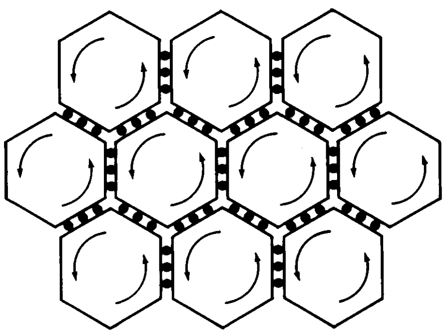
# Module 8
# Maxwell's analogies



UNIVERSITY OF COPENHAGEN

# Introduction

- Maxwell's equations – Physics' Holy Grail

| | | |
|---|---|---|
| (i) | $\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho$ | (Gauss's law), |
| (ii) | $\nabla \cdot \mathbf{B} = 0$ | (no name), |
| (iii) | $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ | (Faraday's law), |
| (iv) | $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$ | (Ampère's law with Maxwell's correction). |

- How do they look in Maxwell´s originals?
    - What models did he have in mind?
    - Which formalism did he use?
    - How was the development that led to Maxwell's equations?

# On Faraday's Lines of Force (Maxwell, 1855)

## Role of [formal] analogies

In order to obtain physical ideas without adopting a physical theory we must make ourselves familiar with the existence of physical analogies. By a physical analogy I mean that partial similarity between the laws of one science and those of another which makes each of them illustrate the other. Thus all the mathematical sciences are founded on relations between physical laws and laws of numbers, so that the aim of exact science is to reduce the problems of nature to the determination of quantities by operations with numbers.

# On Faraday's Lines of Force (Maxwell, 1855)

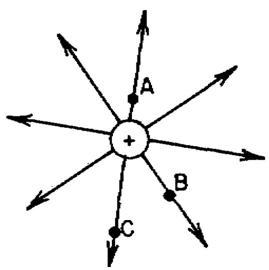
## No experiment / Place before the mathematical mind

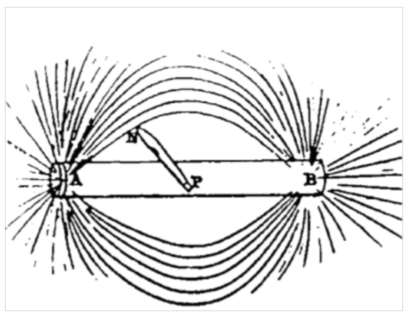
By the method which I adopt, I hope to render it evident that I am not attempting to establish any physical theory of a science in which I have hardly made a single experiment, and that the limit of my design is to show how, by a strict application of the ideas and methods of Faraday, the connexion of the very different orders of phenomena which he has discovered may be clearly placed before the mathematical mind.

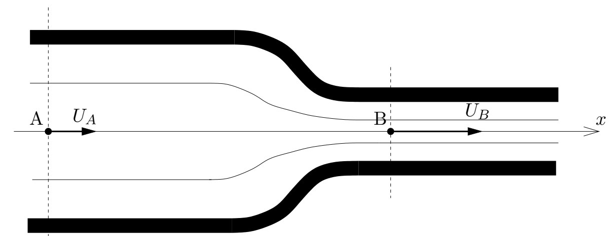
# On Faraday's Lines of Force (Maxwell, 1855)

## Intensity of the force

[…] we might find a line passing through any point of space representing the *direction* of the force acting on a positively electrified particle or on an elementary north pole.




