But this is not the case with the other bright bands. The brightest band of the second order, for instance, corresponds to a division of the wave AI′G into three arcs, the extreme rays of which differ by one-half a wave-length; the effects produced by two of these arcs annul each other. Consequently, this band receives light from only one-third of the incident wave-front, while even the effect produced by this third is somewhat diminished by the fact that there is a difference of one-half a wave-length between the rays from its edges. A similar process of reasoning shows that the middle of the bright band of the third order is illuminated by only one-fifth of the wave-front AI′G, the light of this one-fifth being still further diminished by opposition of phase in its extreme rays.

[*Here are omitted six pages, including a geometrical discussion of the general relations between size of aperture* (*or obstacle*), *distance of screen, distance of luminous point, etc.*]

56. I have just explained the general relations between the size of any particular fringe and the respective distances of the obstacle from the luminous point and from the micrometer. As we have seen, these laws may be derived from theory quite independently of any knowledge of the integral which at each point represents the resultant of all the secondary waves; but in order to find the absolute size of these fringes, it is essential that we compute this resultant, for the positions of maxima and minima of intensity can be determined only by a comparison of the different values of this resultant, or at least by knowing the function which represents it.

In order to do this, we propose to apply to the principle of Huygens the method which we have already explained for computing the resultant of any number of trains of waves when their intensities and relative positions [*phases*] are given.

Application of Theory of Interference to Huygens's Principle

57. Let the waves from any luminous point C be partly intercepted by an opaque body AG. To begin with, we shall suppose that this screen is so large that no light comes around the edge G, so that we need consider only that part of the wave which lies to the left of the point A. Let DB represent the

plane upon which are received the shadow and its fringes. The problem then is to find the intensity of the light at any point P in this plane.

If from C as centre and with a radius CA we describe the circle AMI, it will represent the light-wave at the instant it is partly intercepted by the opaque body. It is from this position of the wave that I have computed the resultant of the secondary waves sent to the point P.. If we consider the wave in an earlier position, say A′M′I′, it then becomes necessary to calculate the effect of the obstacle on each of the secondary waves arising from the arc A′M′I′; and if we consider the wave in a later position, say A″M″I″, it becomes necessary to first determine the intensities of its various points, for they are no longer equal, having been changed by the interposition of the screen. In this case the computation is vastly more complicated, possibly quite impracticable. If, however, we consider the wave at the instant it reaches A, the process is simple; for then all parts of the wave have the same intensity. Not only so, but none of the secondary waves are now affected by the opaque screen. However numerous the subdivisions into which we may consider these elementary waves divided, it is evident that the number will be the same for each, since they are transmitted freely in all directions. And, therefore, we need only consider the axes of these pencils of split rays—*i. e.*, the straight lines drawn from the various points on the wave AMI to the point P. The differences of length in these direct rays are the differences of path traversed by the elementary or partial resultants meeting at P.*

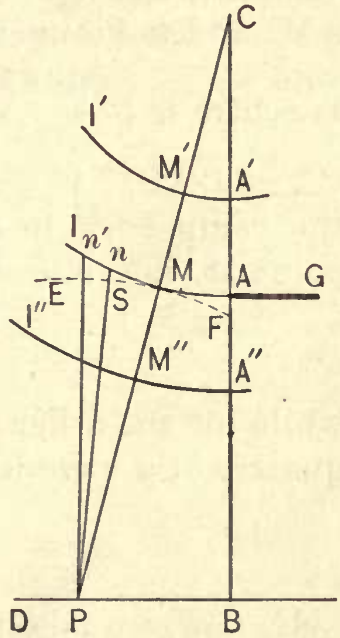

Fig. 19

In order to compute the total effect, I refer these partial resultants to the wave emitted by the point M on the straight line CP, and to another wave displaced a quarter of a wave-length with reference to the preceding. This is the process already employed (p. 101) in the general solution of the interference

* [*A foot-note is here omitted.*]

problem. We shall consider only a section of the wave made by a plane perpendicular to the edge of the screen, and shall indicate by dz an element, nn', of the primary wave, and by z its distance from the point M. These, as I have shown, suffice to determine the position and the *relative* intensities of the bright and dark bands. The distance nS included between the wave AMI and the tangential arc, EMF, described about the point P as centre is $\frac{1}{2}\frac{z^2(a+b)}{ab}$, where a and b are, as before, the distances CA and AB. If we denote the wave-length by λ, we have for the component in question, referred to the wave leaving the point M, the following expression

$$dz \cos\left(\pi\frac{z^2(a+b)}{ab\lambda}\right);$$

while for the other component,* referred to a wave displaced a quarter of a wave-length from the first, we have

$$dz \sin\left(\pi\frac{z^2(a+b)}{ab\lambda}\right).$$

If, now, we take the sum of all similar components of all the other elements, we shall have

$$\int dz \cos\left(\pi\frac{z^2(a+b)}{ab\lambda}\right) \text{ and } \int dz \sin\left(\pi\frac{z^2(a+b)}{ab\lambda}\right).$$

Hence the intensity of the vibration at P resulting from all these small disturbances is

$$\sqrt{\left[\int dz \cos\left(\pi\frac{z^2(a+b)}{ab\lambda}\right)\right]^2 + \left[\int dz \sin\left(\pi\frac{z^2(a+b)}{ab\lambda}\right)\right]^2}.$$

The intensity of the sensation, being proportional to the square of the speeds of the particles, is

$$\left[\int dz \cos\left(\pi\frac{z^2(a+b)}{ab\lambda}\right)\right]^2 + \left[\int dz \sin\left(\pi\frac{z^2(a+b)}{ab\lambda}\right)\right]^2.$$

This is what I have called the *intensity of the light* in order to conform to ordinary usage, while reserving the expression *intensity of vibration* to designate the speed of an ether particle during its oscillation.

* [*These expressions for amplitude follow directly from sec.* 40, *when in the general expression for velocity we make* $a=o$, $a'=dz$, *and* $c=\frac{1}{2}\frac{z^2(a+b}{ab}$.]