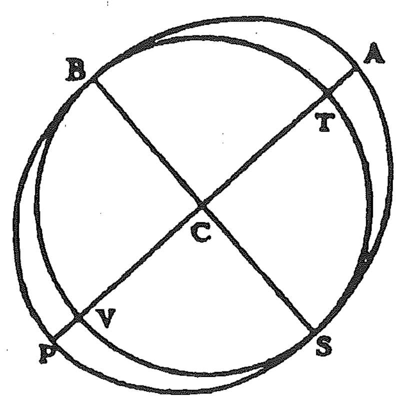


Fig.12.2. From Huygens, *On Light*

spheres (Fig. 12.3). If the incident wave front **AB** is a plane, the extraordinary wavefront **NQ** remains a plane parallel to AB. However, NQ is shifted to the left relative the direction of propagation of light **RA**, which means that the extraordinary ray AV deviates from this direction. On the basis of his theory, Huygens calculated the index of refraction of the extraordinary ray for different directions of incidence and stated (without giving any numerical data) that his theoretical predictions fully agreed with experiment.

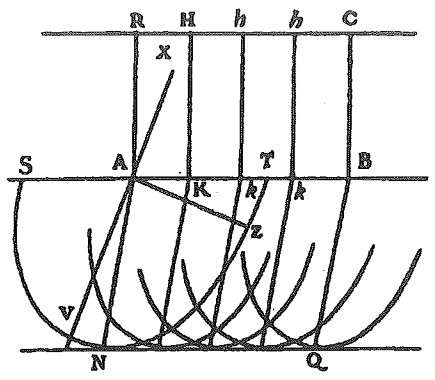


Fig.12.3. From Huygens, *On Light*

Dorothy. This is all interesting, but what does it have to do with polarizing glasses?

Teacher. I am coming to that. Once, Huygens looked at an object through two Iceland crystals and found to his surprise that the number of images varied, depending on the relative position of the crystals. Let us repeat this experiment.

c. Huygens' experiment

lab

Equipment. Two Iceland crystals, index card, scotch tape.

Procedure. Cut a piece of an index card of the size of a side of the crystal, make a 2 mm hole in the middle of it, and attach it to the crystal with scotch tape. Look through the crystal at a light source or a brightly lit surface (the cover must be on the farther side from the eye), and you will see two holes. Place another Iceland spar closer to the eye and slowly rotate it around the line of vision.

John. In most cases, we've observed four images making a parallelogram. The rotation changed their brightness.

Ruth. Apparently the images located on the same diagonal make a pair because they change their brightness at the same time. When one pair is getting brighter the other goes dimmer and the vice versa, and these changes occur four times per revolution.

Mary. We have seen two images, apparently when the principal sections of the crystals were parallel or perpendicular to one another.

history and discussion

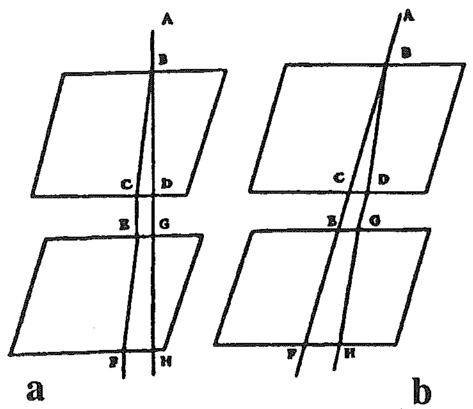


Fig.12.4. From Huygens, *On Light*

Teacher. According to Huygens, two images instead of four appear when the rays emerging from the first crystal lose the ability to split into two other rays in the second crystal. In particular, when the principal sections of the two crystals are parallel, the ordinary ray is transformed into the ordinary one, and the extraordinary ray into another extraordinary one (Fig. 12.4a). On the other hand, when the principal sections are perpendicular, the ray that was the extraordinary in the first crystal becomes the ordinary ray in the

second one, and vice versa (Fig.12.4b). Huygens concluded that both the ordinary and extraordinary rays had a new property absent in natural light, but he could not specify it. (*On Light*, 92-94)

It was Newton who made the first attempt to explain this property. In his view, the transformation of the extraordinary ray into the ordinary one solely by turning the second crystal around the direction of light implies an asymmetry in a light ray relative to this direction. He attributed to a ray two pairs of different sides whose positions relative to the principal plane determine whether the ray will be refracted ordinarily or extraordinarily. (*Opticks*, 358-60)

history and discussion

Dorothy. How can a line have "sides?"

Teacher. I've already mentioned that when speaking of rectilinearity of light not all scientists compared a light ray to a geometrical straight line: in the Middle Ages, some of them attributed to a ray transverse dimensions (the *physical ray*). Since such a hypothesis complicated the explanation of many phenomena, scientists tried to avoid it. However, Newton realized that one **cannot** apply the concept of a light ray to Huygens' experiment without assuming that the ray has *sides*, or, that it resembles a rectangular rod with unequal sides (such as a meterstick) rather than a straight line. Imagine a meterstick being pulled through a lattice made of parallel wires the distance between which **d** is larger that the thickness **a** of the meterstick but much smaller than its width **b** (Fig. 12.5). The meterstick will pass easily between two wires only when its narrow side is parallel to the wires or forms a small angle with them. When this angle increases the wires give way, but their friction will slow the passage of the meterstick. Finally, when the angle approaches 90° the friction is too great to let the meterstick through. This may serve as an analogy to the transfer of light by the second crystal, assuming that the amount of emerging light corresponds to the amount of energy of the moving meterstick transmitted through the lattice.

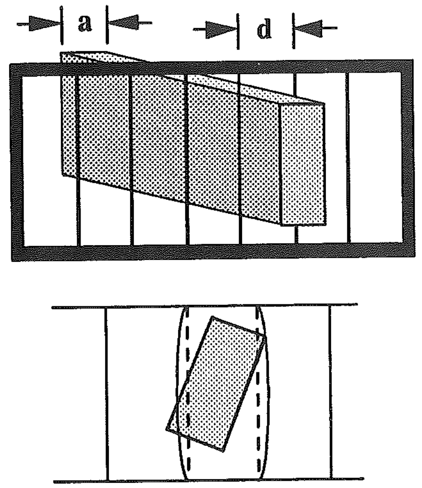


Fig.12.5. A corpuscular model of polarization.

Neither Huygens' theory of double refraction (or *birefringence*) nor Newton's concept of *sides* received much support, and by the end of the eighteenth century both had been practically forgotten. Their revival began in 1802 with experiments of ***William Wollaston*** (1766-1828). Wollaston invented a new method of measuring the index of refraction and applied it to a variety of, substances, including Iceland spar. He knew nothing about double refraction and was surprised to find the index of refraction of Iceland spar depending on the direction of incident light. When he mentioned this fact to his friend Thomas Young the latter referred him to Huygens' theory. Wollaston found his measurements in agreement with Huygens' theory and pronounced this theory to be true. ***Pierre-Simon Laplace*** (1749-1827), a leading French mathematical physicist and a supporter of the emission hypothesis, didn't like Wollaston's conclusion about Huygens' theory, and was looking for someone able to verify it. This role was left for a young military engineer ***Étienne-Louis Malus*** (1775-1812).