$$v_A \cdot S_A = v_B \cdot S_B = const.$$

$$F \propto v$$

$$F \propto \frac{1}{S}$$

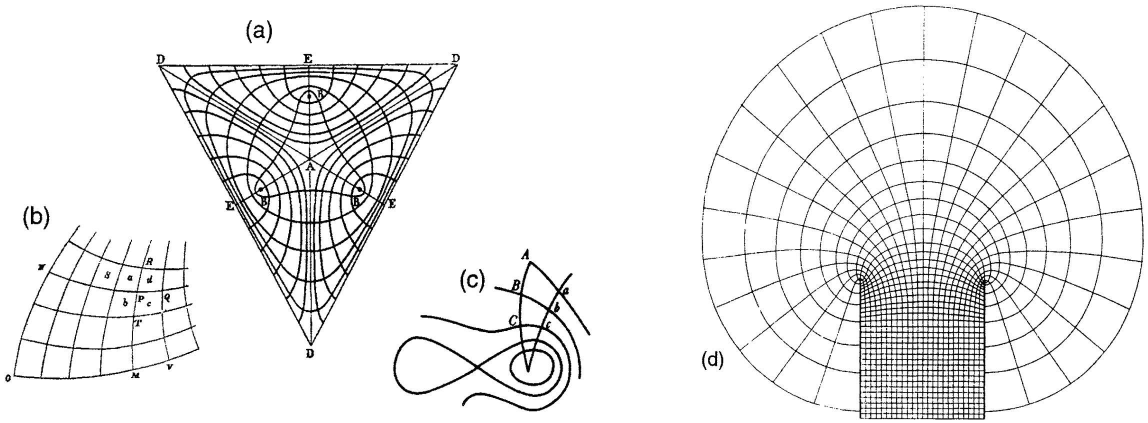
[…] but we should still require some method of indicating the *intensity* of the force at any point. If we consider these curves not as mere lines, but as fine tubes of variable section carrying an incompressible fluid, then we may make the velocity vary according to any given law, by regulating the section of the tube, and in this way we might represent the intensity of the force.

# On Faraday's Lines of Force (Maxwell, 1855)

## Fluid motion in resisting medium

Any portion of the fluid moving through the resisting medium is directly opposed by a retarding force proportional to its velocity.

[…] all the points at which the pressure is equal to a given pressure p will lie on a certain surface which we may call the surface (p) of equal pressure.




Maxwell's geometrical representations in Darrigol (2000, p. 140)

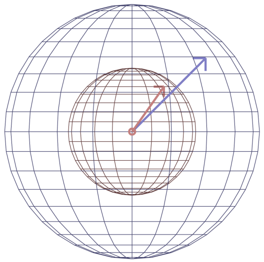
# On Faraday's Lines of Force (Maxwell, 1855)

## Fluid motion in resisting medium

If the velocity be represented by *v*, then the resistance will be a force equal to *kv* acting on unit of volume of the fluid in a direction contrary to that of motion. In order, therefore, that the velocity may be kept up, there must be a greater pressure behind any portion of the fluid than there is in front of it, so that the difference of pressures may neutralise the effect of the resistance.

### Unit point source

Unity of volume flows out of every spherical surface surrounding the point in unit of time



$$v = \frac{1}{4\pi r^2}$$

### Decrease of pressure

$$\frac{k}{4\pi r^2}$$

### Pressure at point *r* (0 at ∞)

$$p = \frac{k}{4\pi r} \qquad p = -\frac{k}{4\pi r}$$

source sink

### For *S* unit sources

$$p = \frac{kS}{4\pi r}$$

Does this seem familiar to you?

# On Faraday's Lines of Force (Maxwell, 1855)

## Analogy between imaginary fluid and electrostatics

$$v = \frac{1}{4\pi r^2}$$

*Velocity* is analogous to *E field*

$$p = \frac{k}{4\pi r}$$

*Pressure* is analogous to *Potential*

$$p = \frac{kS}{4\pi r}$$

*Number of unit sources (+ sources, – sinks)* is analogous to *Charge*

# On Faraday's Lines of Force (Maxwell, 1855)

## It is not a fluid!

The substance here treated must **not be assumed to possess any of the properties of ordinary fluids** except those of freedom of motion and resistance to compression. It is **not even a hypothetical fluid which is introduced to explain actual phenomena**. It is **merely a collection of imaginary properties** which may be employed for establishing certain theorems in pure mathematics in a way more intelligible to many minds and more applicable to physical problems than that in which algebraic symbols alone are used.

# Questions for discussion (Maxwell 1855)

- Maxwell talks about an elementary south pole floating in space, how does he do this, as his own equations are the ones that link magnetism and electricity together?

- I found it quite hard to follow Maxwell's reasoning without any drawing from number (6) in his 1855 paper.

