Roots of quantum mechanics

1900 1913 1925

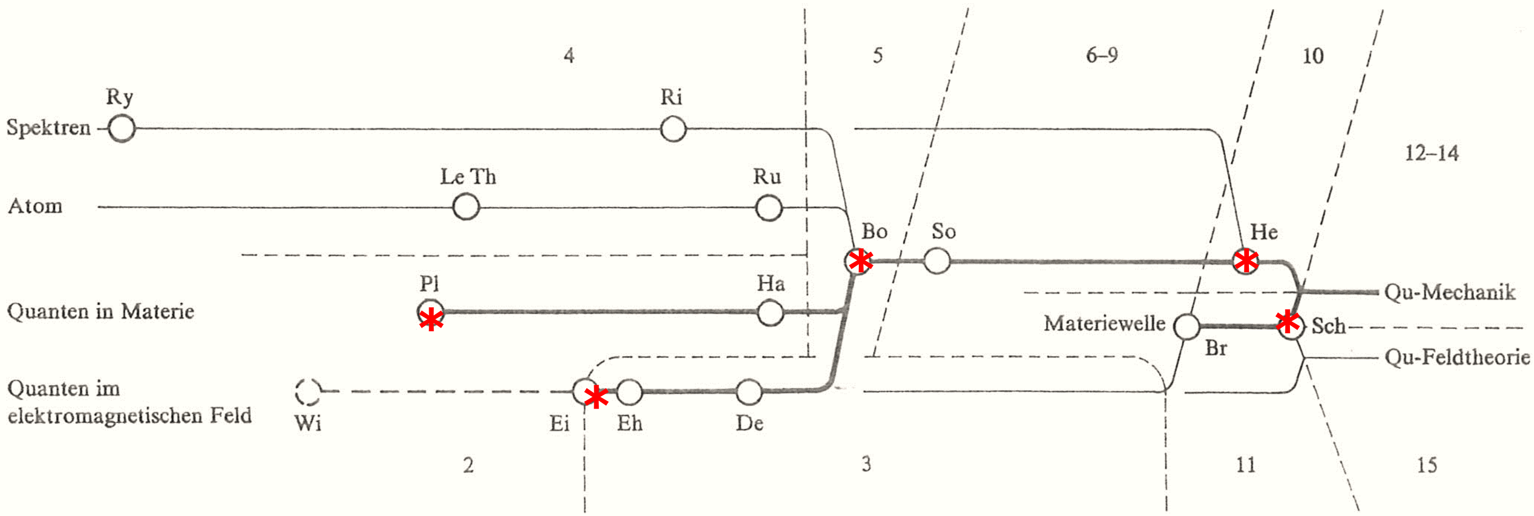


Abb. 1: Schema der Entwicklung der Quantentheorie

Ry = Rydberg; Th = Thomson; Le = Lenard; **Pl = Planck**; Wi = Wien; **Ei = Einstein**; Ha = Haas; De = Debye; Eh = Ehrenfest; Ru = Rutherford; **Bo = Bohr**; **So = Sommerfeld; Br = de Broglie; He = Heisenberg; Sch = Schrödinger**

Planck Radiation Law

- The primary law governing blackbody radiation is the *Planck Radiation Law.*
- This law governs the intensity of radiation emitted by unit surface area into a fixed direction (solid angle) from the blackbody as a function of wavelength for a fixed temperature.
- The Planck Law can be expressed through the following equation.

$$E(\lambda,T)=\frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1}$$

$h = 6.625 \times 10^{-27}$ erg-sec (Planck Constant)

$K = 1.38 \times 10^{-16}$ erg/K (Boltzmann Constant)

C = Speed of light in vacuum

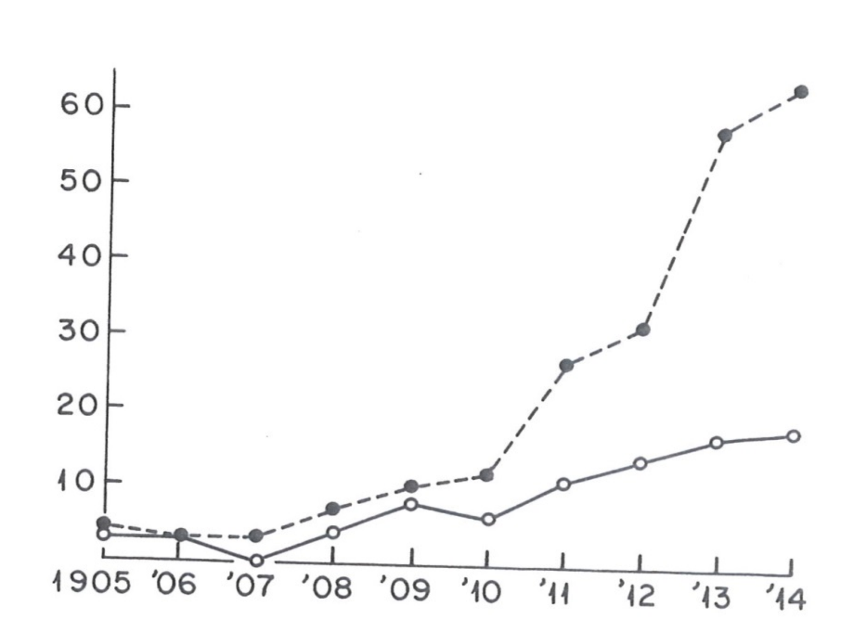
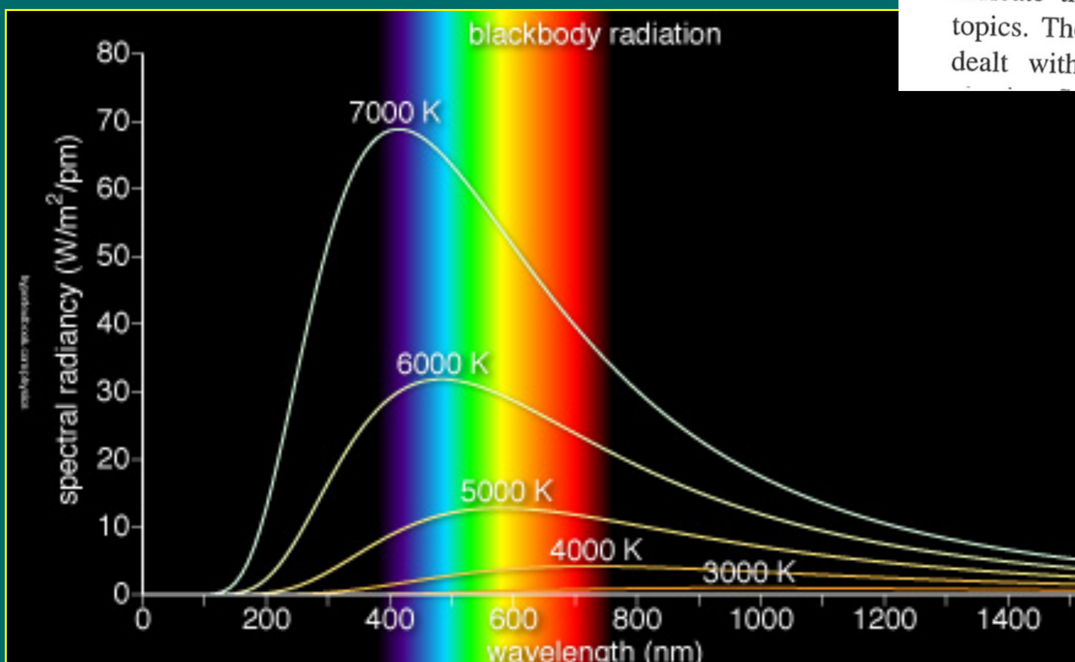


Figure 5.2. The slow rise of quantum theory. The solid circles indicate the number of authors who published on quantum topics. The open circles refer to the number of authors who dealt with blackbody theory, a subset of early quantum



Stefan-Boltzmann law (1879-84)

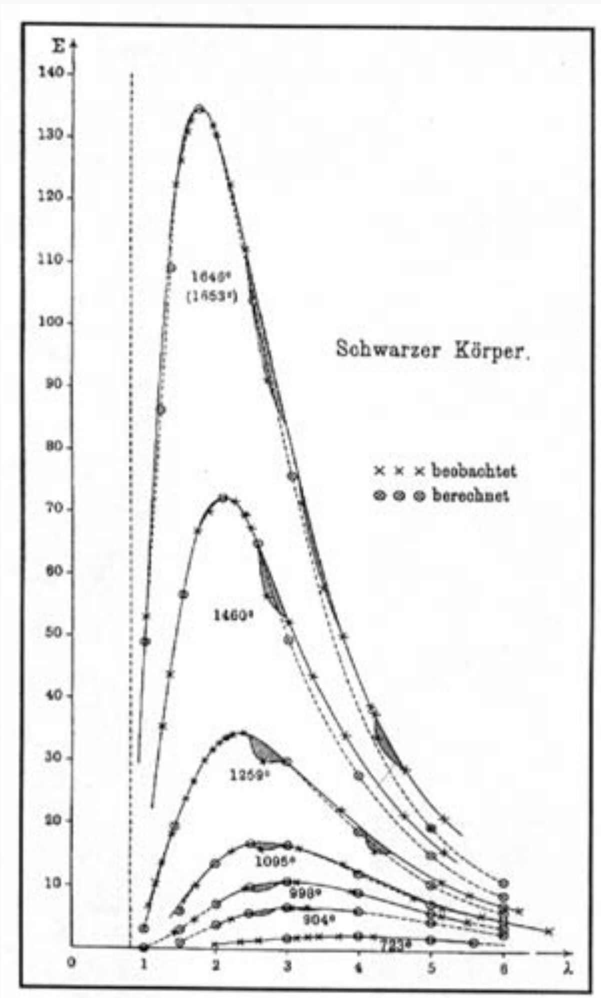
$$\rho = aT^4$$

Wien displacement law (1894)

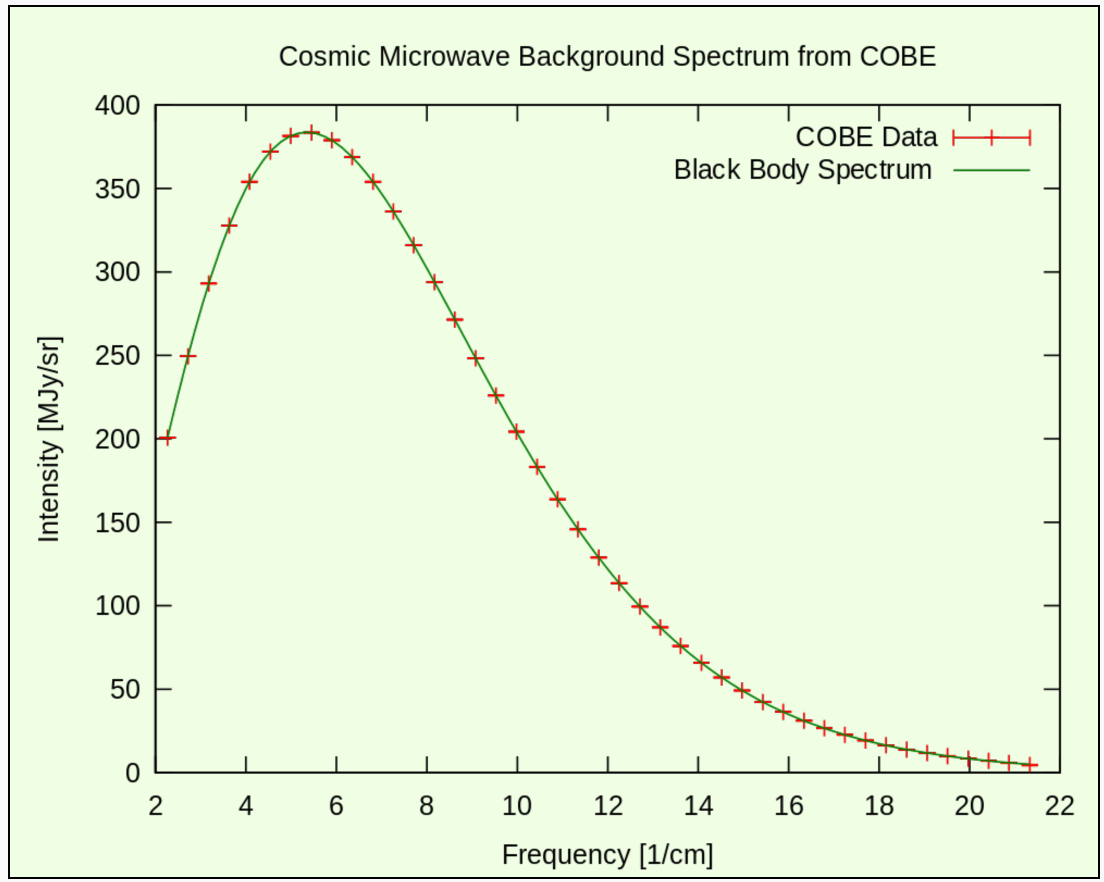
$$\lambda_{\max} T = \text{constant}$$

Measurements of blackbody radiation

laboratory experiments, and astronomical observations



Lummer & Pringsheim, 1900 (PTR)