II. POLARIZATION BY REFLECTION AND REFRACTION

1. Malus and his discovery

history *and discussion*

Teacher. In 1794, a new institution was founded in Paris named *École Polytechnique* (Polytechnical School). Students entered this school at the age of 16 after passing very difficult oral exams and studied there for two years. The main emphasis was on mathematics (among the professors were Monge, Lagrange, Poisson, Lacroix, and Legendre). After graduation, a student could join the army (artillery) or to enroll an engineering school, either civilian or military. Malus was in the first class of the *École Polytechnique* and then became a military engineer. In 1798-1801, he took part in Napoleon expedition to Egypt and Syria (1798-1801), during which he almost died from the plague. It was during his recovery there that he started to think about the nature of light and other optical subjects. Subsequently, Malus served in Strasbourg but he frequently visited Paris where he contributed two optical papers to the Academy of Sciences. Laplace highly regarded these articles and decided that Malus possessed all the necessary skills as experimenter and mathematician to solve the problem of double refraction. Thus, he suggested that the Academy made this topic the subject of its next biannual mathematical contest in 1808. The program of the contest required to build a mathematical theory of double refraction and support it by exact experiment. Malus entered the contest, made a number of exact measurements of the index of refraction of the extraordinary ray in several substances, and found them to confirm Huygens' theory of double refraction. This did not mean an advocacy of the wave hypothesis, for Malus (and Laplace too) found a way to reinterpret Huygens' theory of Iceland crystal so as to base it on the properties of light particles. Following Newton's idea of a transverse asymmetry of light, Malus attributed to light particles two different poles **a** and **b** which were perpendicular to one another and to the direction of propagation **c** (Fig.12.6). He called this asymmetry of light *polarization*. According to him, the ordinary and extraordinary rays were polarized in perpendicular directions.

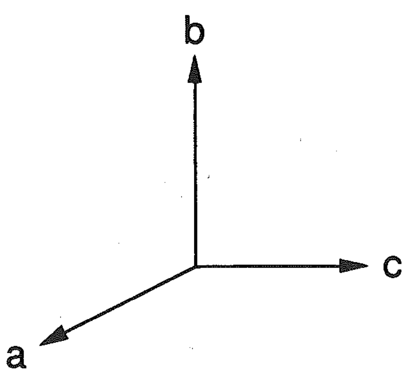


Fig.12.6. Malus' poles.

Dorothy. We don't believe any longer in particles of light having poles, right? If so, what does this *polarization* mean to us?

Teacher. It is simply a word to denote that light has different properties in two directions perpendicular to one another and to the direction of propagation, I've already

described its model in the emission theory. Its wave model is similar if we imagine a vibrating string passing between the wires of a lattice: the vibrations will be transmitted from one side of the lattice to another if their plane is parallel to the threads or makes an acute angle with them. The greater the angle the more energy is absorbed by the lattice, which means less light coming through, and no light at all will pass when the plane of vibrations is perpendicular to the threads (Fig. 12.7).

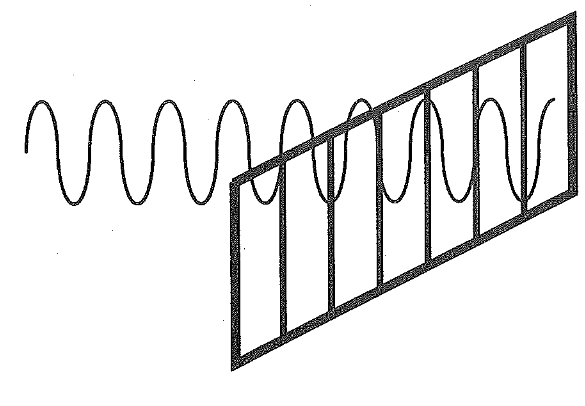


Fig.12.7. A wave model of polarization.

Michael. What does it mean that light is polarized in a specific direction?

Teacher It means a preferential orientation of the poles of light rays (or particles). When speaking of polarized light, we will call *polarizer* and *analyzer* the instruments used to, respectively, produce polarized light and uncover it. The same device can serve as either a polarizer or an analyzer. Prior to Malus, the only known polarizer/analyzer was a double refracting crystal. That changed one evening when, according to Arago, Malus took a look at the image of the setting sun in the windows of Luxembourg Palace through an Iceland crystal. He discovered that the brightness of the two images was different and it changed when he rotated the crystal around the visual line. Malus concluded that since the light reflected by glass behaved similarly to the light transmitted by an Iceland crystal, it had to be polarized. Malus found that other substances, including marble and water (but not metals), can also polarize light, and the effect is more pronounced at a specific angle of reflection, such as 52°45' for water and 56°30' for glass. He tried another glass plate as an analyzer and observed that the image he saw in it changed its brightness with the angle between the two glasses, being the darkest when the two were perpendicular On the basis of such observations, Malus formulated the following law for the intensity of reflected (or transmitted) polarized light ("Malus' law"):

$$I = I_0 \cos^2\alpha; \qquad (12.1)$$

where I_0 is the intensity of the incident light, **I** is the intensity of the emergent light, and α is the angle between the planes of polarization of light of the two mirrors, or between two planes of reflection of light (Fig.12.8). In Malus' view, each ray has its specific orientation of the pole **a** in space, and polarization refers to a statistical distribution of these orientations. When light is natural, the poles are randomly oriented; if light is fully polarized, they have the same orientation, Malus called *partially polarized* the type of

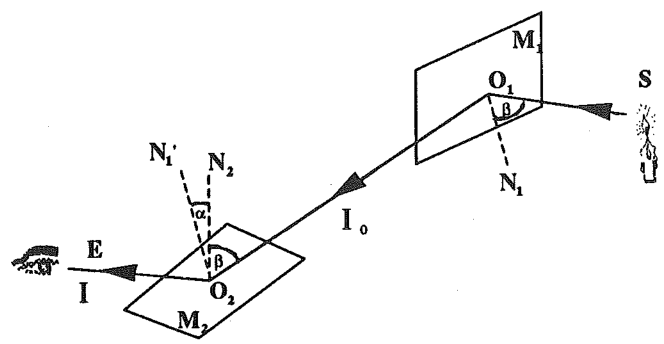


Fig.12.8. Polarizing and analysing light by reflection.

light that shows a preferential orientation of poles in a specific plane: in his view, it was a mixture of fully polarized and unpolarized light. He believed, that a polarizing device selects of all randomly distributed rays only those that have a specific orientation, which implied that the remaining rays will no longer retain a random distribution. For instance, at the boundary of two media the incident light is either reflected or refracted, thus if the reflected portion is polarized, so must be the refracted one, and the two planes of polarization must be perpendicular. Malus' experiment confirmed that the refracted light is partially polarized in the plane perpendicular to that of reflected light. He also found that the degree of polarization of refracted light can be increased by transmitting it through a number of parallel glass plates (Fig.12.9)

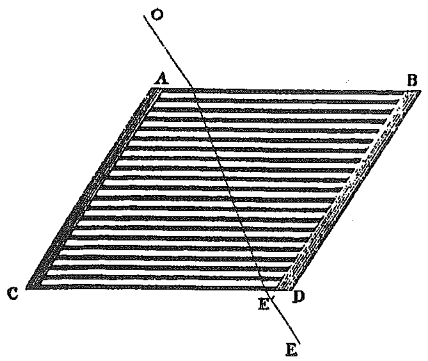


Fig.12.9. Polarization of refracted light. From Brewster, *Phil. Trans.*,1814, 239-46.