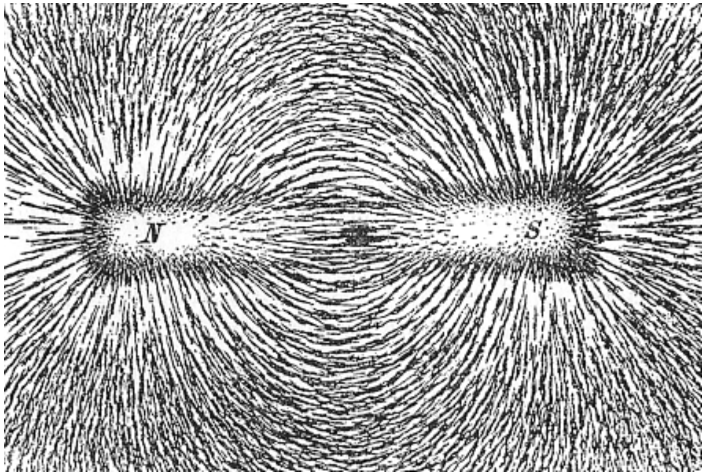
- Maxwell mentions in his 1855 paper, that the theory we are presenting should not be committed to any other theory in physics, is this a reference to the failed attempts of understanding electrodynamics through mechanics?

- Did Faraday make explicit arguments for his method or was it just a method which Maxwell deduced from Faraday's papers?

# On Physical Lines of Force (Maxwell, 1861-62)

## Mechanical explanation

Let us now suppose that the phenomena of magnetism depend on the existence of a tension in the direction of the lines of force;




[…] what mechanical explanation can we give of this inequality of pressures in a medium?

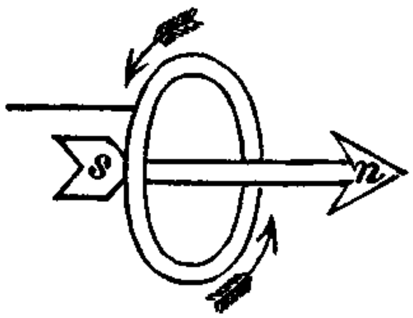
# On Physical Lines of Force (Maxwell, 1861-62)

## Molecular vortices

The explanation which "most readily" occurs to the mind is that the excess of pressure in the equatorial direction arises from the centrifugal force of vortices in the medium having their axes in directions parallel to the lines of force.

LXXVI. *On the Centrifugal Theory of Elasticity, as applied to Gases and Vapours.* By WILLIAM JOHN MACQUORN RANKINE, *C.E., F.R.S.E., F.R.S.S.A. &c.**

(1.) THE following paper is an attempt to show how the laws of the pressure and expansion of gaseous substances may be deduced from that which may be called the *hypothesis of molecular vortices*, being a peculiar mode of conceiving that theory which ascribes the elasticity connected with heat to the centrifugal force of small revolutions of the particles of bodies.




* See "Dynamical Illustrations of the Magnetic and the Helicoidal Rotatory Effects of Transparent Bodies on Polarized Light." By Prof. W. Thomson.—*Proceedings of the Royal Society*, June 12, 1856.

# On Physical Lines of Force (Maxwell, 1861-62)

## Stress tensor

In a circular vortex, revolving with uniform angular velocity, if the pressure at the axis is $p_0$ that at the circumference will be $p_1 = p_0 + \frac{1}{2}\rho v^2$, where $\rho$ is the density and $v$ the velocity at the circumference. […] A medium of this kind, filled with molecular vortices having their axes parallel, differs from an ordinary fluid in having different pressures in different directions.

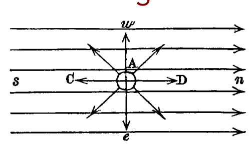
$$\left.\begin{aligned} p_{xx} &= \frac{1}{4\pi}\mu\alpha^2 - p_1, & p_{yz} &= \frac{1}{4\pi}\mu\beta\gamma \\ p_{yy} &= \frac{1}{4\pi}\mu\beta^2 - p_1, & p_{zx} &= \frac{1}{4\pi}\mu\gamma\alpha \\ p_{zz} &= \frac{1}{4\pi}\mu\gamma^2 - p_1, & p_{xy} &= \frac{1}{4\pi}\mu\alpha\beta \end{aligned}\right\}$$