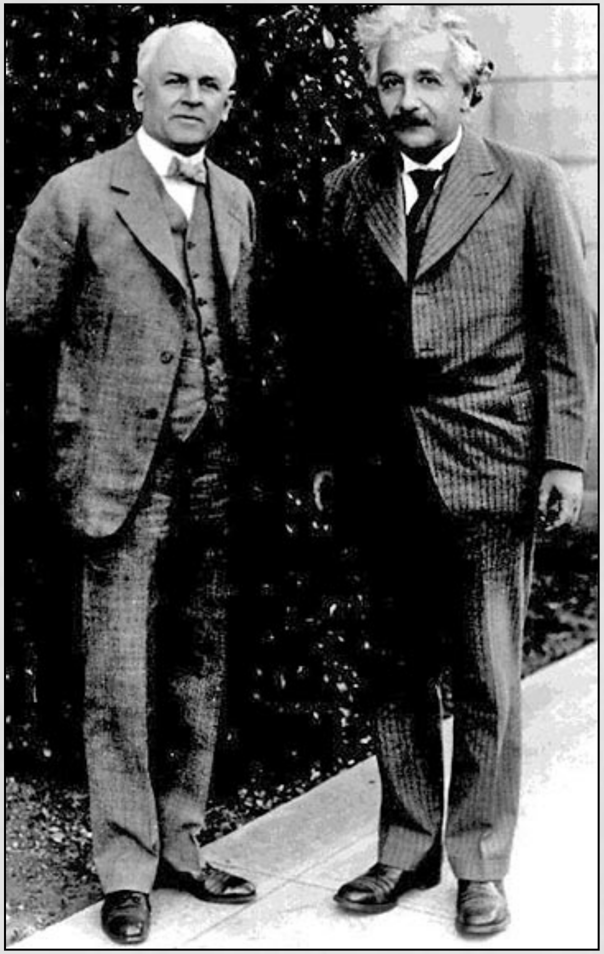


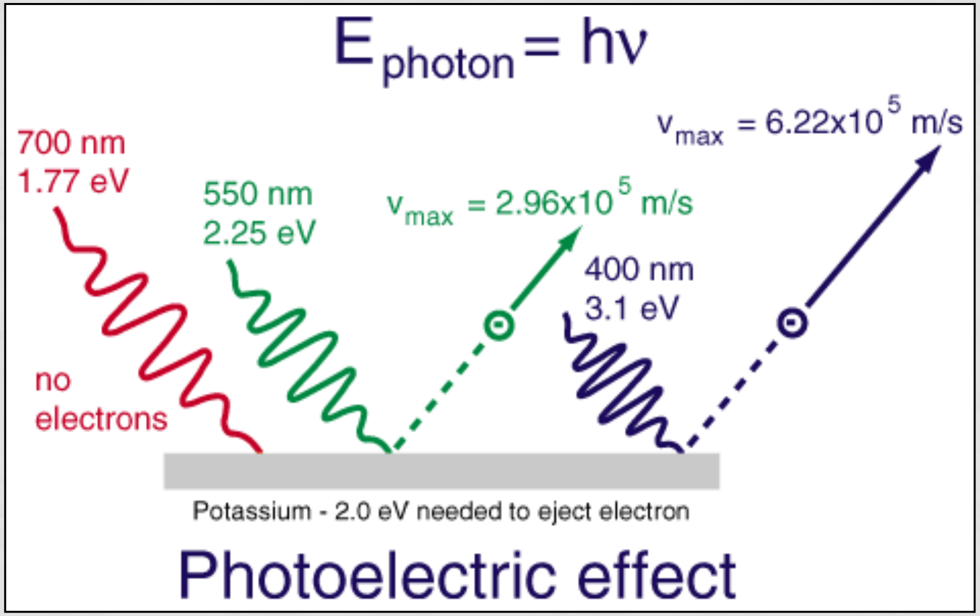
COBE data of CMB-radiation, 1990

Einstein's law of photo-electricity:

$$E_{\text{electron}} = h\nu - P$$

i.e., linear (E, ν) relationship





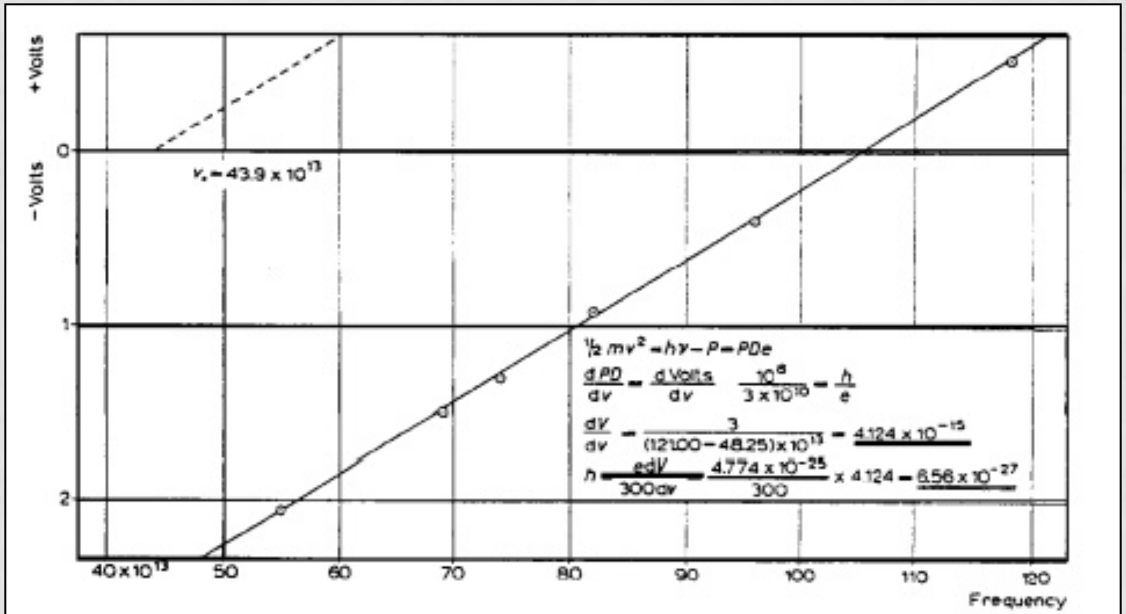
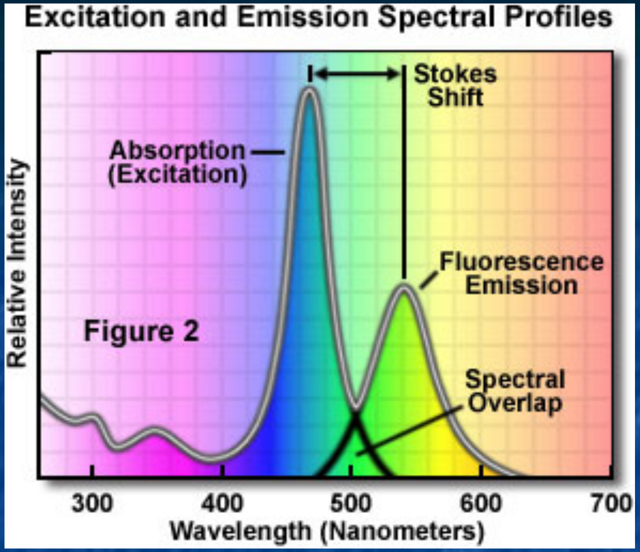
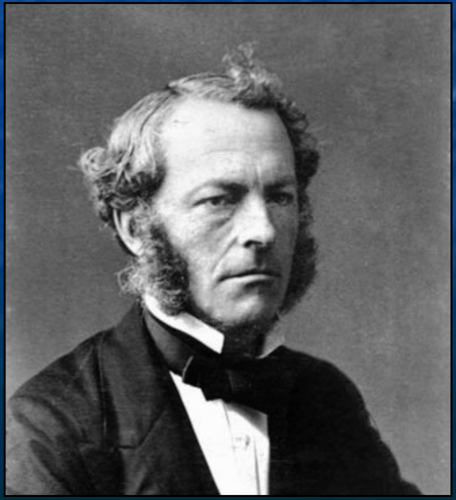


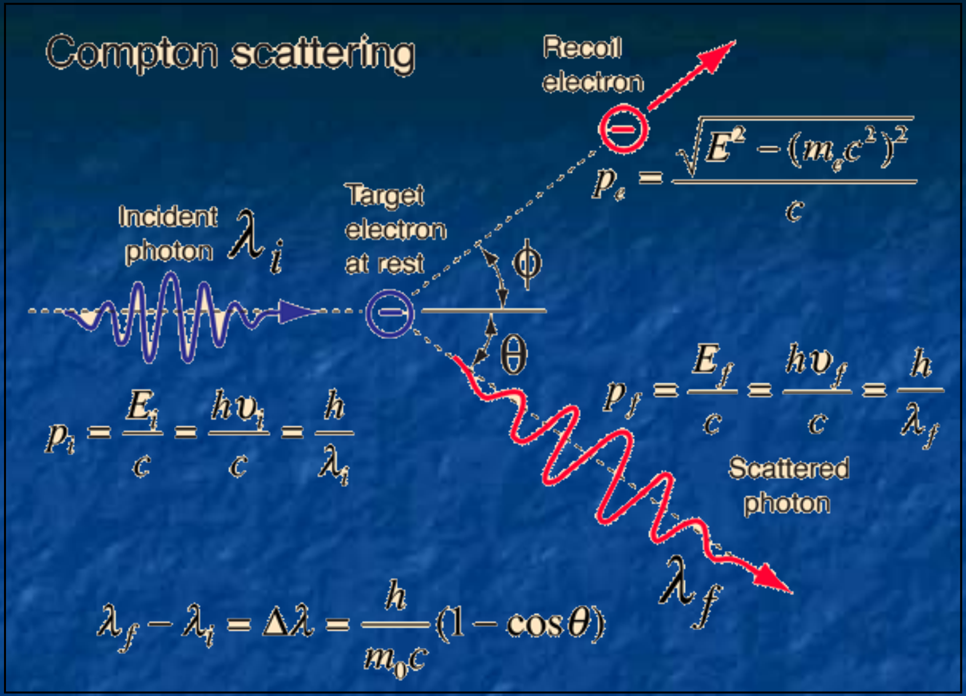
Fig. 4.

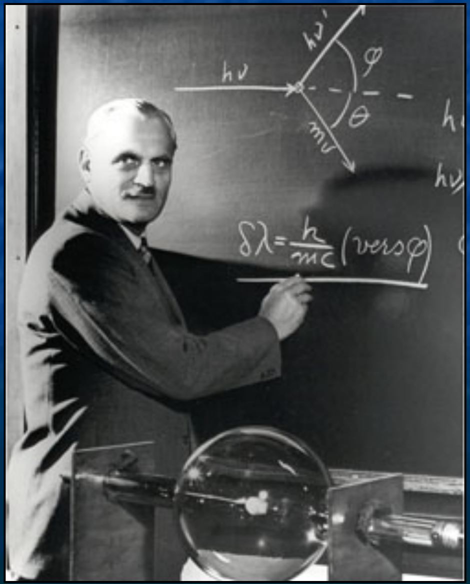


Fluorescence (1852)

Stokes' rule: $\lambda_{\text{incident}}$ > λ_{emtited}







Acceptance of the photon: Compton effect, 1923-24.

Specific heat capacity of solid bodies (Einstein 1907)

Dulong-Petit law (1819): $c \times A \cong 6$ cal/K

Boltzmann (1876): $E = 3NkT = RT$, $c = dE/dT = d(3RT)/dT = 3R$

Anomalies (diamond; low temperatures)

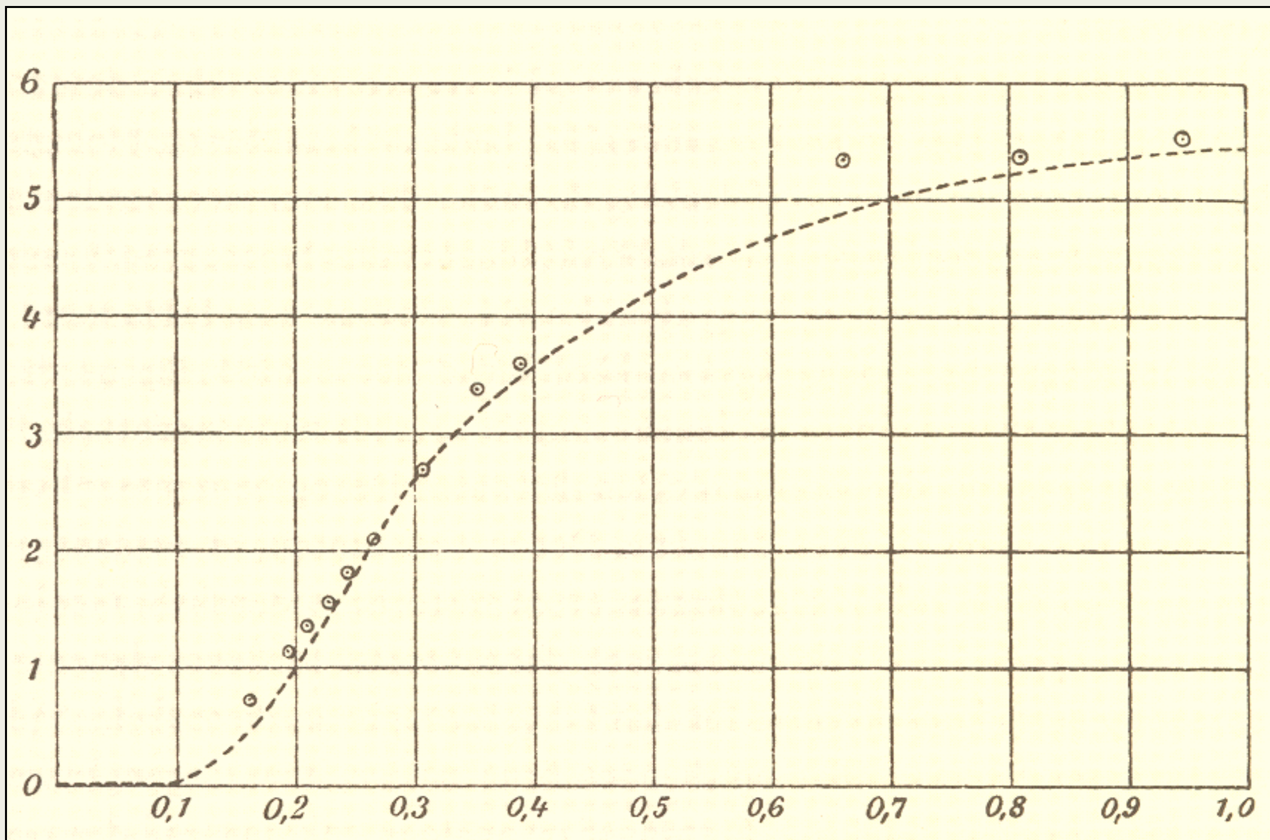
Einstein (1907):

$$kT \rightarrow \frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

$$E_{oscillators} = \frac{3h\upsilon N_A}{e^{h\upsilon/kT} - 1} mole^{-1}$$

$c \rightarrow 3R$ for large T

$c \rightarrow 0$ for $T \rightarrow 0$



Quantum theory: 1st Solvay Congres, 1911

Physics anno 1910 (*Beiblätter zu den Annalen der Physik*)