2. Brewster

history and discussion

Teacher. Although Malus was certain that each substance has its peculiar angle of full polarization, he could not derive any rule for it. This was accomplished in 1815 by ***David Brewster*** (1781-1868) who discovered that the angle of the full polarization β (later, it became known as *Brewster's angle*) for a specific substance depends on its index of refraction **n** as follows:

$$\tan\beta = n; \tag{12.2}$$

Brewster was born in Scotland in a teacher's family. He attended the University of Edinburgh and graduated as a divinity student. However, although he had a license to preach in the Church of Scotland, he was never ordained a minister. His first interest in optics was in improving microscopes and telescopes, but he became famous for his investigations of double refraction and polarization. A war between Britain and France delayed the news about Malus's discovery, and Brewster's first results in this field (published in 1813) repeated those of French physicists. Brewster's real strength was in experiment, his theories were unsuccessful. Among his best achievements, apart from Brewster's angle, were discoveries of artificial double refraction (1814-15) and of biaxial crystals (1818). During his most productive years, Brewster could not obtain a teaching position (there were few of them in Britain), and he made his income by writing popular books and articles and editing scientific journals and encyclopedias. He was involved in a number of scientific controversies, either on the priority of his discoveries, or on the nature of light and colors. He remained a staunch opponent of the wave theory of light even when it became generally accepted (since 1830s).

Mary. How could he object the theory that was accepted by all scientists?

Teacher. Well, first of all, he had a personal stake in the emission hypothesis, which was the basis of his own theory of biaxial crystals. Second, although the wave theory was accepted as providing a better explanation of double refraction and polarization, there were other phenomena unexplained in this theory, even in the 1850s.

3. Repeating Malus' experiments

lab

Teacher. Let us go outside and repeat Malus's original experiment with light reflected by windows. You will use an Iceland spar as an analyzer: first try it as is, then cover one side of it with paper in which you make a small (2-3 mm) hole. If you turn the opposite side to the eye, you will see two holes instead of one, and you should observe the polarizer through both images of the hole. After completing this experiment you may do other experiments involving reflection and refraction. In addition to an Iceland spar, use a polaroid, and other analyzers. If indoors, use a window or a desklight as a light source; outdoors, watch the image of the sun or the sky. Devise experiments to verify Malus' law.

Michael. We observed a big difference in brightness between two images of a second floor window seen through an Iceland spar. This means that a window can polarize sunlight not only at sunset. The angle was around 50°, so I am not sure whether it was a full polarization.

Mary. We were watching the window reflecting clouds. At some positions of an Iceland crystal one image was dark and the other was bright. When we turned the crystal by 90°, the bright image became dark and the dark image changed to light.

Dorothy. I observed a piece of glass lying on the ground using a polaroid as an analyzer. Every time I turned it by 90° the image of the sky in the glass changed from bright to dark or vice versa.

John. I used an Iceland spar as an analyzer and watched water in a pool reflecting clouds. It didn't work at first, but eventually I found a position from which the reflected light appeared to be quite polarized, for one image was much darker than the other. This occurred at four different positions of an Iceland spar.

David I've tried a glass mirror as an analyzer. First, I found a position where I saw the reflected light at about 56° which is the angle of full polarization. I measured the angle by means of a sighting device provided with a protractor. Then, I turned 90° to my left and, holding a glass plate before me, found in it an image of the polarizing mirror. I started turning the analyzer around the horizontal axis and noticed that the image was the brightest when the analyzer was horizontal and the darkest when the analyzer was vertical . Finally, while holding the analyzer vertically, I moved it to the right to increase the angle of reflection. The image first darkened then lightened, the darkest being at the angle of full polarization. It means that given the same angle of incidence, two glass plates reflect the most light when they are parallel, and the least, when they are perpendicular. Since the angle between the two mirrors is the same as the one between their planes of polarization, the experiment conforms to Malus' law.

Ruth. I've tried to combine a polarizer based on refraction in a glass plate using a polaroid analyzer. One plate showed some difference in brightness of the sky when I turned the analyzer 90°. Five glass plates put together produced a greater difference, and ten plates made the image quite dark.

discussion

Teacher. Very good! You've learned how to polarize and analyze light and you even confirmed Malus' law. He never described how he derived this law or tested it. Since there was no means then for measuring the intensity of light for an arbitrary angle of reflection, he only could do what you did: comparing the orientation of an analyzer at which one sees the image the darkest or the brightest. Since a maximum occurs when the planes of polarization of the polarizer and the analyzer are parallel, and the minimum is perceived when the planes are perpendicular, it would have been natural to suggest the involvement of $\cos\alpha$. According to Biot, Malus probably selected the simplest function containing $\cos\alpha$ and providing positive values, which is $\cos^2\alpha$. Subsequently, the truth of Malus' law was established with high precision.

Michael. If glass polarizes light at about 56°, why was I able to see polarization at other angles?

Teacher. Malus emphasized that some polarization occurs at any angle of reflection, but at one angle the reflected light is fully polarized. The greater the degree of polarization the greater the difference in brightness of the two images seen when turning the analyzer by 90°.

David. I wonder why Malus' discovery was called accidental. If you look through an Iceland spar at various objects, such as windows, water, cars, and others, it is difficult to miss a difference in the brightness of the two images. And it was this difference which reminded Malus of the Huygens experiment with two crystals.

Teacher. You are right: for a person well familiar with the properties of two beams polarized by an Iceland spar, it is easy to see the analogy between them and the light reflected by windows (forget about cars!). As ***John Herschel*** (1792-1871), an English astronomer and physicist, said, "***the seeds of great discoveries are everywhere present and floating around us, but they fall in vain on the unprepared mind, and germinate only where previous inquiry has elaborated the soil for their reception, and awakened the attention to a perception of their value.***"("Light," 505) On the other hand, Malus did **not** expect light to be polarized by reflection; from that perspective, the discovery was accidental. There are other aspects as well. For instance, if you didn't place a screen with a small hole before an Iceland spar, you would not see the effect. Thus, to succeed Malus needed a proper experimental procedure, but designing such a procedure is to some extent a matter of luck.

important

Dorothy. The Iceland spar is not easy to use: if the hole is too small, you don't see much through it, and if it is large, the two images overlap. Why didn't Malus use a polaroid?