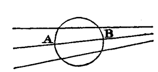
Normal and tangential stresses

$$X = \frac{d}{dx}p_{xx} + \frac{d}{dy}p_{xy} + \frac{d}{dz}p_{xz}$$

Law of equilibrium of stresses
Rankine's *Applied Mechanics*

$$X = \alpha\underbrace{\frac{1}{4\pi}\left\{\frac{d}{dx}(\mu\alpha) + \frac{d}{dy}(\mu\beta) + \frac{d}{dz}(\mu\gamma)\right\}}_{\text{Amount of magnetic matter}} + \underbrace{\frac{1}{8\pi}\mu\frac{d}{dx}(\alpha^2+\beta^2+\gamma^2)}_{\text{A body is urged towards places of stronger magnetic intensity}} \underbrace{- \mu\beta\frac{1}{4\pi}\left(\frac{d\beta}{dx} - \frac{d\alpha}{dy}\right) + \mu\gamma\frac{1}{4\pi}\left(\frac{d\alpha}{dz} - \frac{d\gamma}{dx}\right)}_{\text{Electric currents parallel to z and y, respectively}} - \underbrace{\frac{dp_1}{dx}}_{\text{Force in the (-) direction of pressure gradient}}$$



# On Physical Lines of Force (Maxwell, 1861-62)

## How are the vortices set in rotation?

We have as yet given no answers to the questions, "How are these vortices set in rotation?" and "Why are they arranged according to the known laws of lines of force about magnets and currents?"

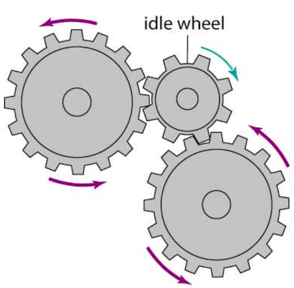
I have found great difficulty in conceiving of the existence of vortices in a medium, side by side, revolving in the same direction about parallel axes. The contiguous portions of consecutive vortices must be moving in opposite directions; and it is difficult to understand how the motion of one part of the medium can coexist with, and even produce, an opposite motion of a part in contact with it.

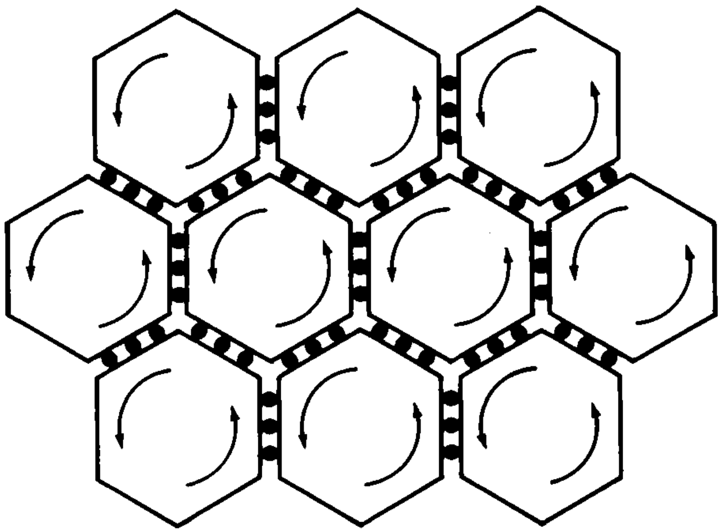
The only conception which has at all aided me in conceiving of this kind of motion is that of the vortices being separated by a layer of particles, revolving each on its own axis in the opposite direction to that of the vortices, so that the contiguous surfaces of the particles and of the vortices have the same motion.

# On Physical Lines of Force (Maxwell, 1861-62)

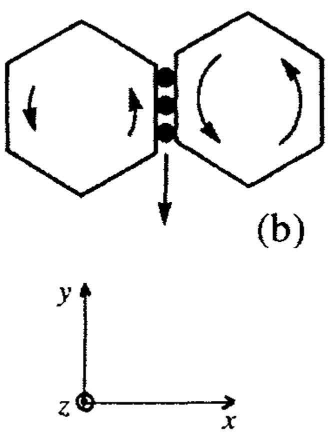
## Idle wheels

In mechanism, when two wheels are intended to revolve in the same direction, a wheel is placed between them so as to be in gear with both, and this wheel is called an **"idle wheel"**.