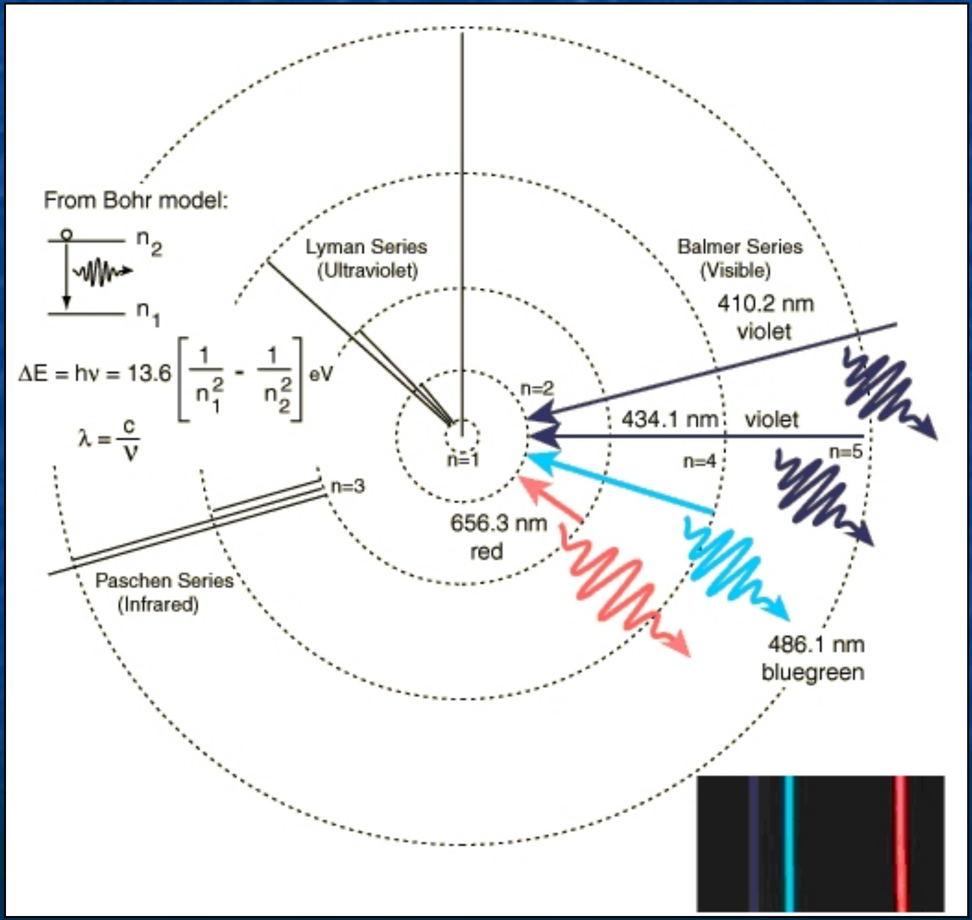
Subject	*Number*	*%*
Electricity & magnetism	913	31
Optics	488	16
Constitution & structure of matter	351	12
Cosmic physics	321	11
Heat, thermodynamics	293	10
Mechanics	264	9
Radioactivity	182	6
Measure and weight	60	2
General	59	2
Acoustics	27	1
History & biography	26	1

Bohr's 1913 explanation of the H-spectrum and Balmer's law

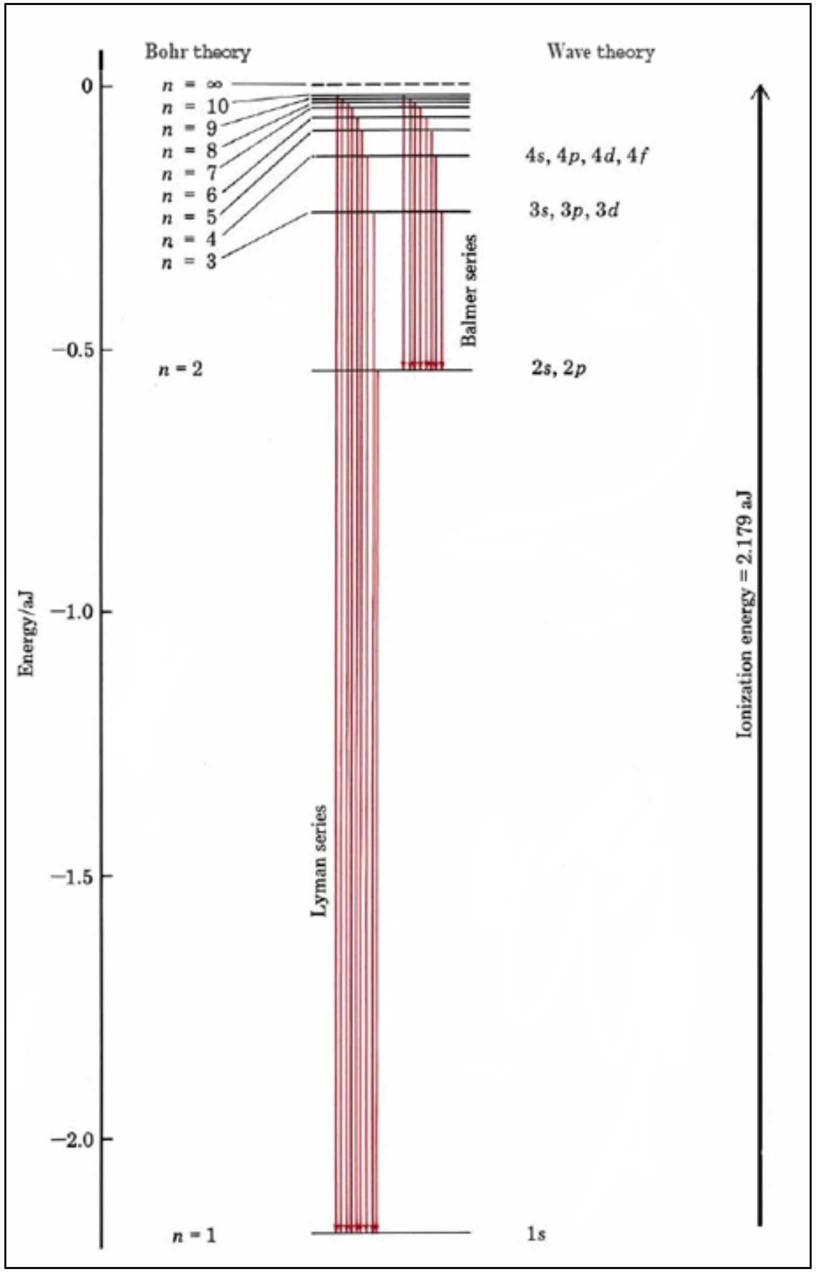
$$\nu = cR\left(\frac{1}{2^2} - \frac{1}{n^2}\right), n = 3,4,\ldots$$

$$R = \frac{2Z^2\pi^2 me^4}{h^3}$$

$$m_{red} = \frac{mM}{m+M} = \frac{m}{1+m/M}$$



H_α line:
Transition from n=3 to n=2



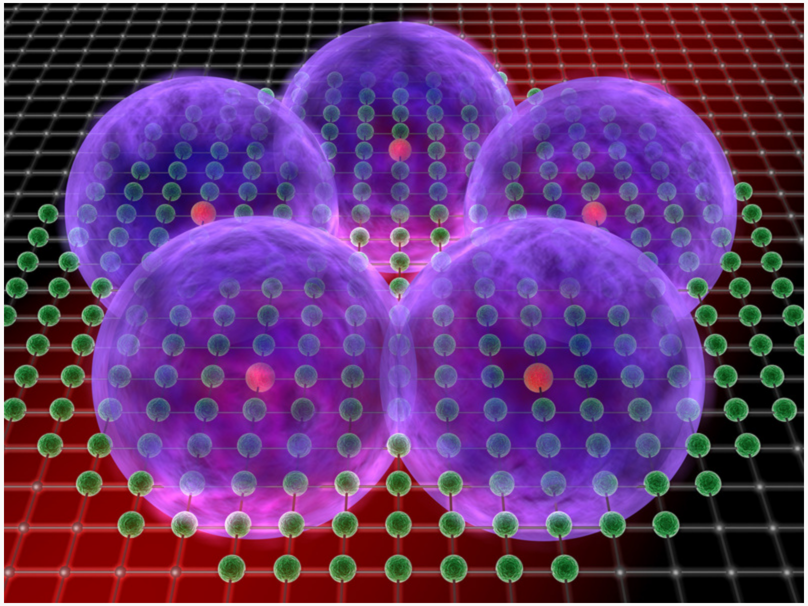
Rydberg (Bohr-Rydberg) atoms

$a_n = n^2 a_0$, n = 1,2,3,...

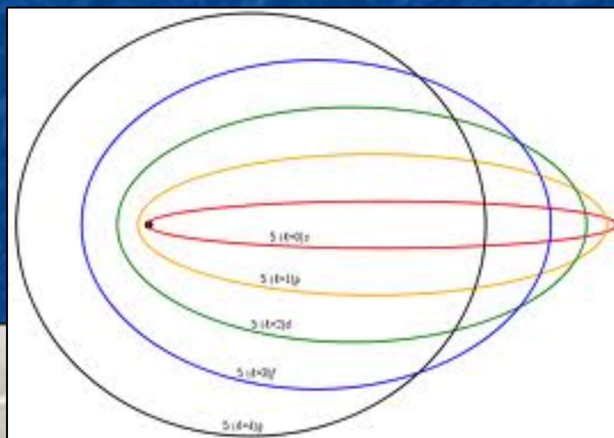
$a_0 = 0.53 \times 10^{-8}$ cm (Bohr radius)

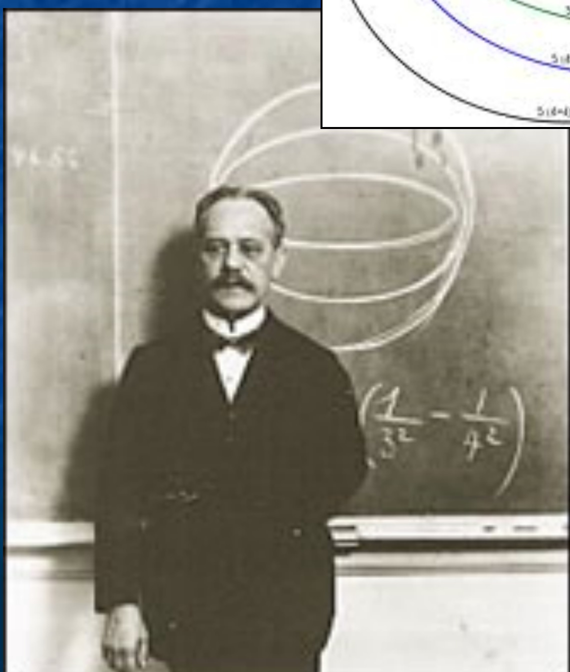
The diameter of an H-atom with n = 300

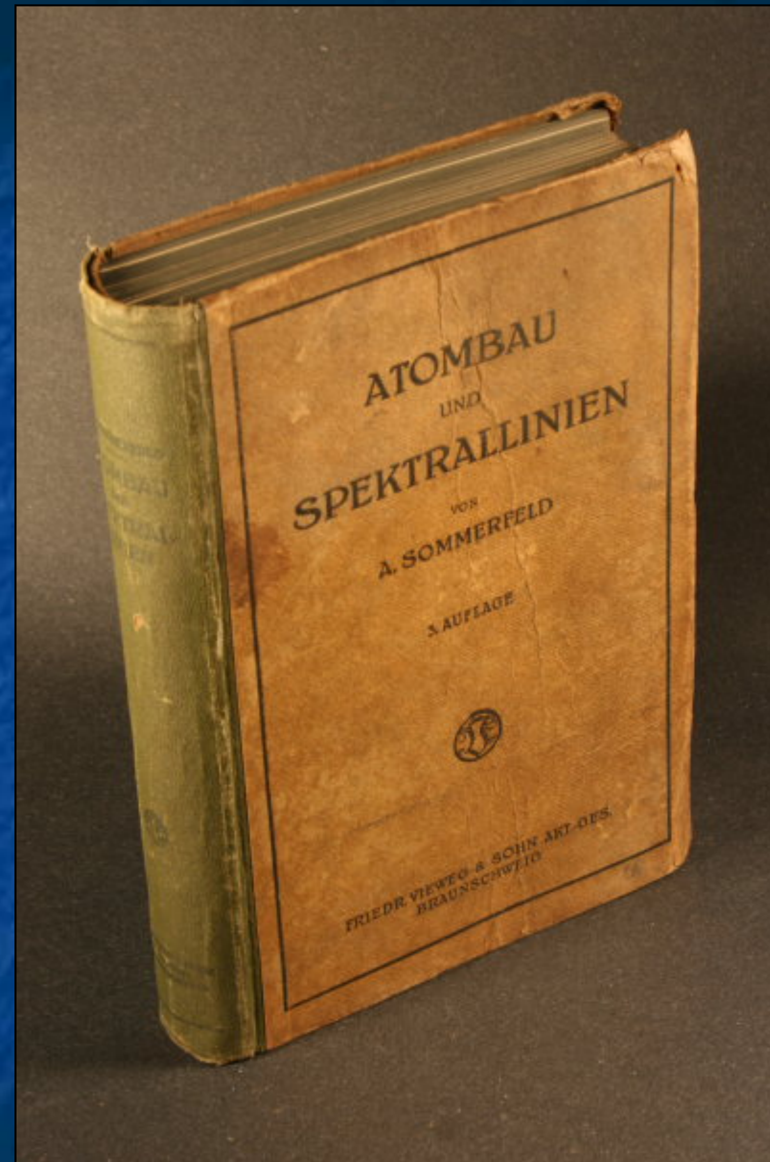
is of the order 0,001 cm!