Teacher You are right: in some cases, an Iceland crystal is not very convenient to use because the two polarized beams are difficult to separate from one another. However, there was nothing better available at Malus' time. Later, different means were devised to get rid of one of the two refracted rays. One of the simplest was to use a crystal (tourmaline, for instance) which absorbs one of the rays. The polaroid film used in sunglasses has the same property, although it is made not of a single large crystal but of many elongated microcrystals

imbedded in an organic substance so that they all have the same direction.

John. With a polaroid film one can detect even a small degree of polarization. Could physicists do it before the invention of polaroid?

Teacher. Yes, they had found a way to do it using the phenomenon of *chromatic polarization*.

III. CHROMATIC POLARIZATION

1. Arago

history

Teacher. A premature death from tuberculosis in 1812 terminated Malus' brilliant but short career in optics and left a rich new field of polarization to other investigators. At the moment, the general attention was attracted to the phenomenon of *chromatic polarization*, discovered in 1811 by ***François-Dominique Arago*** (1786-1853). As a child, he received a traditional classical education, but he dreamed of becoming an officer in the army. Once, he saw a very young officer and learned from him that students of the *École Polytechnique* obtain a promotion soon after the graduation. Arago abandoned his literary studies and began preparing himself for the entrance exams that were very demanding. In mathematics, he went beyond the secondary school textbooks and studied works of Euler, Lagrange, Laplace, and Poisson. He passed the exams with distinction and was enrolled at the *École Polytechnique* in 1803. Upon graduation in 1805, Arago intended to join the artillery but was talked into taking a position at the Observatory of the Paris Academy of Sciences. He was involved in two optical projects measuring refraction of gases, together with ***Jean-Baptiste Biot*** (1774-1862), and verifying whether the speed of light coming from heavenly bodies depends on their movement. In 1806, Biot and Arago were sent by the Academy of Sciences to Spain to continue measuring the earth meridian. Soon Biot returned to Paris, and Arago continued to do the work alone. The assignment was difficult in itself and it was especially endangered by highway bandits and the war situation between Spain and France. In 1808, Arago was arrested under suspicion of being a French spy. He managed to escape to Algiers. Disguised, with a false passport Arago embarked a merchant ship heading for Marseilles, however, on its way the ship was intercepted by a Spanish corsair, and the passengers were detained. Three months later, the ruler of Algiers threatened to start a war if the Spanish government would not release the ship. Arago was set free and again boarded a ship for Marseilles. The second attempt was no more successful than the first: a severe storm forced them to return to the African coast. During that season there were no sea communications with Algiers, and Arago decided on a very dangerous journey by land joining an Arab caravan. To save his life he faked an intention to convert to Islam. Finally, after some more adventures in Algiers, Arago arrived at Marseilles in July 1809. When the news that Arago is alive and preserved the results of measurements reached the Academy of Sciences, it considered filling the vacancy in the section of astronomy. The Academy had 40 positions in five sections, and each position was for life. The membership carried great prestige and some salary as well, consequently the elections had been very competitive. Some very good scientists had been waiting for decades for a vacancy. Although Arago was only 23 years old and had only two papers to his credit, the academicians decided that his ability to handle so difficult an assignment made him a good candidate and decided to delay the elections until his return. In September1809, Arago was elected to the Academy, and he fully justified the credit given to him. Arago is most known for his work in optics and electromagnetism. Since 1830, he was an active member of the Parliament expressing democratic and liberal views. After the February revolution of 1848, he became a member of a provisional government, serving as a Minister of War and Navy.

Arago's road to fame began in 1811 with his discovery of colors of crystalline plates. While experimenting with Newton's rings produced in thin films by a polarized light, he decided to check whether a thin film of mica produces the same phenomenon as a pair of lenses or glass plates pressed together. While looking at a thin plate of mica through an Iceland crystal, Arago accidentally placed the mica against the blue sky. To his surprise, he found the two images colored.

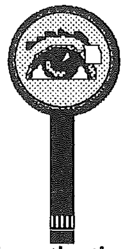
David. Is the "blue sky" a figure of speech or a physical factor?

Teacher. You have an opportunity to discover this yourself. Go outside, repeat Arago's original experiment and then try to modify it.

2. Colors of mica

Preliminary part

Experiments



investigation

Dorothy. First, I couldn't see any colors at all, so I made the hole in the screen somewhat larger and then I noticed a slight coloration but only when I looked at some parts of the sky.

Michael. We didn't see any colors against clouds or in the part of the sky opposite the sun.

Ruth. Apparently, the colors are seen only in the direction perpendicular to that to the sun. When they do appear, the colors of the two images appear to be complementary, such as greenish and purplish, for instance. The colors changed when I rotated the crystal. Sometimes, I was not sure about the color, but I noticed that one image appeared darker than the other, and when I turned the crystal by 90° the darker one became the brighter of the two.

John. We've got much brighter colors by tilting the mica plate to the line of vision.

David. A thick plate of mica (thicker than 1 mm) didn't show any colors, and neither did very thin plates. While working with a plate of a proper thickness we've discovered that colors changed when we rotated the plate in its own plane. Since the rest was kept the same, the cause could be in the plate's thickness, because the crystal was not uniform.

Teacher. O.K. Let us now summarize the results of your experiments in order to formulate a problem and determine the factors that affect the appearance of colors and, thus, could be the variables.

Formulating a problem

To find the cause of colors in mica.

Selecting variables

Mary. The significant factors affecting the colors of mica are of two kinds, some of them refer to mica (its orientation and thickness) while others point to an external cause, apparently connected with the condition of the sky.

Selecting a procedure

David. To have two distinct images in an Iceland spar we need a screen with a hole on the way of light. However, since our mica crystals are not uniform, I suggest attaching this screen to the mica rather than to the Iceland spar: by doing this we will always look through the same region of the mica (Fig. 12.10)

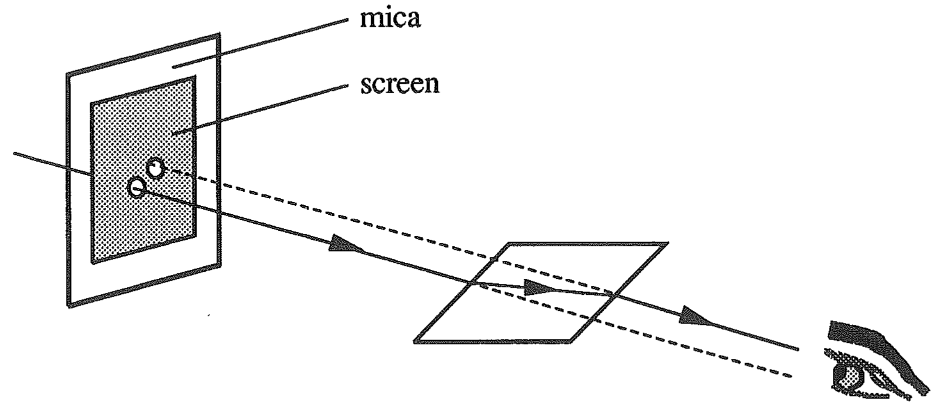


Fig.12.10. Arago's experiment..

Main part

a. the external cause

Preliminary experiments

Teacher. What can be different in different parts of the sky that could affect colors?

