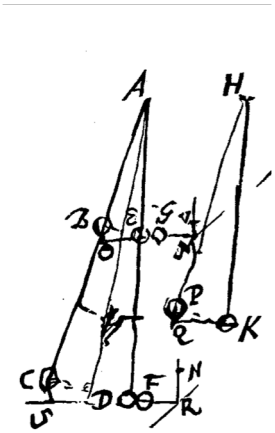
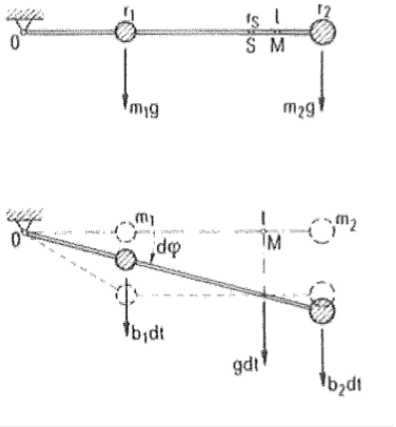
Sample Exam Questions - Module 2

1) What are the core developments in theoretical mechanics that happened from the publication of Newton's *Principia* (1687) until Lagrange's *Mécanique Analytique* (1788)? What were the main challenges faced by the actors of this process?

2) The problem of determining the center of oscillation of a compound pendulum has a major historical importance. Figure 1 contains sketches of the solutions proposed by (a) Huygens and (b) James Bernoulli. Explain the basic ideas of these solutions. How would you solve this problem today and what are the main differences from these two methods?



(a) Solution by Huygens



(b) Solution by James Bernoulli

Figure 1: Center of oscillation problem

3) D'Alembert's principle is praised by Lagrange as being a "general and direct method to solve all the problems of dynamics that can be imagined". Formulate this principle with your own words and explain its basic ideas. Exemplify how d'Alembert's principle was applied to solve the inelastic collision and center of oscillation problems.

4) What is Fermat's principle and the Brachystochrone problem? How are they related with the principle of least action and the calculus of variations?

5) Consider Maupertuis's original publication of the principle of least action from 1746. Explain his original concept of action and how he applies the "principle of least action" to solve the problems of inelastic and elastic collisions as well as the equilibrium of a lever with one body at each of its extremities.

6) Compare Maupertuis (1746) and Euler (1744) texts regarding their methodological and metaphysical approaches to the principle of least action.

7) In 1744 Euler proposes that the quantity $\int vds$ is minimized for the actual path taken by particles acted upon by (conservative) forces, given their initial and final positions. What were the arguments for this proposal? Show that minimizing this integral enables one to determine the trajectory of i) a moving particle free of external forces and ii) a moving particle acted upon by a downwards force of constant acceleration g.

8) Compare the original formulations of d'Alembert's principle and the principle of least action with the way we learn/teach these things today. What was gained and what was lost?