Arnold Sommerfeld: electrons move in elliptic orbits. Relativistic correction (fine-structure)

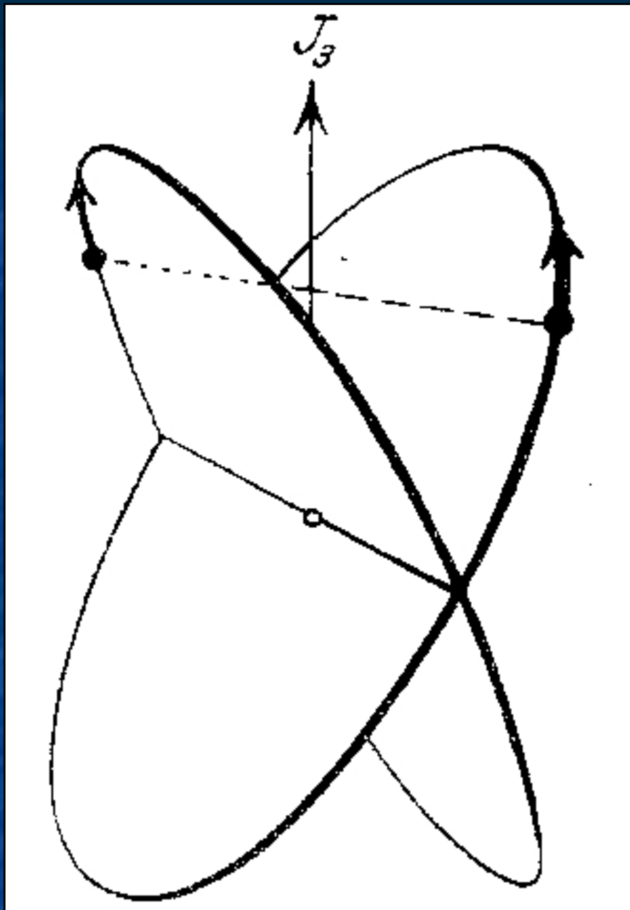


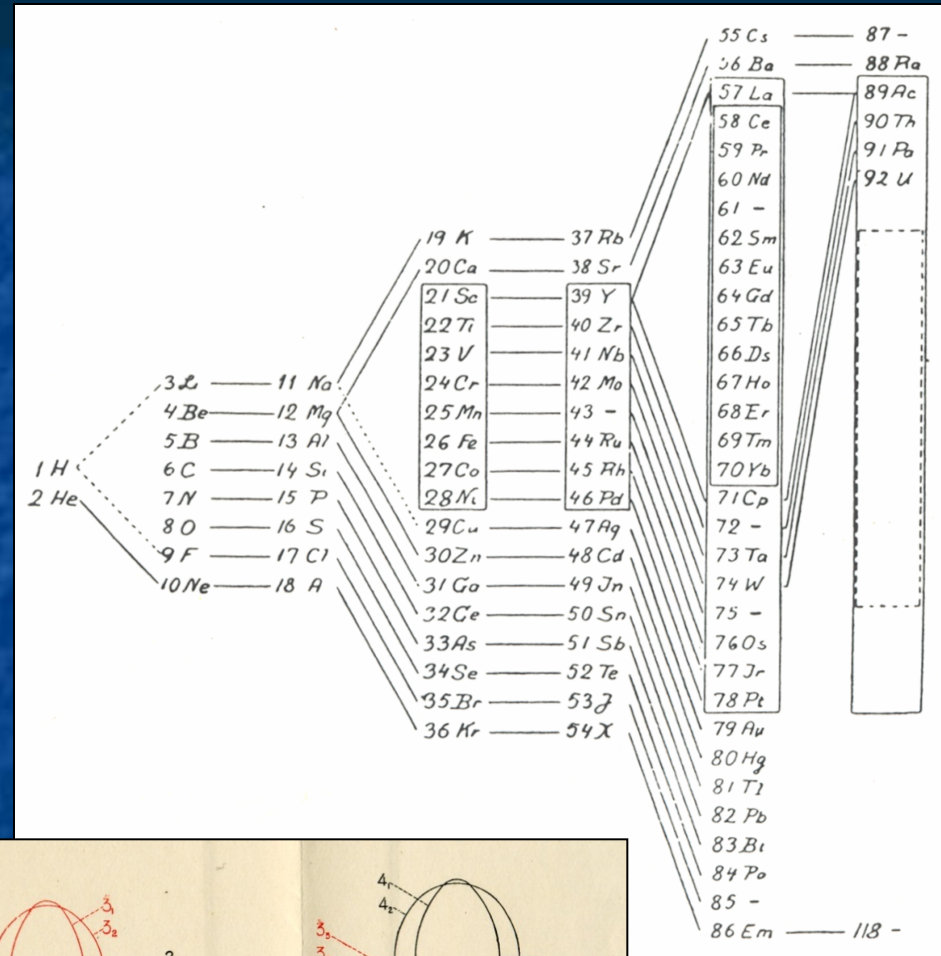


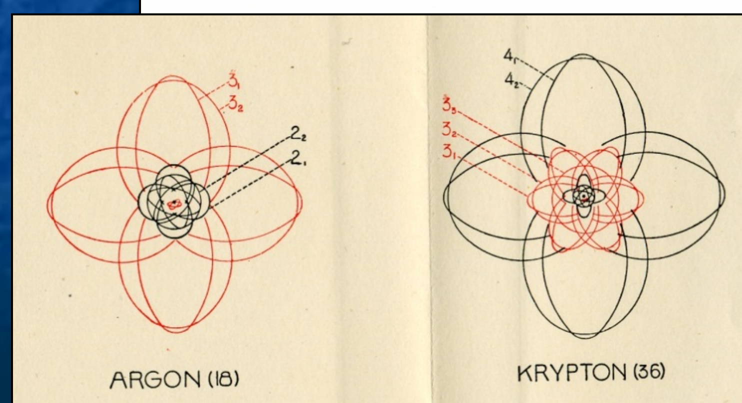


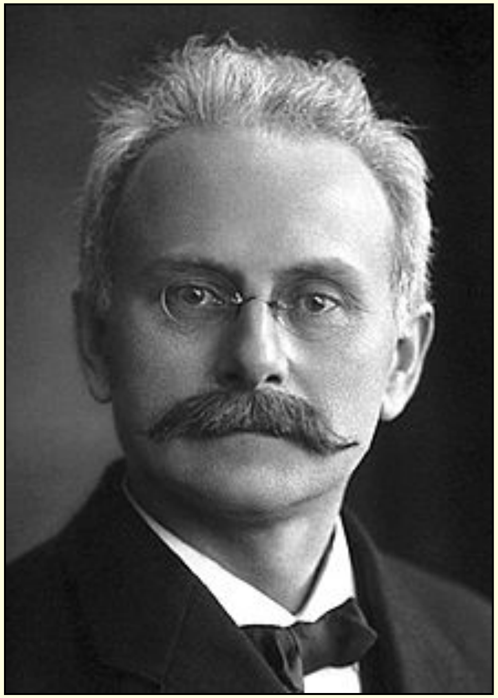
"The Bible of Atomic Physics"

The helium atom puzzle





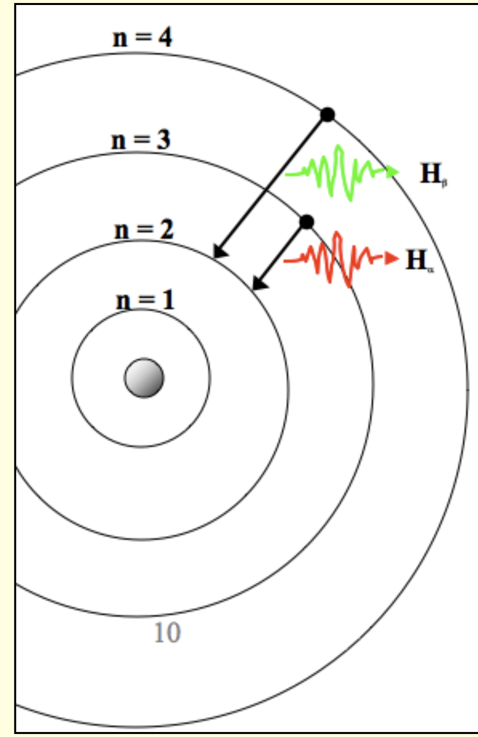


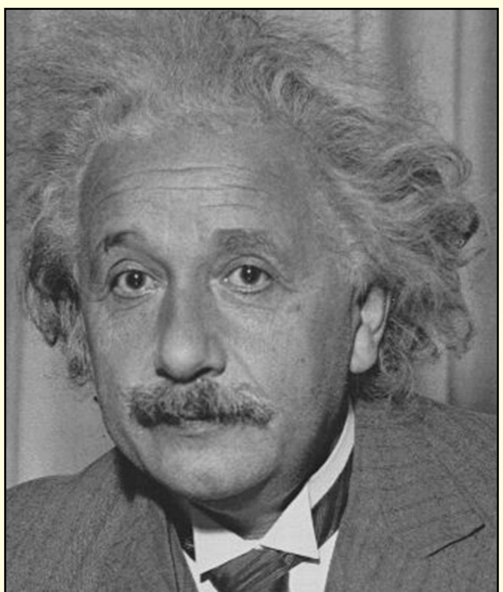


Philosophical problems

How can an elektron pass the "forbidden area" between two orbits?

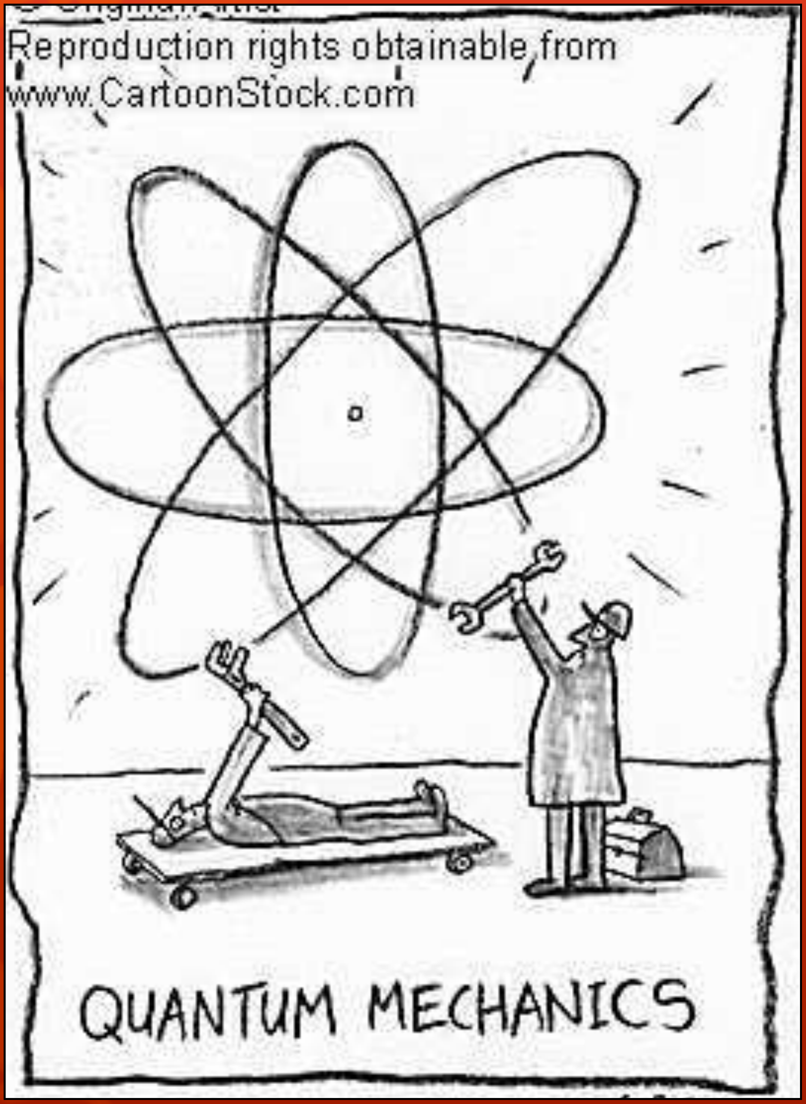
How can it "know" where to end?