John. How about the color of light? A blue light from a clear sky works, while white light coming from clouds doesn't.

Michael. Isn't it true, though, that not all areas of the blue sky produce colors?

Ruth. You're right, of course. Listen, I noticed that the two images in an Iceland crystal were of different brightness. Can't we suppose, as Malus did, that light from blue sky resembles the one emerging from a double refracting crystal, which means it is polarized?

Mary. Perhaps, and if so, the polarization must be partial, because I never saw very dark images.

Hypothesis

The colors of mica are produced by a partially polarized light.

Test

David. Since a window glass partially polarizes the light from the sky (even if overcast), let us look at a mica plate set against a window.

Dorothy. I looked at a window at a large angle, and saw wonderful colors, much better than when I looked at the sky.

Michael. We couldn't find a properly oriented window, so we laid down a glass plate on the ground and watched the reflection of the sky. The colors were very distinct.

Ruth. First, I made a bundle of ten glass plates, taped them together and used them as a polarizer. When holding the bundle in my left hand (it made about 30° with the horizon) and the Iceland spar with mica in my right hand I looked at the sky through both of them and saw nice colors.

conclusion ➡ | The colors of mica are produced by polarized light

David. We see that Arago discovered, among other things, that light from the blue sky is partially polarized. Apparently, the degree of polarization reaches a maximum in the direction perpendicular to that of the sun.

b. orientation of mica

Preliminary experiments

Teacher. We've already found that the orientation of mica affects the colors. To investigate this in detail we need a more convenient polarizer-analyzer system. Take two square polaroids and tape them to the opposite sides of a book, or a box, or a table so that the polaroids were parallel, about 10 - 20 cm apart, and have their planes of polarization crossed (perpendicular).

John. Before stopping to use the Iceland spar as an analyzer I decided to check whether the two images are indeed complementary. The slightly polarized light from the sky is not very good for this experiment, so I used a polaroid to make light fully polarized. I also enlarged the hole in the screen so as to make the two images overlap in half. Finally, I selected a piece of mica of apparently the same thickness. With this arrangement, I observed that whatever the colors of the two images, the overlapping part was white, proving that the colors were complementary.

Dorothy. I placed a crystal of mica between the polaroids parallel to them and saw a reddish-purplish color. Then I started slowly turning it around a vertical axis, and the color changed in the following sequence: blue, green, yellow, red, and so on.

Michael. I turned the mica around a horizontal axis and observed a sequence of colors similar to that of Dorothy.

Ruth. We rotated the mica in its own plane, which is difficult to do without giving it a tilt, so we pressed it to the analyzer. We observed the same color (red-purple) but its intensity changed, and in two positions of the mica the image became uncolored. We marked these two directions in the vertical plane and found them to be perpendicular.

Teacher. The existence of such two directions implies that, unlike Iceland spar, mica has two optic axes, and along each of them the velocity of both refracted rays is the same. The existence of two optic axes is not an unusual phenomenon: Brewster discovered in 1818 that most double refracting crystals are biaxial. It became clear that Huygens' theory could cover only a limited class of crystals. This problem was solved by Fresnel in 1821-23.

At this point, given the time constraint and the simplicity of our instruments, we have reached the limit for our investigation of the orientation. As to the thickness, we will resume studying its role with other substances that are more uniform than our samples of mica.

3. Biot and the emission theory

history

Teacher. In 1812-1816, Brewster's main rival in the field of polarization and double refraction was Biot. He was a very thorough investigator intent on deriving quantitative laws of phenomena. Although some of the phenomena he studied were discovered by others, this did not diminish the importance of Biot's contribution. Like Malus, Biot entered the *École Polytechnique* soon after its foundation. After graduation, he taught mathematics and physics first in province and then in Paris. In 1803, he became a member of the Academy of Sciences (mathematics section). Late in his life, for his contributions to the history of ancient astronomy and his writings, he was elected to two other French academies devoted to antiquities and literature. Biot was the leading champion of the emission theory of his time, and he devoted much effort to reintroduce Newton's quantitative method into physics and revive his theory of fits. His first interest in polarization was excited by Arago's discovery of colors of mica. In1812-13, Biot. found that a relation between the colors of thin crystalline plates and their thicknesses was very similar to that discovered by Newton for thin films of air or water, and that the ratio of the thicknesses of Newton's and Biot's plates which displayed the same color was constant for all colors. Thus, after determining this ratio experimentally for one color, aided by Newton's table (Newton, *Opticks*, 233), Biot correctly predicted the color of various plates given their thickness (between 0.03 mm and 0.45 mm). To Biot, that was another confirmation of Newton's theory of fits of easy reflection and easy transmission. To explain chromatic polarization, Biot supposed that when polarized ray penetrated a thin crystalline plate, its plane of polarization oscillated between the initial one and another plane which made the angle **2i** with it (**i** was the angle between the planes of polarization of the polarizer and of the crystal). The oscillations took place at equal intervals, which were different for light of different colors. This theory satisfactorily explained a number of phenomena but later was superseded by Fresnel's theory (1821).

At the same year 1811, Arago discovered that sometimes colors could be produced by thicker plates: a quartz plate 6 mm thick cut perpendicularly to its optic axis exhibited colors similar to those of mica. However, the similarity was not complete, because when Arago rotated the quartz plate in its own plane only the brightness of the transmitted colored light changed but not its hue. Two years later, Biot studied the colors of quartz in detail. He found that when the plate was perpendicular to the beam its color depended on the plate's thickness. While rotating the analyzer from right to left Biot observed a succession of all spectral colors and supposed that quartz polarized light of different colors in different planes. He explained this as follows: while passing a quartz plate the light particles rotated their poles from right to left and the angular speed of this rotation depended on color, This theory suggested that the angle of rotation is proportional to the plate's thickness and depends on the wavelength (or, the interval of fits) if light is monochromatic. Biot proved these hypotheses by exact experiments. He also found a different variety of quartz, which produced the same phenomenon with the analyzer rotating from left to right. In 1815, Biot discovered that some liquids, such as oil of turpentine and citric acid, can also rotate the plane of polarization, the former to the right, and the latter to the left. Later, he found that sugar syrup does the same and suggested measuring the angle of rotation of the plan of polarization in human urine as a test for diabetes.

Dorothy. This "rotation of the plane of polarization" staff literally makes me dizzy. Is there a simpler way to understand it?

Teacher. Let us do an experiment, then the subject will become clear.

4. Experimenting with optical rotation

lab

Teacher. Let us experiment with corn syrup. Use small rectangular containers that we've utilized in experiments on refraction. They are convenient when one needs to measure the thickness of the optically active medium. We will investigate the variables studied by Biot: the thickness of a medium and the color of light. The experimental arrangement consists of two crossed polaroids set at a sufficient distance from one another to place the container between them. Attach a red filter to the outer surface of the polarizer. What do you see?

Dorothy. The visual field is dark because the polaroids are perpendicular.

Teacher. What do you see when you insert the container?

Michael. The syrup appears red.

Teacher. Why do you see light now?