Electric current




*Illustrations from Siegel (1991) and Darrigol (2000)*

# Explaining induced currents (Basil Mahon, 2003)

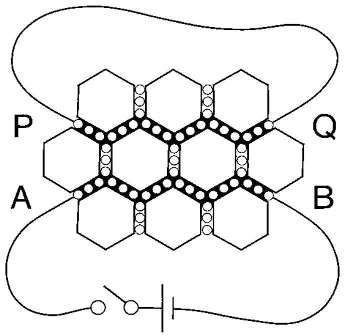



**Figure 2a. Switch open**

- All cells and idle wheels stationary
- No currents
- No magnetic fields

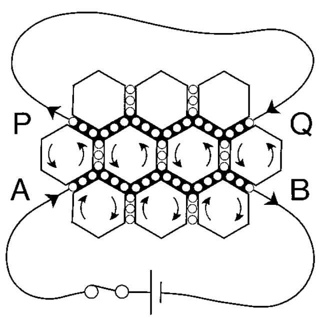



**Figure 2b. Switch first closed**

- AB current flows from left to right
- PQ current flows from right to left
- Cells below AB rotate clockwise, causing a magnetic field pointing away from the viewer
- Cells between AB and PQ rotate anticlockwise, causing a magnetic field pointing towards the viewer (in three dimensions, a circular field envelopes AB)
- Cells above PQ still stationary

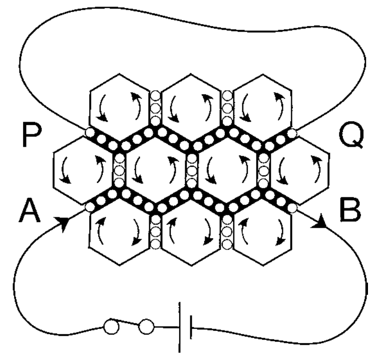



**Figure 2c. Shortly after switch closed**

- PQ current slows, then stops
- Cells above PQ start to rotate anticlockwise, and by the time the current stops are rotating at the same rate as those in the row below PQ

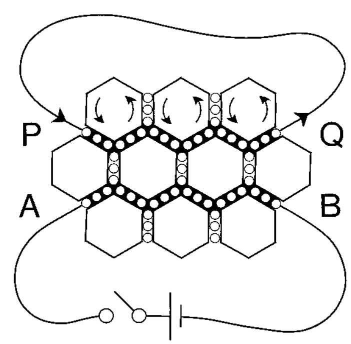



**Figure 2d. Switch opened again**

- AB current stops
- Cells in rows above and below AB stop rotating
- PQ current flows from left to right
- The current will slow, then stop; the situation will then be as in Figure 2a

# An "akward" hypothesis…

The conception of a particle having its motion connected with that of a vortex by perfect rolling contact **may appear somewhat awkward**. […] It is, however, **a mode of connexion which is mechanically conceivable**, and easily investigated, and it serves to bring out the actual mechanical connexions between the known electromagnetic phenomena; so that I venture to say that anyone who understands the provisional and temporary nature of this hypothesis, **will find himself rather helped than hindered** by it in his search after the true interpretation of the phenomena.

# What happened to the models?

Dynamical Theory of the EM field (1865) and Treatise on Electricity and Magnetism (1873): Abstract field concept

“Maxwell striped away all the imagery of the (vortices) model until all that remained was the mathematics. **The mathematics was the model**.” (Raymond Flood)

“Maxwell's turn away from mechanical models was one of the precipitating events in the **decline of the mechanical worldview** and the **transition to the more abstract physical formalisms** of the 20th century.” (Daniel Siegel)

# 20(!) Maxwell´s equations (1865)

$$\left.\begin{aligned} p' &= p + \frac{df}{dt} \\ q' &= q + \frac{dg}{dt} \\ r' &= r + \frac{dh}{dt} \end{aligned}\right\} \text{(A)}$$

$$\left.\begin{aligned} \mu\alpha &= \frac{dH}{dy} - \frac{dG}{dz} \\ \mu\beta &= \frac{dF}{dz} - \frac{dH}{dx} \\ \mu\gamma &= \frac{dG}{dx} - \frac{dF}{dy} \end{aligned}\right\} \text{(B)}$$

$$\left.\begin{aligned} \frac{d\gamma}{dy} - \frac{d\beta}{dz} &= 4\pi p' \\ \frac{d\alpha}{dz} - \frac{d\gamma}{dx} &= 4\pi q' \\ \frac{d\beta}{dx} - \frac{d\alpha}{dy} &= 4\pi r' \end{aligned}\right\} \text{(C)}$$