"I find the idea quite intolerable that an electron exposed to radiation should choose *of its own free will*, not only its moment to jump off, but also its direction. In that case, I would rather be a cobbler, or even an employee in a gaming-house, than a physicist."

(Einstein to Born, 1924)



1925: The new quantum mechanics – electron orbits demounted.

Paul Dirac and Werner Heisenberg, co-founders of QM.

Heisenberg, July 1925 (matrix mechanics); age 24

Dirac, October 1925 (q-number algebra), age 23

$$\boldsymbol{pq} - \boldsymbol{qp} = \frac{h}{2\pi i}\mathbf{1}$$

Physical quantities do not generally commute – what does it mean?

Zero-point energy, harmonic oscill.

$E_n = (n + \frac{1}{2})h$[illegible]

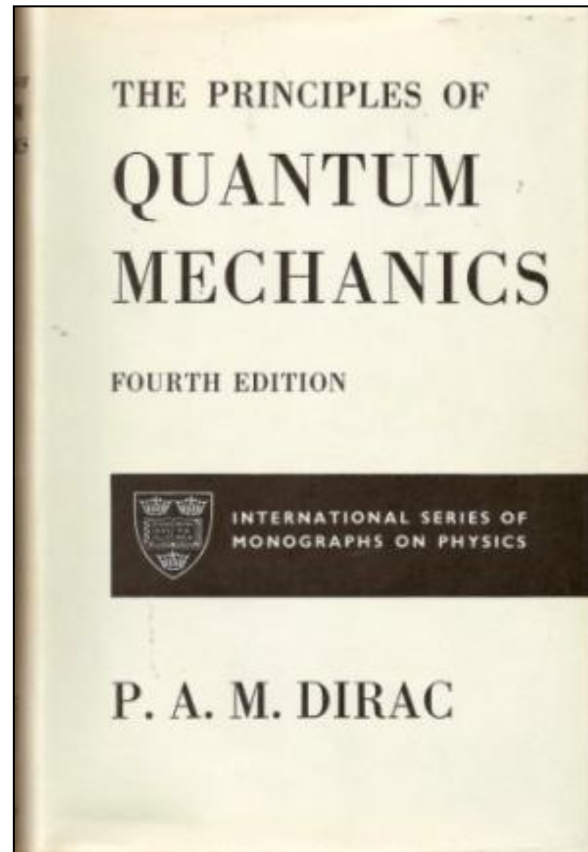
Dirac realized that non-cummutation appeared in the Poisson brackets introduced by Siméon Poisson in 1809. For generalized position and momentum variables *q* and *p*, the brackets are

$$[x,y] = \sum_j \left(\frac{\partial x}{\partial q_j} \frac{\partial y}{\partial p_j} - \frac{\partial x}{\partial p_j} \frac{\partial y}{\partial q_j} \right)$$

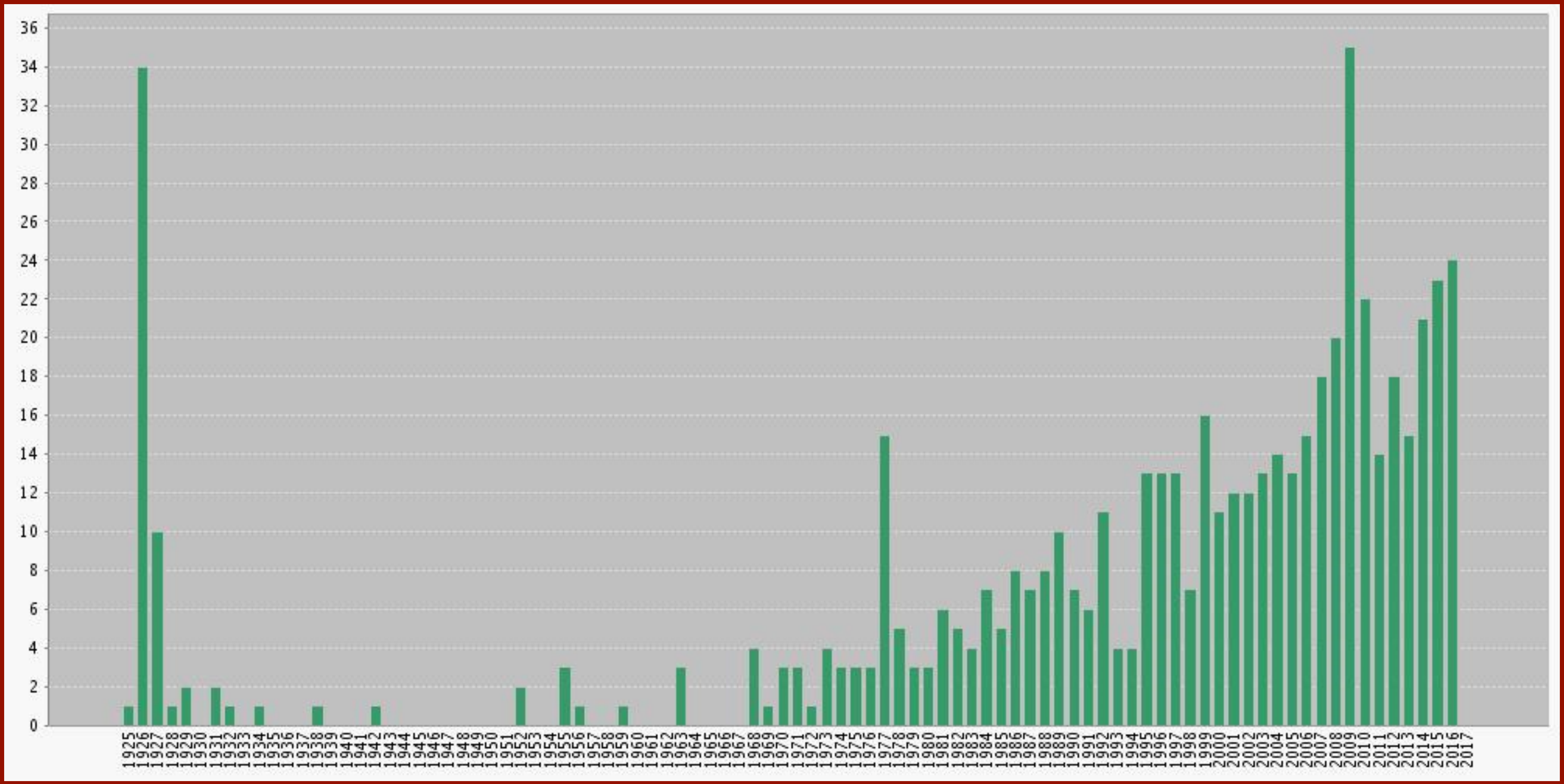
$$[q_j, q_k] = 0; [p_j, p_k] = 0; [p_j, q_k] = \delta_{jk}$$

$$\delta_{jk} = 1\ (j = k), 0\ (j \neq k)$$

The analogy led him to an abstract theory expressed in terms of non-commutative ”*q*-numbers” and classical ”*c*-numbers.”



Dirac’s classical and remarkably successful textbook of 1930, several later editions and still widely sold and read.



Citations to Heisenberg’s 1925 *Umdeutung* paper

The phenomenal growth of early quantum mechanics

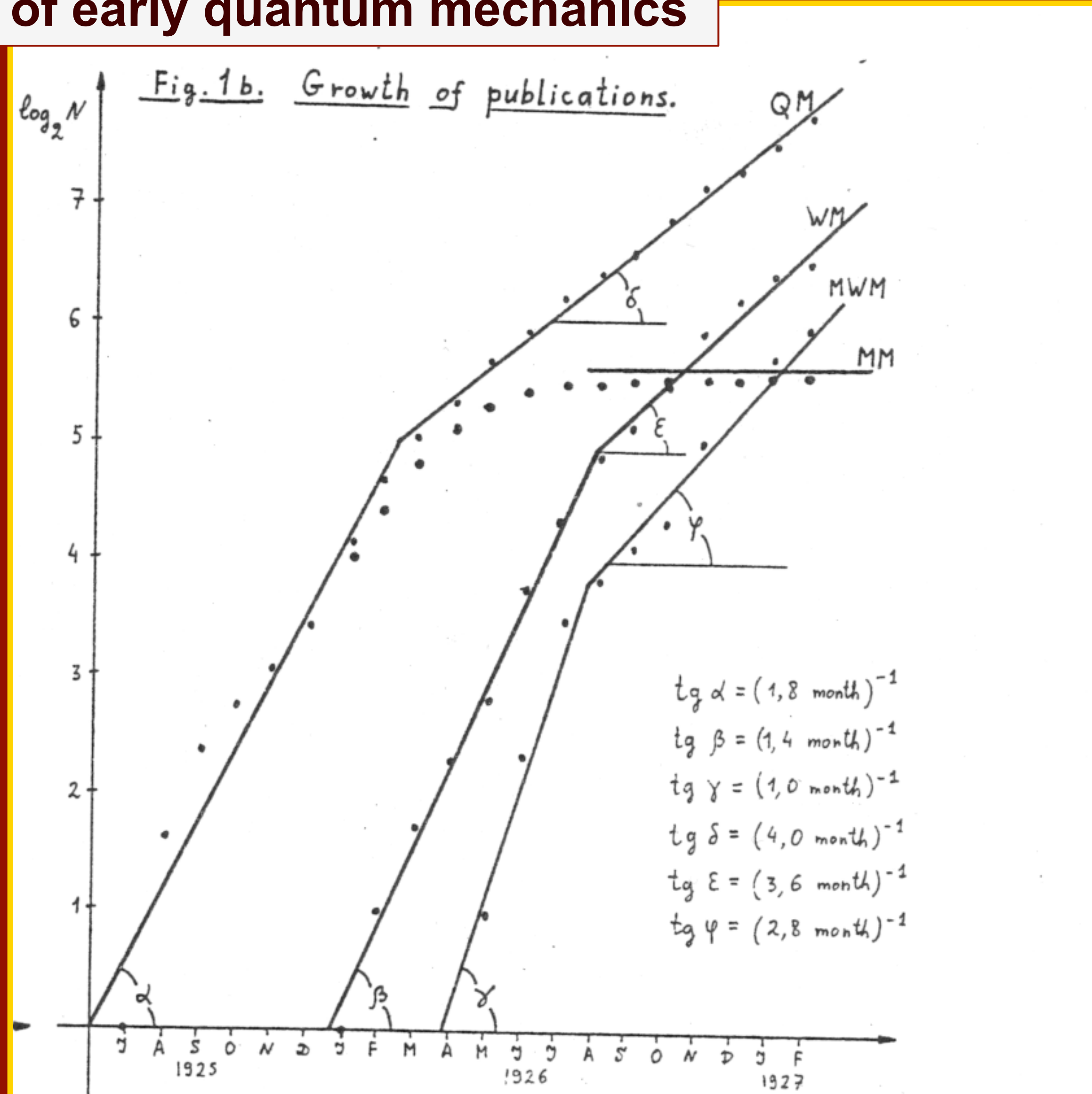
Just 2 years after Heisenberg's breakthrough, QM was essentially complete.

QM = all versions

WM = wave mechanics

MWM = modified wave mechanics

MM = matrix mechanics

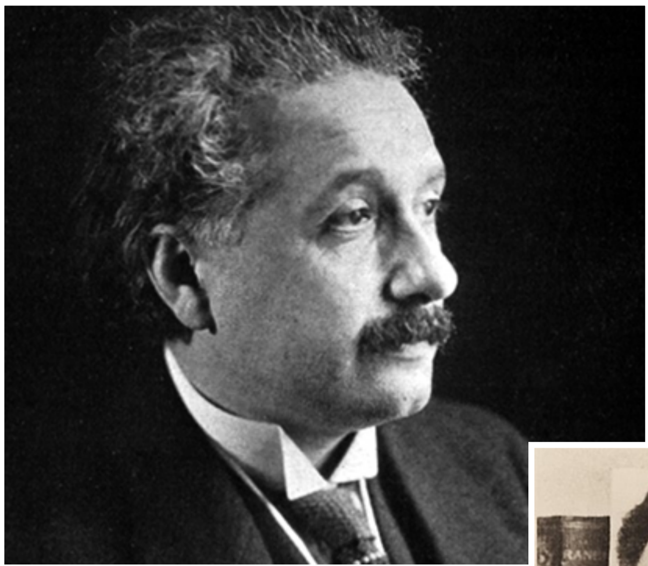


	J	A	S	O	N	D	J	F	M	A	M	J	J	A	S	O	N	D	J	F
MM	1	3	5	6	8	10	16	21	28	36	38	40	45	45	47	48	49	49	49	50
WM							1	2	3	5	7	13	20	28	35	48	60	74	86	93
MWM											2	5	10	14	17	20	35	47	52	60
Σ	1	3	5	6	8	10	17	23	31	41	47	58	75	87	99	116	144	170	187	203

Publications and authors in QM, July 1925 to March 1927

Country	No. of authors	Papers written in	Papers published in
Germany	19	54	120
Britain	6	18	30
USA	19	26	27
France	2	12	14
USSR	11	11	9
Denmark	4	17	1
Total	81	182	203

Photons: energy, hence mass?



$$m^2c^4 = E^2 - p^2c^2$$

Einstein

$E = mc^2$ (particle)

$E = h\nu$ (wave)

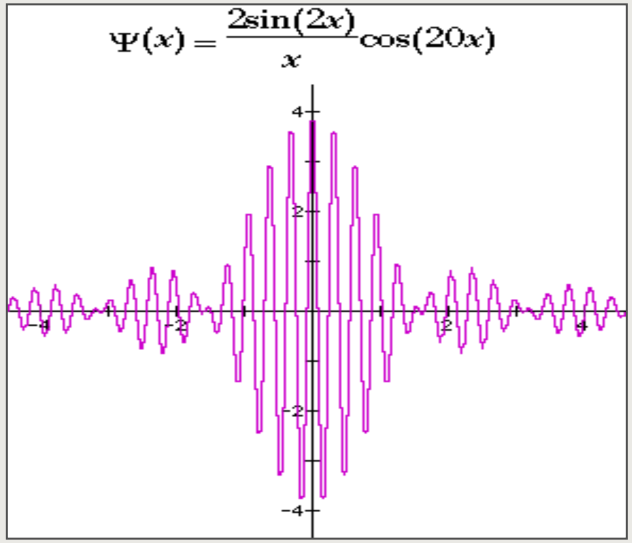
$mc^2 = h\nu$ (?!?)