Mary. You said that after passing an active medium light rotates its plane of polarization. This implies that the angle between this plane and the plane of polarization of the analyzer is other than 90°, thus some light must come through.

Teacher. Fine. How can we measure the angle of rotation of the plane of polarization?

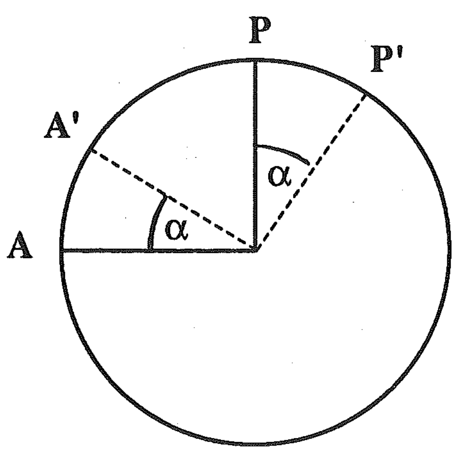


Fig.12.11. Aeasuring the optical rotation (the idea)

David. Suppose that the original position of the plane of polarization of light is **P** and the final position is **P'** (Fig. 12.11). It is clear from this diagram that if we rotate the analyzer **A** to a position **A'** where we regain darkness, its angle of rotation α will be equal to the angle we are seeking.

Teacher. Let us determine how the angle of rotation depends on the thickness for a specific color, and then how this angle depends on color. To reduce the range of transmission of your color filters tape several identical filters together. Take a circular analyzer and mark on its circumference the position of its plane of polarization. Make a transparency photocopy of a protractor, whose inner diameter is 5 mm larger than the diameter of the analyzer (make sure the plastic does not polarize light). Make a cardboard ring of 2-3 mm thick and 5 mm wide, whose inner diameter is the same as that of the analyzer. Fix both the scale and the ring to a glass plate with "Tak" and place the analyzer inside the ring (Fig. 12.12).

Michael. With a red filter, one container placed lengthwise (6 cm) produced the rotation of 30°, and two containers aligned lengthwise (12 cm) rotated the plane of polarization by 60°.

John. I've tried a larger container of 10 cm long (also with a red filter) and found the angle of 48°.

Ruth. With a blue filter, one small container produced 60° of rotation, while two such containers aligned made the angle of 120°. It looks like the angle of rotation is proportional to the thickness of the liquid, although the constant of proportionality differs for different colors.

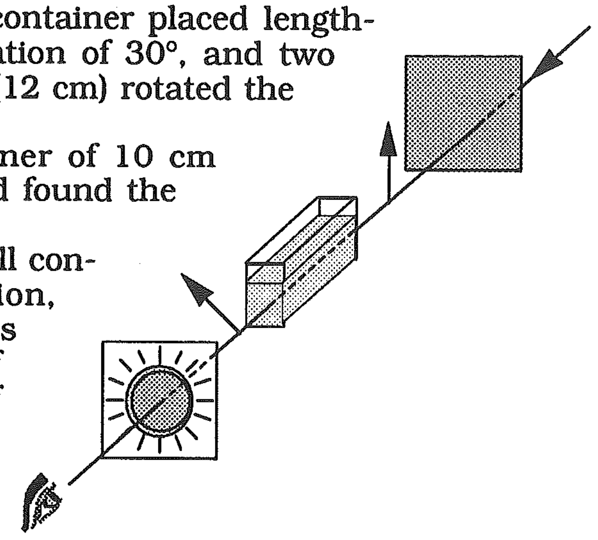


Fig.12.12. Measuring the optical rotation (the experiment)

Mary. We experimented with a small container and three different filters. We obtained 32° for a red filter, 47° for a green filter, and 61° for a blue one. This is consistent with the previous results and shows that the angle of rotation of the plane of polarization increases when the wavelength of light diminishes.

David. This means that when analyzer stood at 32° it eliminated red light, at 47° it absorbed green light, and at 61° the blue one. If so, what can we expect to see in white light while rotating the analyzer? I guess, that at these three positions we would see, respectively the remains of the spectrum, that is, blue-green, blue-red or purple, and yellowish-red.

Dorothy. Let me check your prediction. Yes, it comes quite close for these angles.

John Our group became interested in your remark about measuring the concentration of sugar. We diluted the syrup two and four times with water and found that the angle of rotation decreased 1.9 times with the 50% solution and 3.8 times with 25% solution. We checked that water does not have any effect on polarization. Thus, apparently the angle of rotation is proportional to the concentration of the active substance (sugar).

Teacher. Your results are similar to Biot's, which is very good, taking into account the simplicity of your equipment.

5. Artificial double refraction

history *and discussion*

Teacher. We have studied a variety of materials that polarize the transmitted light. What do they all have in common?
Dorothy. Some of them are crystals (Iceland spar, quartz, mica), others are liquids. I can't see what these two groups have in common.

lab home

Ruth. These two groups don't exhaust all polarizing materials. I placed a piece of a clear scotch tape between the crossed polarizers, and it exhibited colors like mica.
Michael. We've tried some wrapping cellophane, and it did the same when several layers were used.
Teacher. Did you notice any directional properties in these materials?
Ruth. My material was certainly double refracting, because when I rotated it the colors changed.
Michael. So it was in my case too.
Mary. I understand that crystals are asymmetrical, you can see it. Probably, that is why they transmit light with different speed in different directions. But how about plastics or liquids?
David. Perhaps the active liquids have asymmetrical molecules. As to plastics, isn't possible that they made asymmetric during their manufacturing, for instance, stretched in one direction.
Teacher. Excellent! You understand that the cause of double refraction is an asymmetry in the physical properties of a body, which can be achieved in various ways. Why don't we produce an artificial asymmetry and see whether it leads to double refraction.
Dorothy. I stretched cellophane wrapping and observed colors near its edges.
Michael. We've observed the same when stretching plastic lunch bags.
Mary. I've bent a plastic ruler, and it displayed colors.
Teacher. Very good! That mechanical deformations create double refraction was discovered by ***Thomas Johann Seebeck*** (1770-1831) and Brewster (1815). Glass was the first material utilized for this purpose (Fig. 12.13) Glass, however, required a considerable force, and Brewster was glad to find that a jelly, which is easily deformed, shows signs of double refraction caused by pressure or induration (12.14) I want you to do this experiment at home. Take a packet of a clear unflavored gelatin. Put it in boiling water (take one half of the amount of water recommended per packet) and stir until completely dissolved. Pour the liquid in a plastic container and place it in a refrigerator. When the jelly is set cut a rectangular piece out of it and place it between the crossed polaroids.

lab home

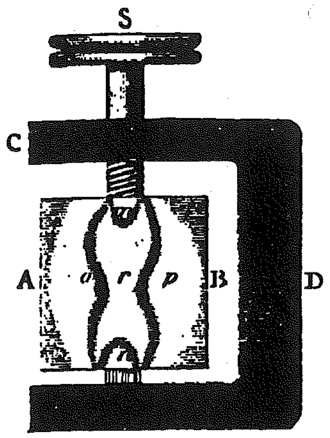


Fig.12.13. Double refraction in glass produced by a compression. From D. Brewster, *Phil. Trans.*, 1816

Watch the image while pressing or squeezing the jelly and try to relate the figures produced to the type of deformation. You have to remember that a coloration is not the only sign of double refraction, because it is observed only for a small range of thicknesses. A more general sign, true for every thickness, is a lightening of the dark visual field after inserting the sample between the polaroids.

history *and discussion*

Fig.12.14. Double refraction in jelly produced by induration. From D. Brewster, *Phil. Trans., 1816*

Teacher. Good! Now, is it possible to discover a deformation that is invisible to a naked eye? Such deformations are produced, for instance, during manufacturing, either by mechanical forces or by a heating/cooling of the object.
Ruth. It is possible if the body retains the deformation. You place it between the crossed polaroids, and if you see a coloration only in some parts of it, it is probably the result of a deformation. I've checked that with cellophane.