(70) In these equations of the electromagnetic field we have assumed twenty variable quantities, namely,

| | |
|---|---|
| For Electromagnetic Momentum .................................... | $F$ $G$ $H$ |
| For Magnetic Intensity .................................................. | $\alpha$ $\beta$ $\gamma$ |
| For Electromotive Force .............................................. | $P$ $Q$ $R$ |
| For Current due to true Conduction ................................ | $p$ $q$ $r$ |
| For Electric Displacement ............................................. | $f$ $g$ $h$ |
| For Total Current (including variation of displacement) .................................................... | $p'$ $q'$ $r'$ |
| For Quantity of Free Electricity ...................................... | $e$ |
| For Electric Potential .................................................... | $\Psi$ |

Between these twenty quantities we have found twenty equations, viz.

| | |
|---|---|
| Three equations of Magnetic Force ............................ | (B) |
| Three equations of Electric Currents ........................... | (C) |
| Three equations of Electromotive Force ...................... | (D) |
| Three equations of Electric Elasticity .......................... | (E) |
| Three equations of Electric Resistance ........................ | (F) |
| Three equations of Total Currents ............................... | (A) |
| One equation of Free Electricity .................................. | (G) |
| One equation of Continuity ......................................... | (H) |

These equations are therefore sufficient to determine all the quantities which occur in them, provided we know the conditions of the problem. In many questions, however, only a few of the equations are required.

# Treatise on Electricity and Magnetism - Maxwell (1873)

A most important distinction was drawn by Hamilton when he divided the quantities with which he had to do into Scalar quantities, which are completely represented by one numerical quantity, and Vectors, which require three numerical quantities to define them.

The invention of the calculus of Quaternions is a step towards the knowledge of quantities related to space which can only be compared for its importance, with the invention of triple co-ordinates by Descartes. The ideas of this calculus, as distinguished from its operations and symbols, are fitted to be of the greatest use in all parts of science.

$$\left.\begin{aligned} 4\pi\mu u &= \frac{dJ}{dx} + \nabla^2 F. \\ 4\pi\mu v &= \frac{dJ}{dy} + \nabla^2 G, \\ 4\pi\mu w &= \frac{dJ}{dz} + \nabla^2 H. \end{aligned}\right\}$$

$$\left.\begin{aligned} a &= \frac{dH}{dy} - \frac{dG}{dz}, \\ b &= \frac{dF}{dz} - \frac{dH}{dx}, \\ c &= \frac{dG}{dx} - \frac{dF}{dy}. \end{aligned}\right\}$$

*Quaternion Expressions for the Electromagnetic Equations.*

618.] In this treatise we have endeavoured to avoid any process demanding from the reader a knowledge of the Calculus of Quaternions. At the same time we have not scrupled to introduce the idea of a vector when it was necessary to do so. When we have had occasion to denote a vector by a symbol, we have used a German letter, the number of different vectors being so great that Hamilton's favourite symbols would have been exhausted at once. Whenever therefore, a German letter is used it denotes a Hamiltonian vector, and indicates not only its magnitude but its direction.

619.] The equations (A) of magnetic induction, of which the first is,

$$a = \frac{dH}{dy} - \frac{dG}{dz},$$

may now be written $\mathfrak{B} = V\nabla\mathfrak{A}$,

where $\nabla$ is the operator

$$i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz},$$

and $V$ indicates that the vector part of the result of this operation is to be taken.

Since $\mathfrak{A}$ is subject to the condition $S\nabla\mathfrak{A} = 0$, $\nabla\mathfrak{A}$ is a pure vector, and the symbol $V$ is unnecessary.

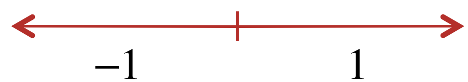
**What are quaternions? How are they related to vectors?**

# Analytical representation of direction – Wessel (1798)

*Change in direction should be represented by algebraic symbols*

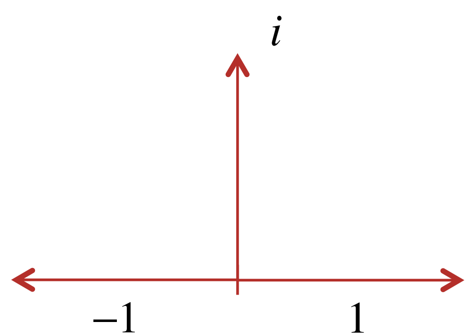
*Problem: direction cannot be changed by algebraic operations except to the opposite*