("photon mass" = $h\nu/c^2$)

($\nu = c/\lambda$)

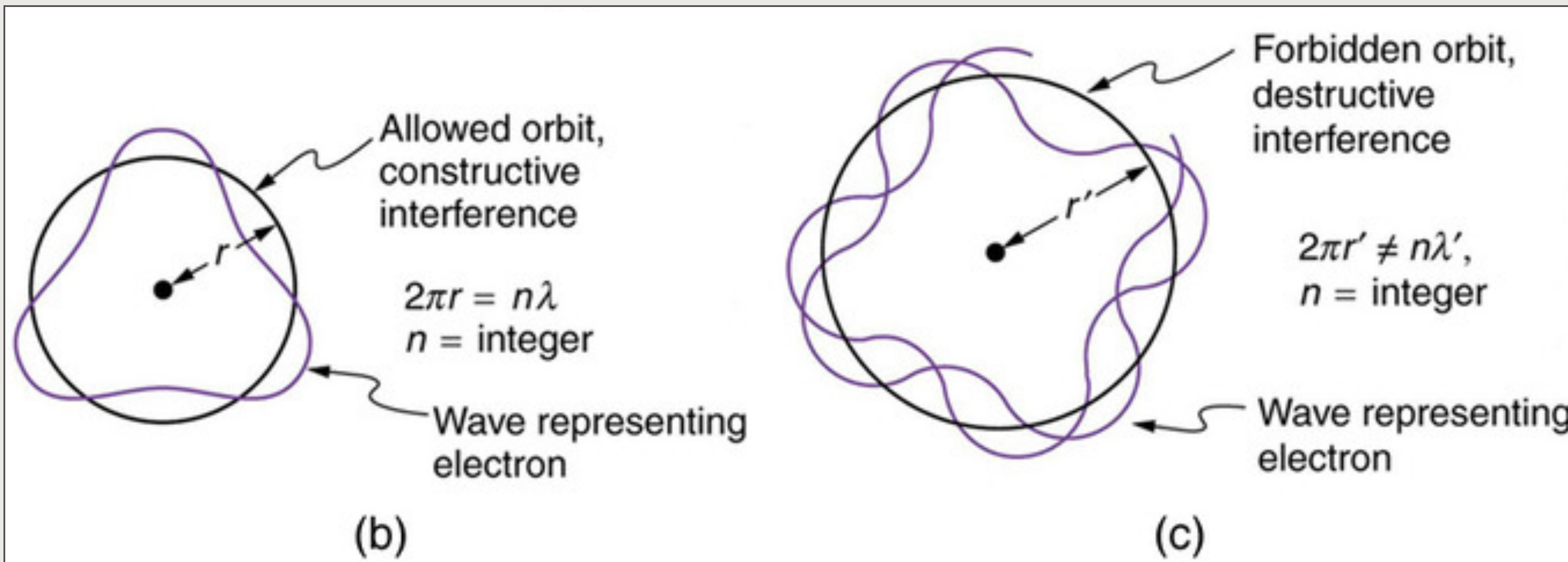
$$\lambda = h/p \cong h/mv$$

Louis de Broglie: particles as waves?

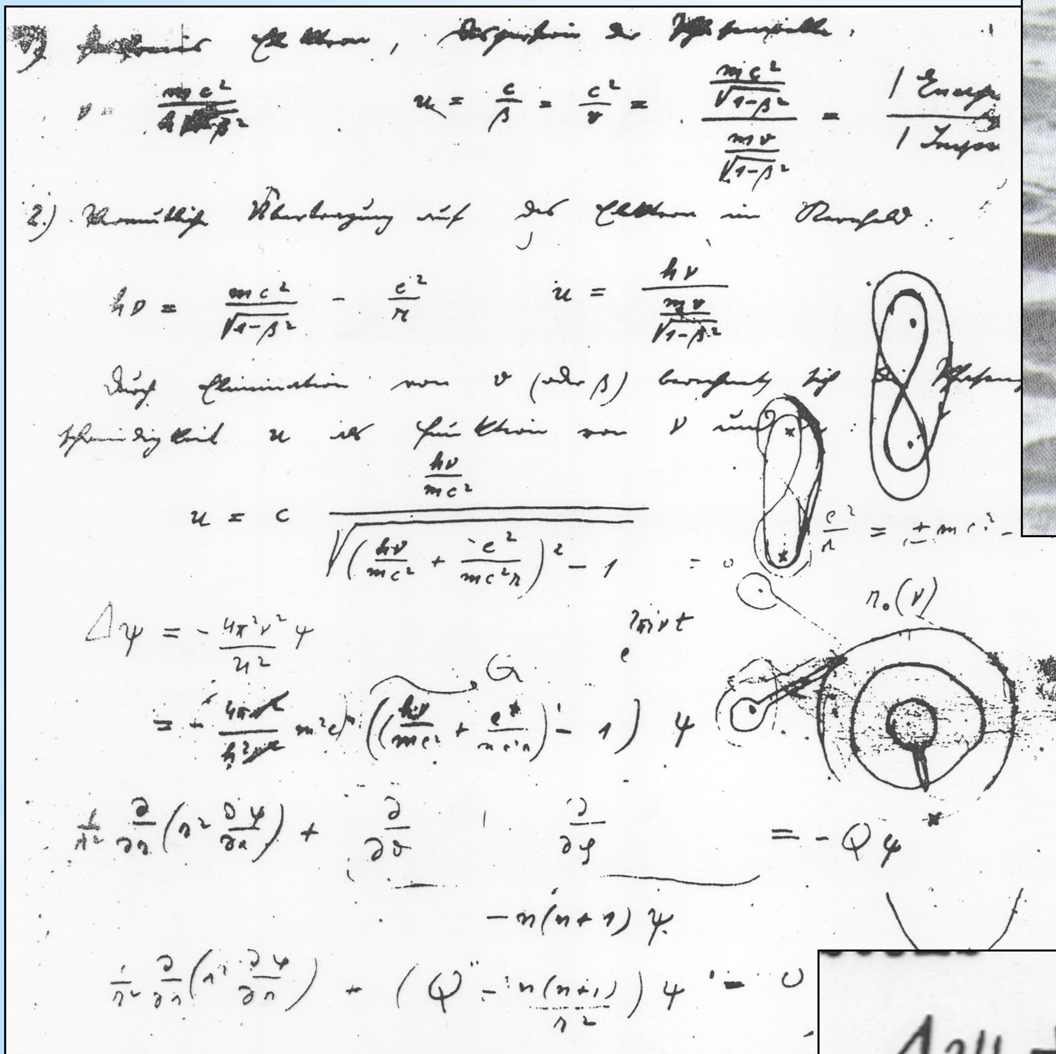


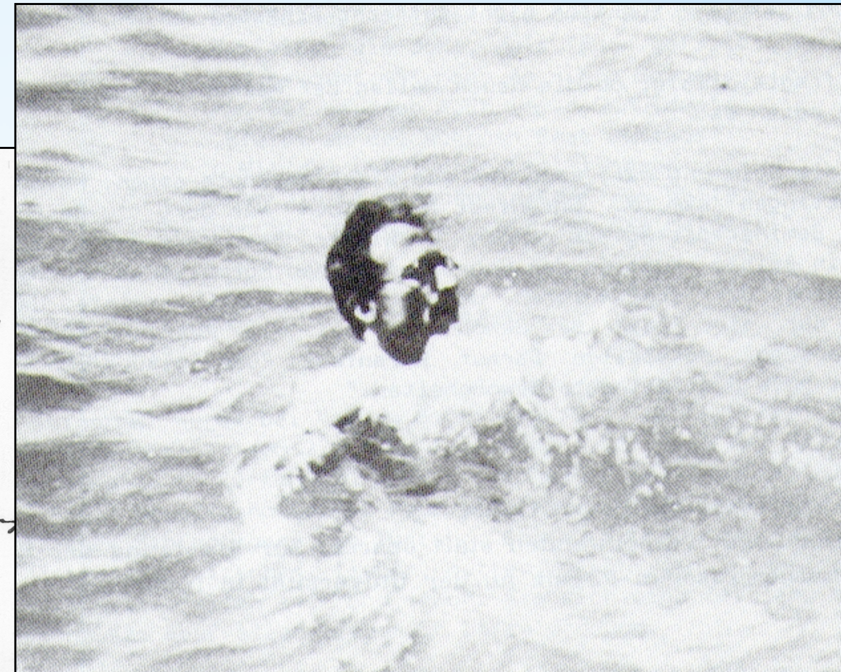
$$\oint p\, d\varphi = n_\varphi h$$

$$\oint p_r\, dr = n_r h$$



Schrödinger makes matter waves





$$\Delta\psi + \frac{2m}{K^2}\left(E + \frac{e^2}{r}\right)\psi = 0$$

"Im Augenblick plagt mich eine neue Atomtheorie. *Wenn ich nur mehr Mathematik könnte!* … Vorläufig muss ich noch Mathematik lernen, um das Schwingungsproblem ganz zu übersehen – eine lineare Differerenzialgleichungen, der Bessel'schen ähnlich, aber weniger bekannt und mit merkwürdigen Randbedingen."

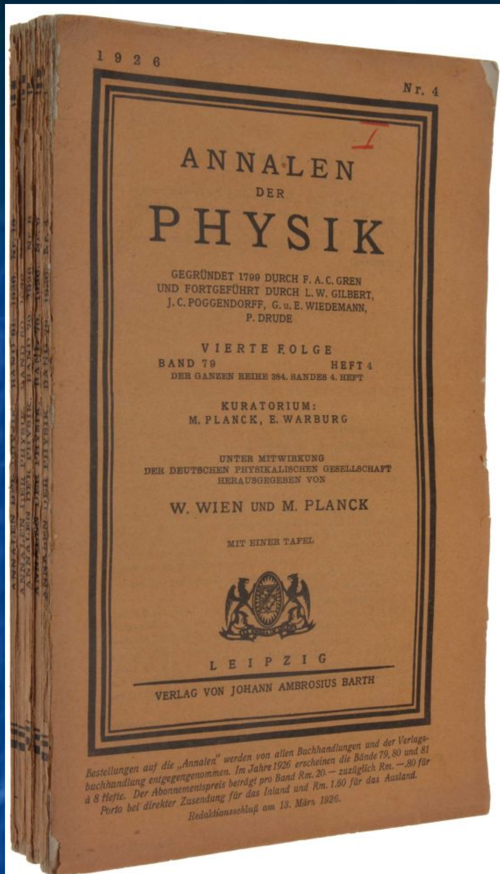
Letter to W. Wien, 27 December 1925

Based on de Broglie's ideas of matter waves, Schrödinger found a relativistic wave equation for the H-atom. He found it very hard to find the energy eigenvalues. He got a fine-structure separation, *BUT* not quite the right one:

Sommerfeld (1917): $\square f^* = R\square^2/16 \quad (f = 1/\lambda = \nu / c)$

Schrödinger (1926): $\square f = (8/3)\, R\square^2/16$

For this reason, he only published the wave equation in its non-relativistic approximation (no fine-structure).



3. *Quantisierung als Eigenwertproblem;*
von E. Schrödinger.

(Erste Mitteilung.)

§ 1. In dieser Mitteilung möchte ich zunächst an dem einfachsten Fall des (nichtrelativistischen und ungestörten) Wasserstoffatoms zeigen, daß die übliche Quantisierungsvorschrift sich durch eine andere Forderung ersetzen läßt, in der kein Wort von „ganzen Zahlen" mehr vorkommt.