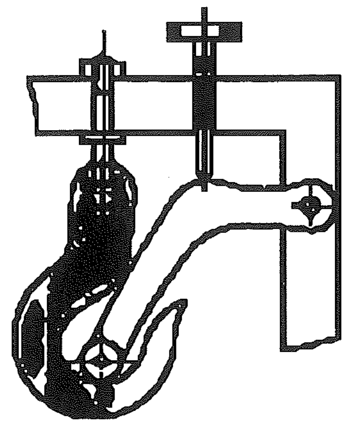


Fig. 12.15. Studying mechanical deformations by means of polarized light.

Teacher. Fine. This idea found a practical application. If one makes a clear plastic model of a working device and studies the colors displayed in polarized light, one can get a qualitative picture of the distribution of stresses in the device, which is important for improving its design (Fig. 12.15). Brewster and Seebeck also discovered in 1814 that glass can be made double refracting by heating it (Fig. 12.16). I want you to repeat this experiment at home. Heat a piece of glass over a candle flame and insert it between the crossed polaroids. Check whether it retains the double refraction after cooling off.

lab home

David. I remember hearing a crackling sound when I put an ice cube in water. This implies that the ice had some residual stresses acquired probably during solidifying. Is it possible that ice has a double refracting capability?
Teacher. I want all of you to check this at home. Compare slices of ice of different thickness. To make ice more clear polish it with your fingers.

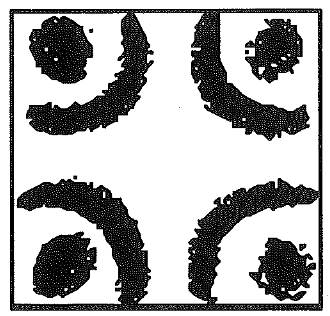


Fig.12.16. Double refraction in glass produced by heating. From T.Seebeck, *Journ. für Chem. u. Physik*, 12 (1814).

IV. WAVE THEORIES

1. Chromatic polarization

history

In 1814, Young suggested that Biot's experiments could be explained through interference of the ordinary and extraordinary waves. He was unaware then that light waves polarized in perpendicular directions cannot interfere. This was demonstrated by Augustin Fresnel and François Arago in 1816. In the two-slit experiment, they placed two crystals before the slits and found that the interference fringes appeared when the two beams were polarized in the same direction and disappeared when they were polarized in perpendicular directions. Thus, they discovered a new condition of coherence: two independent rays which are polarized in perpendicular directions cannot interfere. They also found that two rays polarized perpendicularly to one another could interfere if they originate from the same polarized ray and are reduced to the same plane. Fresnel expressed a light wave by a vector whose length represented the amplitude of light vibrations and the angle this vector formed with a selected axis displayed the phase of these vibrations. He inferred from the experiments on interference that the light vector must be perpendicular to the direction of propagation of light, or, in other words, that light wave is transversal. Mathematically, it means that an arbitrary light vector can be resolved into two (not three!) components that are perpendicular to one another and to the direction of propagation of light. For instance, let **P** be a polaroid with the axis of polarization **x**, (Fig.12.17), and the incident light wave have the amplitude **A** and is polarized in the direction **k**, which forms an angle α with the direction **x**, Then, according to Fresnel, this wave can be replaced with a sum of two waves of the same amplitude but polarized in perpendicular directions. If **x** is one of these directions, **y** being the other, the polaroid will transmit light of the amplitude a=Acosα. By squaring both sides of this equation we obtain Malus' law, because A^2 and a^2 are the intensities I_0 and **I** of, respectively, incident and emergent light.

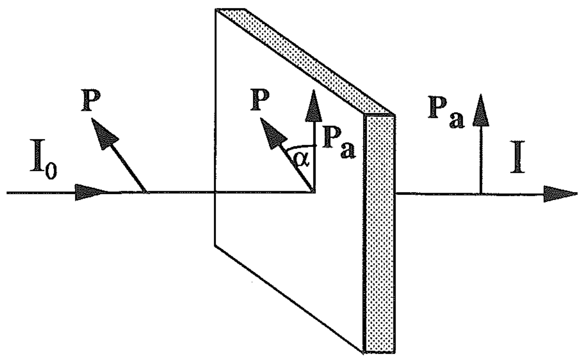


Fig.12.17. Wave explanation of chromatic polarization of light.

theory

Colors of crystalline plates result, according to Fresnel, from an interference of two polarized waves that originated from the same polarized wave, acquired perpendicular polarizations and a path difference when passing a plate, and finally were reduced to the same plane of polarization in an analyzer. For instance, let **P** (Fig. 12.16) be a uniaxial crystal of the thickness **d** cut parallel to its optic axis **O**, and let **x** and **y** be the respective directions of polarization in this crystal of the ordinary and extraordinary rays. Let the incident wave be of the amplitude A and polarized at the angle α to the axis **x**. When entering the crystal the incident light is divided in two waves whose vibrations are directed along the axes **x** and **y** and can be expressed as x_0=acosωt and y_0=bsinωt, where a=Acosα and b=Asinα. These waves travel along the same direction **O** but with different velocities $v_e=c/n_e$ and $v_o=c/n_o$. As the result,

when exiting the crystal the two waves will have the phase difference

$$\theta = \frac{2\pi d(n_o - n_e)}{\lambda} \tag{12.3}$$

and the projections of the light vector on the two axes will be, for instance, x=acosωt and y=bcos(ωt−θ). To find the trajectory of the end of this vector we have to eliminate time from our equations, which can be done as follows:

$$y = b(\cos\omega t\cos\theta + \sin\omega t\sin\theta); \tag{12.4}$$

$$\cos\omega t = x/a; \tag{12.5}$$

$$\sin\omega t\sin\theta = y/b - x\cos\theta/a; \tag{12.6}$$

Square both sides of equation (12.5) and multiply them by $\sin^2\theta$:

$$\cos^2\omega t\sin^2\theta = x^2\sin^2\theta/a^2; \tag{12.7}$$

After squaring both sides of equation (12.6) and adding to equation (12.7) we obtain

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\theta = \sin^2\theta \tag{12.8}$$

This is an ellipse **ABCD** (Fig. 12.18), the shape and orientation of which depend on the phase difference θ.