Schrödinger's astounding creativity

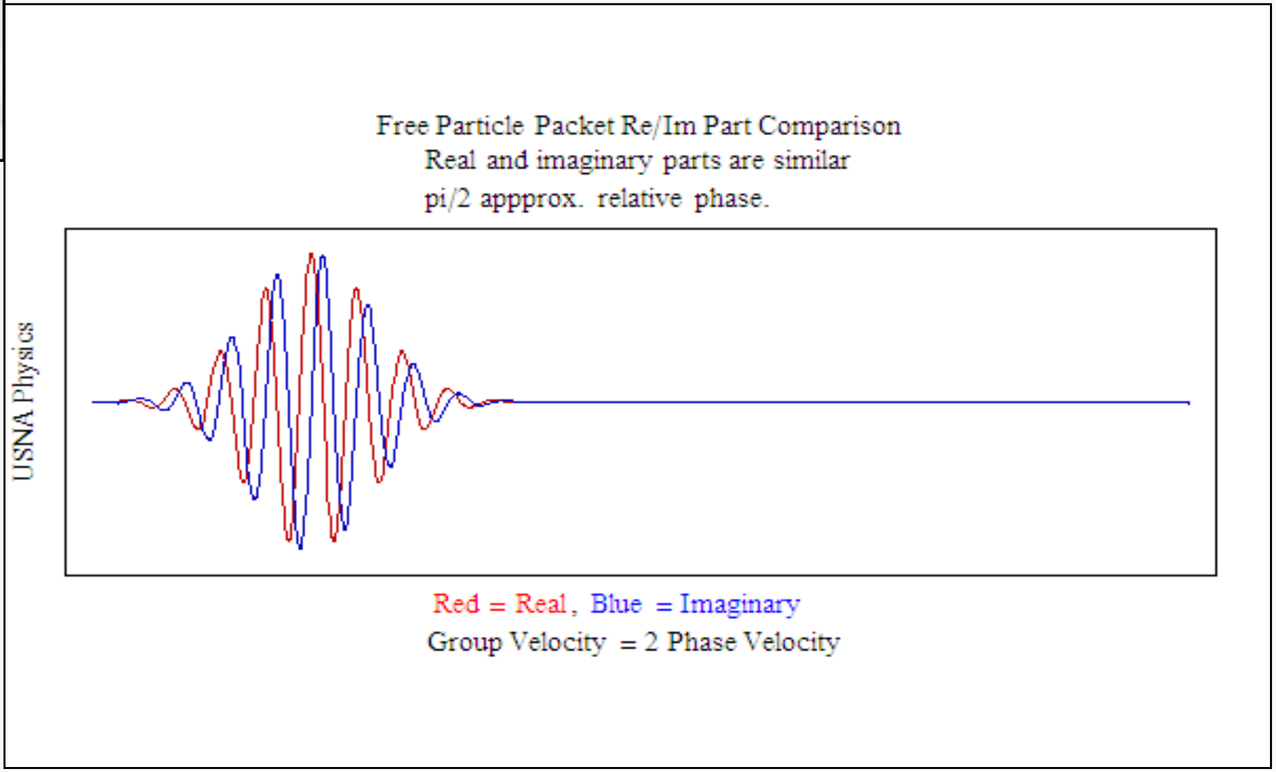
4 articles with same title, in same journal, Jan-June 1926. Moreover:

"Das Verhältnis der Heisenberg-Born-Jordanschen Quantenmechanik zu der meinen" (also *AdP*)

$$[\frac{-\hbar^2}{2m}\nabla^2+V]\Psi=i\hbar\frac{\partial}{\partial t}\Psi$$

"If we are to keep these damned quantum jumps (*verdammte Quantenspringerei*), then I regret that I became involved in quantum theory."

Schrödinger to Lorentz, 1926

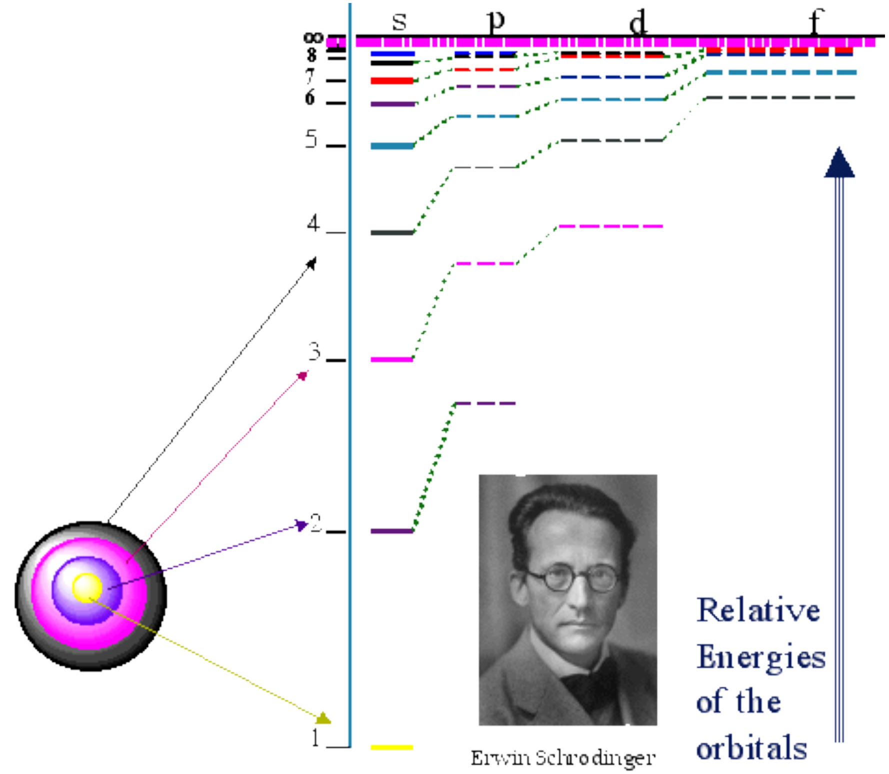


Schrödinger was *"abgeschrecht, um nicht zu sagen abgestossen"* by Heisenberg's theory because of its lack of *Anschaulichkeit*.

$$E\psi = -\frac{\hbar^2}{2m}\nabla^2\psi - \frac{e^2}{4\pi\varepsilon}\frac{1}{r}\psi$$

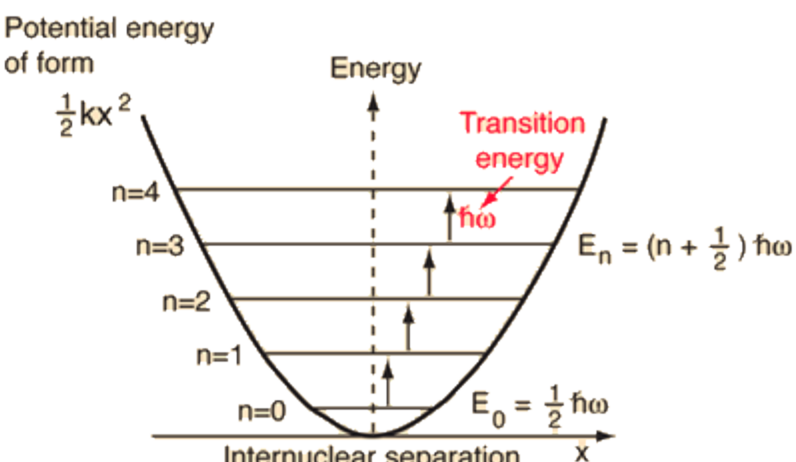
$$\Psi(r,\theta,\phi) = R(r)P(\theta)F(\phi)$$

n	ℓ	m_ℓ
principal quantum number	orbital quantum number	magnetic quantum number



Erwin Schrodinger

Hydrogen atom



Harmonic oscillator

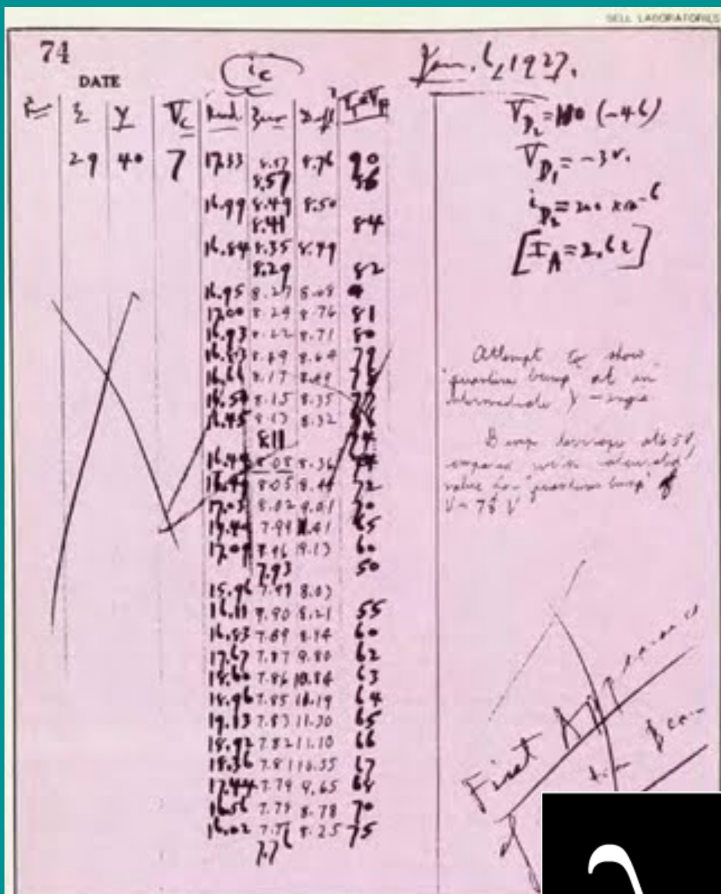
(with zero-point energy)

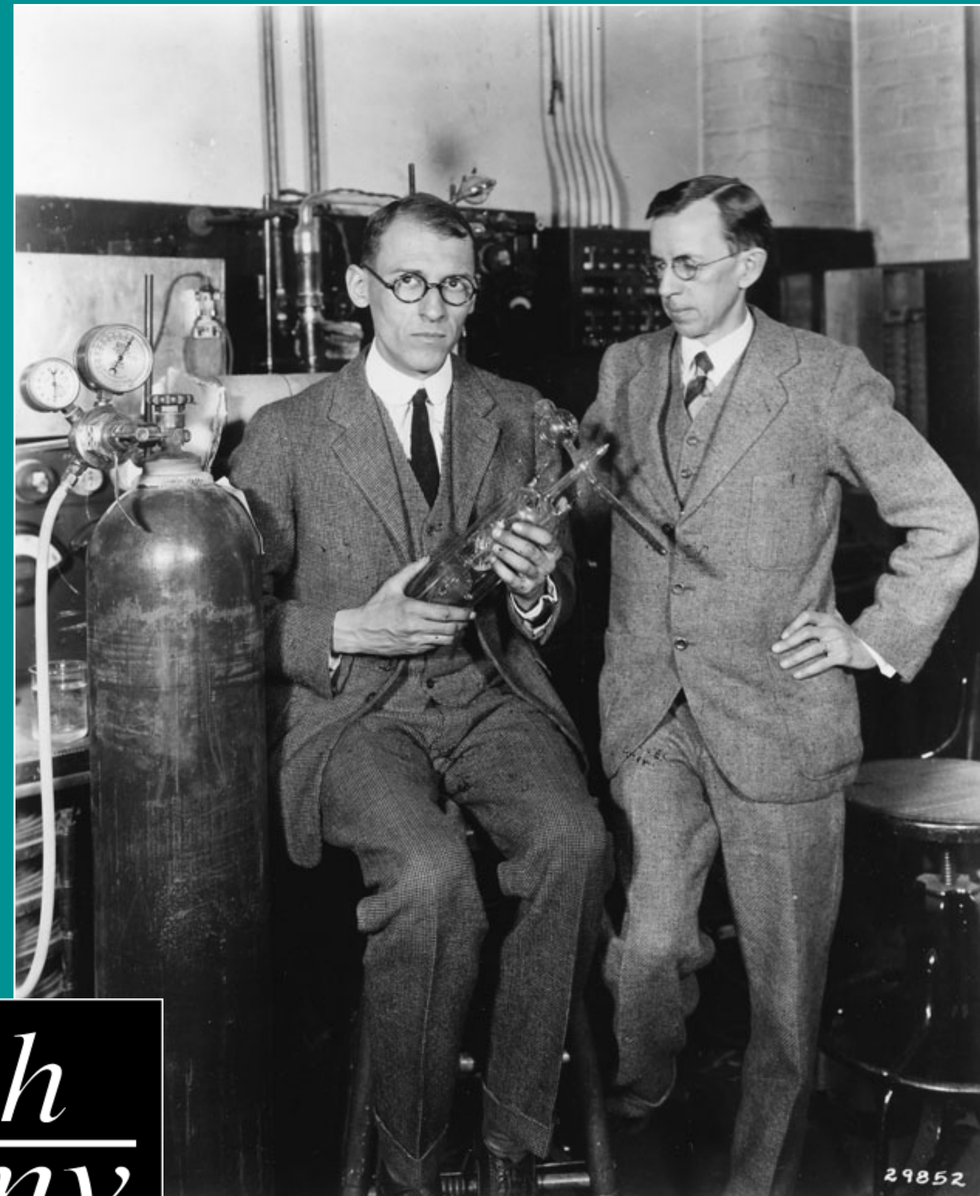
$$\frac{-\hbar^2}{2m}\frac{d^2\Psi(x)}{dx^2} + \frac{1}{2}m\omega^2x^2\,\Psi(x) = E\Psi(x)$$

Electron diffraction:

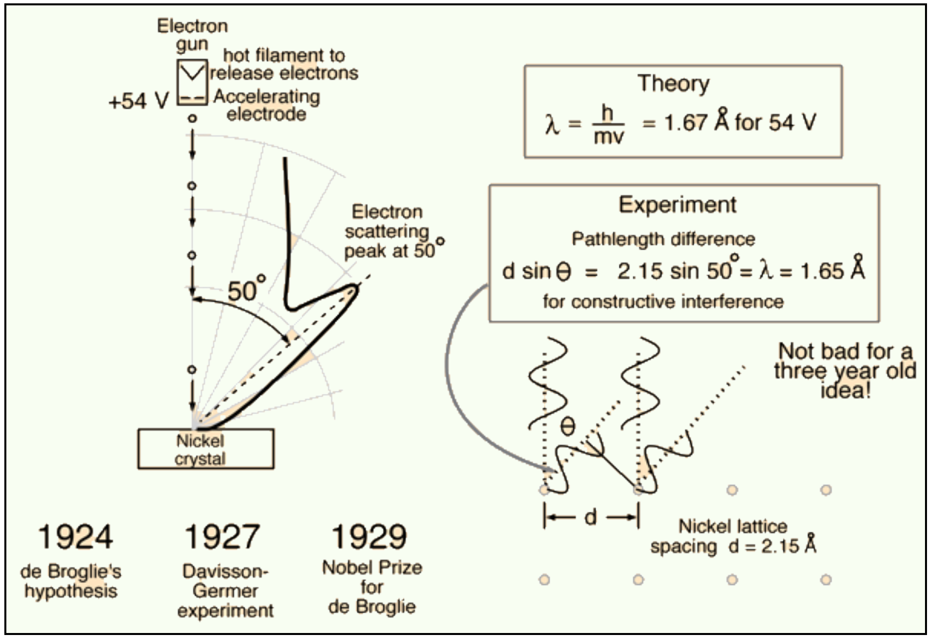
Electrons exhibit wave properties

(Davisson & Germer, 1927)





$$\lambda = \frac{h}{mv}$$



A chance discovery made at an industrial laboratory (Bell Labs). Originally Davisson was unaware of theory of matter-waves; he worked on the surface of Ni-crystals for technical purposes.

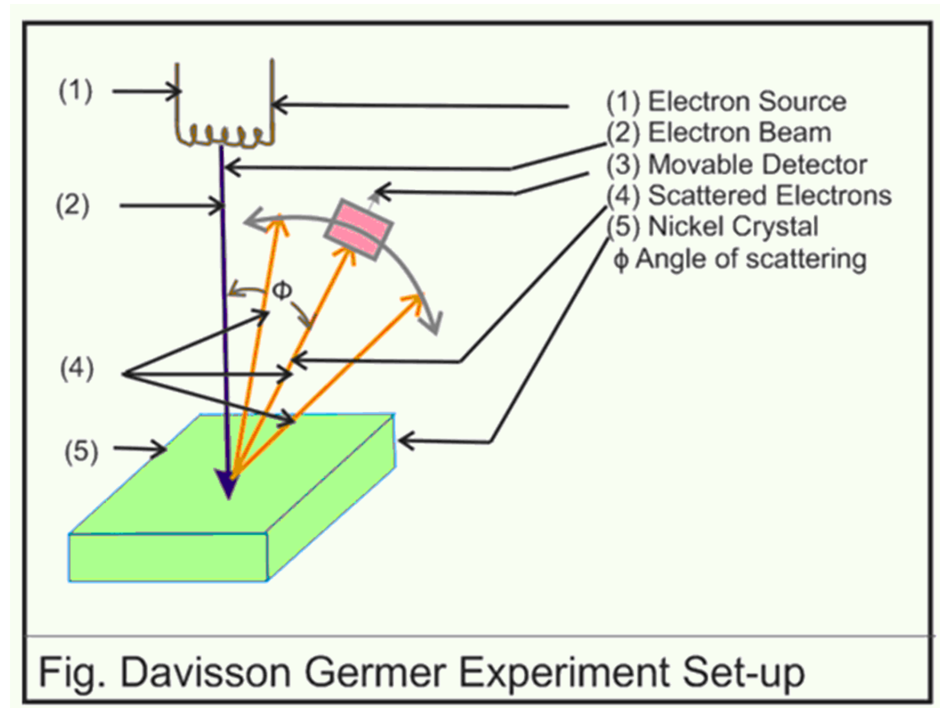


Fig. Davisson Germer Experiment Set-up

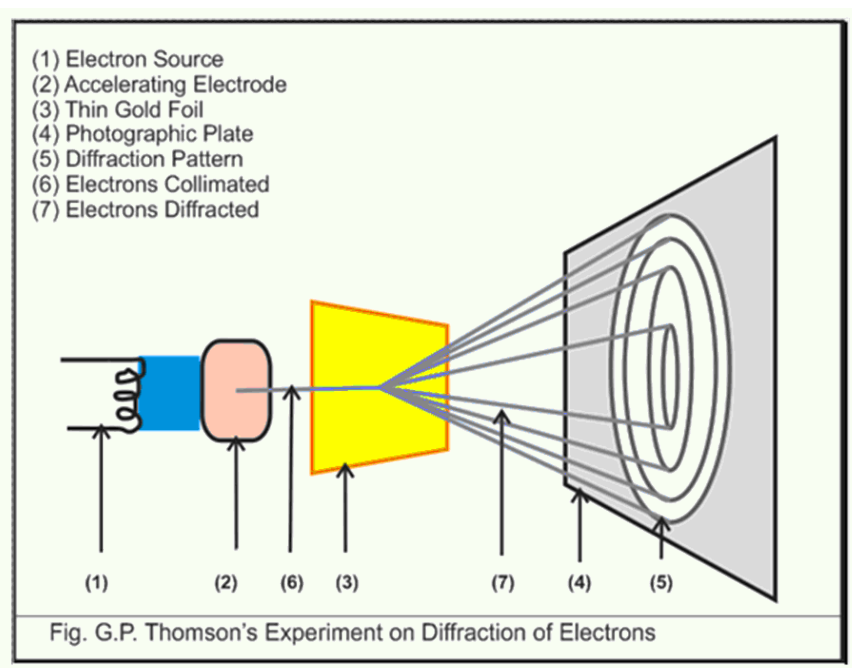
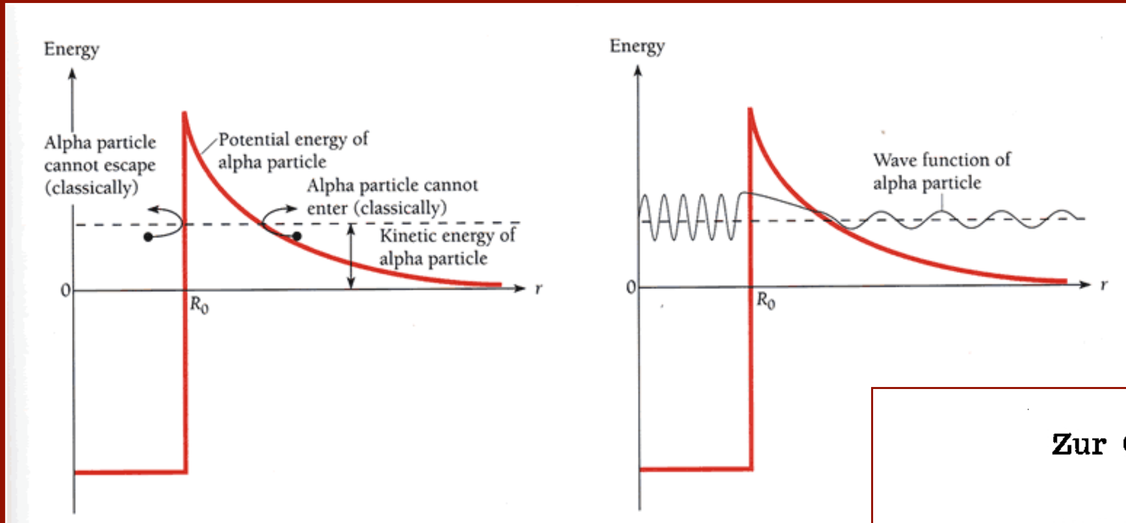


Fig. G.P. Thomson's Experiment on Diffraction of Electrons

Fundamental physics & industrial research: 14 Nobel Prizes (!) to Bell Labs physicists, 1937-2014.



1928: QM valid also for the tiny atomic nucleus ($r \sim 10^{-15}$ m)

Zur Quantentheorie des Atomkernes.

Von **G. Gamow**, z. Zt. in Göttingen.

Mit 5 Abbildungen. (Eingegangen am 2. August 1928.)

Es wird der Versuch gemacht, die Prozesse der α-Ausstrahlung auf Grund der Wellenmechanik näher zu untersuchen und den experimentell festgestellten Zusammenhang zwischen Zerfallskonstante und Energie der α-Partikel theoretisch zu erhalten.

. . . .

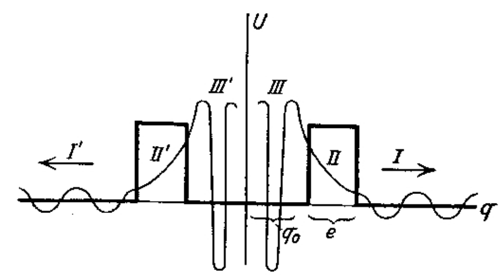


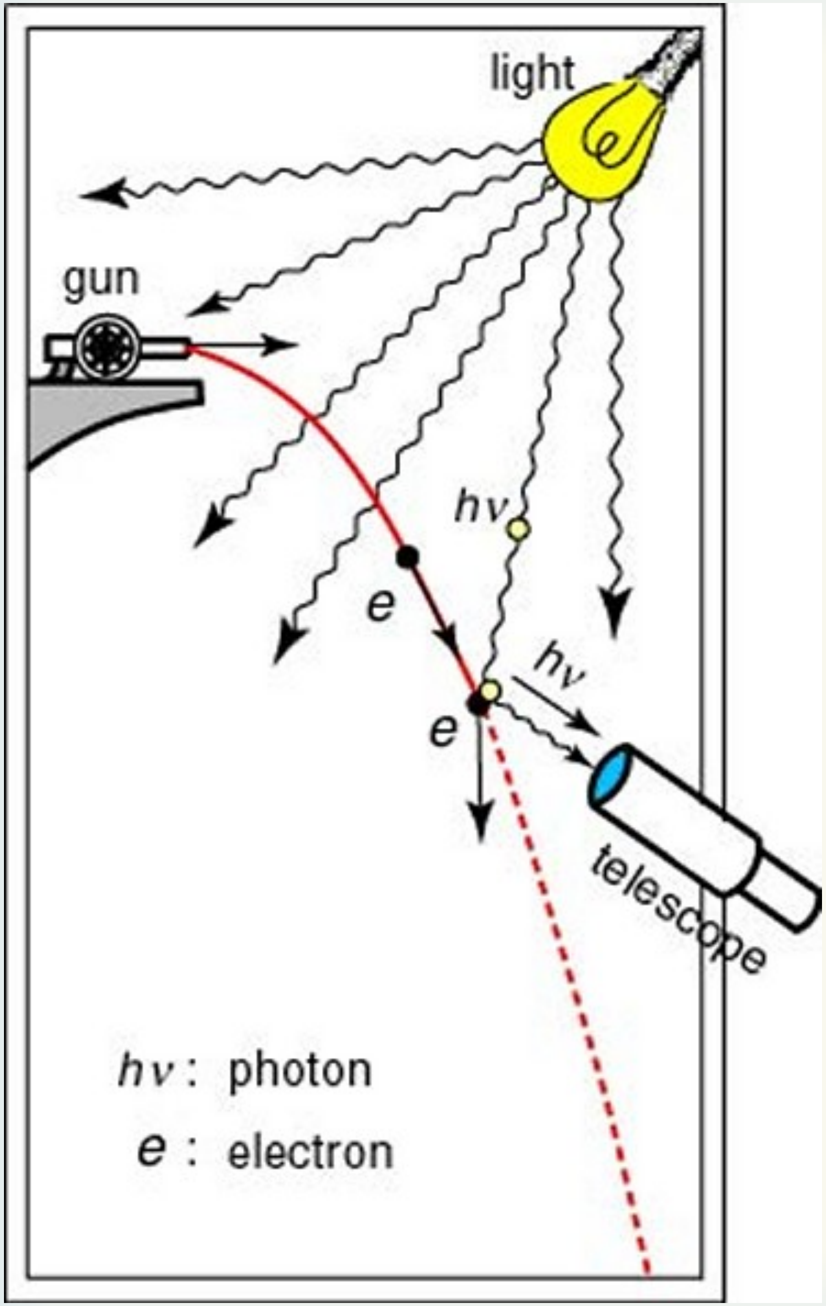
Fig. 3.

erfüllen und nur eine Konstante α zur Verfügung. Die physikalische Ursache dieser Unmöglichkeit ist, daß die aus diesen zwei Lösungen konstruierte ψ-Funktion dem Erhaltungssatz

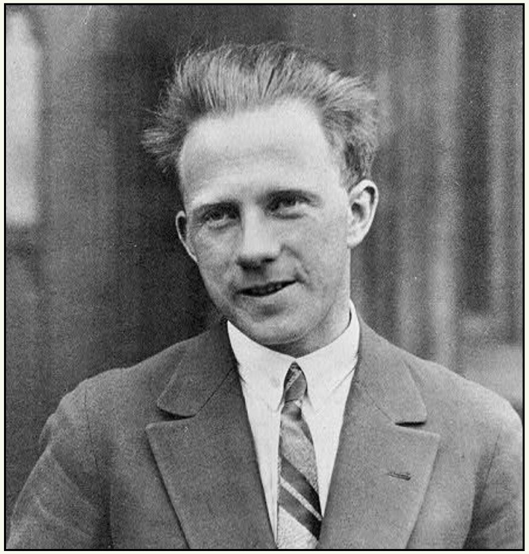
$$\frac{\partial}{\partial t}\int_{-(q_0+l)}^{+(q_0+l)} \psi\,\overline{\psi}\,dq = 2\cdot\frac{-h}{4\pi i m}\left[\psi \operatorname{grad}\overline{\psi} - \overline{\psi}\operatorname{grad}\psi\right]_{\mathrm{I}}$$

nicht genügt.

George Gamow



$$\Delta p \, \Delta x \geq \frac{1}{2} \hbar$$



Although Schrödinger equation has same structure as classical equation of motion, QM is fundamentally indeterministic.

(Nobel lecture)