For instance, if θ=(2m+1)π/2, we have

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; \tag{12.9}$$

This is an ellipse **A'B'C'D'** with its axes oriented along **x** and **y**, and with its semi-axes **a** and **b** representing the amplitudes of vibrations. A rotation of the ellipse will thus mean a change in the amplitude (and intensity) of emergent light. Such polarization is called *elliptical*. If a=b, we have a *circular polarization*.

If θ=mπ (m=0, 1, 2,...), we have

$$y = \pm\frac{b}{a}x; \tag{12.10}$$

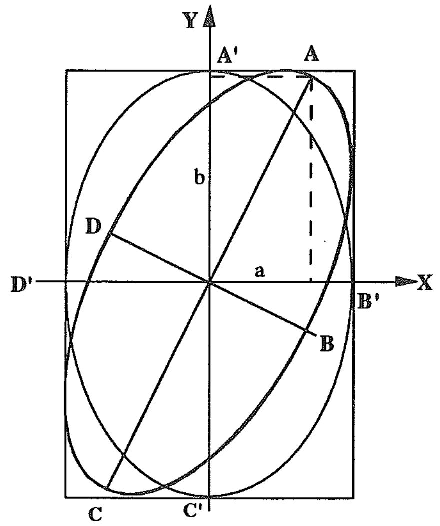


Fig.12.18. Elliptical polarization.of light..

math is a friend !

which is the equation of a straight line. This means that after exiting the plate light remains linearly polarized.

discussion

Dorothy I understand the math but not the physical meaning of this elliptical (or circular) polarization. Does it mean that we see colored ellipses?

Teacher. No. Imagine we can slow down optical vibrations so much that we are able to watch the end of the optical vector. If light is linearly polarized, it will move back and forth along the same line; if light is circularly polarized, it will move along a circumference, either clockwise or counterclockwise.

John. How do we discover that a light is polarized elliptically or circularly? Can we use a polaroid or an Iceland spar?

Teacher. Let us have an analyzer whose direction of polarization makes an angle α' with the axis **x**. The two waves will produce along this direction vibrations with amplitudes $a'=a\cos\alpha'$ and $b'=b\sin\alpha'$ and phase difference θ. Their sum will have the amplitude A', and the intensity of emergent light will be $I'=A'^2$ such that

$$A'^2 = a'^2 + b'^2 - 2ab\cos\theta; \qquad (12.11)$$

Thus, for an elliptically polarized light if we rotate the analyzer, changing the angle α', the total intensity will change between a non-zero minimum and a maximum. If light is circularly polarized, the amplitude of the emergent light will not change during the rotation. Thus, a circularly polarized light can be confused with natural light, while an elliptical polarization can be perceived as a partial linear polarization. One can avoid this confusion by transforming an elliptically polarized light into a linearly polarized light. To achieve this we have to change the phase difference by $(4m+1)\pi/2$, which corresponds to the path difference of $(m+1/4)\lambda$. To achieve this, we need a crystal plate of such a thickness **d** that

$$(n_o - n_e)d = (m+1/4)\lambda; \qquad (12.12)$$

The thinnest of such plates (m=0) is called a *quarter-wave plate.*

Michael. How does this theory explain colors of mica?

Teacher. For a plate of a constant thickness the phase difference θ depends only on the wavelength λ. For each position of the analyzer, light of different colors will have different amplitudes, which will result in one color dominating over others.

Mary. If we keep the analyzer steady and rotate the plate in its own plane, neither the phase difference θ nor the angle α' change. If so, why does the field of view change its brightness?

Teacher. Keeping these two factors permanent removes the coloration. However, the rotation of the plate is equivalent to a rotation of the axes **x** and **y**, which produces the same effect as a rotation of the ellipse in Fig. 12.18: the projections of the ellipse' s semi-axes change, and so does the intensity of transmitted light.

Ruth. We see that a linearly polarized light is quite an ordinary phenomenon because it is produced at every reflection of light by a smooth surface. Where can we find an elliptically polarized light?

Teacher. Fresnel discovered in 1817 that when a linearly polarized light experiences two consecutive internal reflections at Brewster's angle it becomes circularly polarized when the plane of incidence forms an angle of 45° with the plane of polarization. A circularly polarized light

can be also obtained from a linearly polarized light transmitted through a quarter-wave plate whose direction of polarization makes 45° with the axes of the crystal. Light reflected by metals acquires an elliptical polarization.

2. Double refraction

Like Huygens, Fresnel's connected the variation of the velocity of light in different directions inside a crystal with a change of elasticity of the ether. His problem was more difficult, however, for he had to account for both uniaxial and biaxial crystals. To model the distribution of velocities of light in a crystal, Fresnel used the *ellipsoid of elasticity* with three unequal perpendicular principal axes. For every direction of incident light there are two refracted waves with different velocities (Fig. 12.19). To determine them, it is necessary to draw a plane through the center of the ellipsoid perpendicularly to the direction of propagation of light: this plane cuts the ellipsoid along an ellipse, whose axes give the directions of polarization of the two waves and the magnitudes of their velocities. Among all elliptical sections two are circular, and the perpendiculars to their planes are the optical axes of the crystal. In an uniaxial crystal the two optical axes coincide. The electromagnetic theory of light did not change the essence of this theory, it only gave a different cause for changing the velocity of light in different directions: different elasticity was replaced with different dielectric constant of the material.

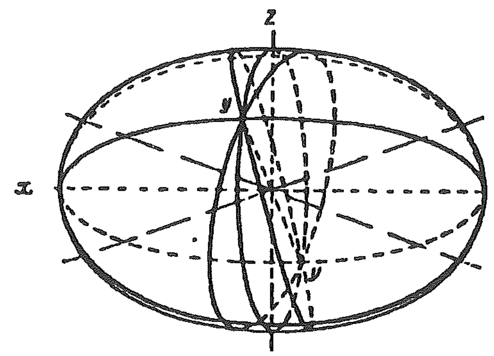


Fig.12.19. Wave model of double refraction.

Thus to explain chromatic polarization and double refraction Fresnel assumed a complete transversality of light waves. This idea initially occurred to him in 1816 after the discovery of the non-interference of light polarized in perpendicular planes. However, it took Fresnel five years to adopt this concept because he could not overcome two difficulties in establishing a mechanical foundation for such a theory (optics was thought of then as a part of mechanics). First, it was known that mechanical waves when crossing a boundary of two media produce both longitudinal and transversal refracted waves. Secondly, the ether was treated then as a fluid, but a fluid could not transmit transversal waves. Thus, Fresnel had to attribute to the ether the properties of a solid body. The idea was perceived as wild, and even such champions of the wave theory as Young and Arago refused to adopt it. Fresnel's alternative consisted of either adopting transversal light waves together with the solid ether, or abandoning a comprehensive quantitative explanation of chromatic polarization and double refraction. He opted for the former. Later, electromagnetic waves replaced the mechanical ones, but the core of Fresnel's theory - the transversality of light waves - remained intact.

history

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