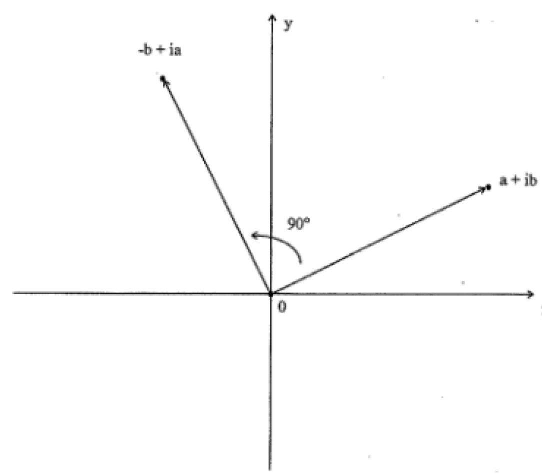
*.(-1)*
*means 180°*




*How to express 90°?*

*Hint: $i^2$ = -1*




*i means 90° rotation!*

$$v = a + bi$$

# Analytical representation of direction – Wessel (1798)

*Multiplication of directed line segments is analogous to real numbers*

*The product of two straight lines should be formed from the one factor, in the same way as the other from the positive unit line that is set = 1*

*real numbers* ⟷ *directed lines*

$$2 \cdot (-3) = -6$$

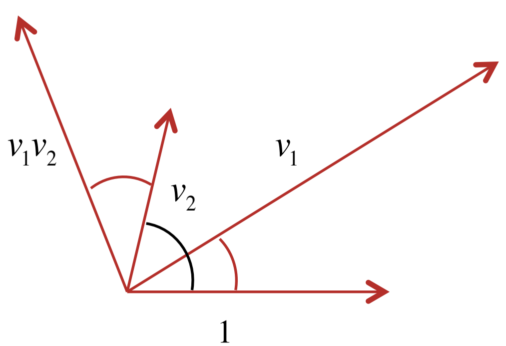
$$\frac{-6}{2} = \frac{-3}{1} \qquad \frac{-6}{-3} = \frac{2}{1}$$

$$\frac{v_1 v_2}{v_2} = \frac{v_1}{1}$$

*i) The factors and the product are in the same plane as the unit*

*ii) The length of the product is the product of the lengths of the factors*

*iii) The product must deviate as many degrees from the one factor, as the other factor deviates from the unit so that the directional angle of the product is the sum of the directional angles of the factors.*




*Directed lines can be analytically represented*

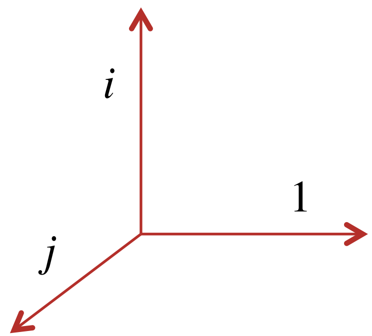
$$v = a + bi \qquad v = |v|(\cos\alpha + i\sin\alpha)$$

$$\boxed{v_1 v_2 = |v_1||v_2|[\cos(\alpha+\beta) + i\sin(\alpha+\beta)]}$$

# Analytical representation of direction in 3D – Hamilton (1843)

## Hamilton's path to quaternions

*A plausible 2D-3D analogy: "it seemed natural"*

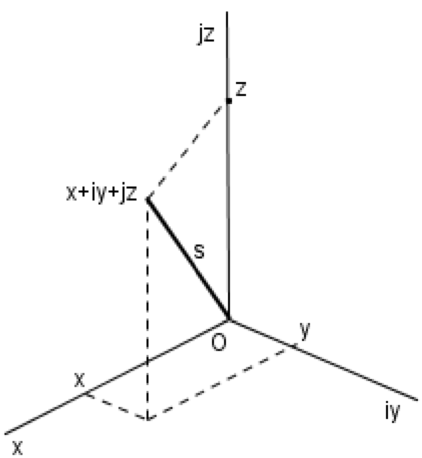



*Since i is in a certain well-known sense, a line perpendicular to the line 1, it seemed natural that there should be some other imaginary to express a line perpendicular to the former.*

$$j^2 = i^2 = -1$$

*… I tried the effect of assuming also $j^2 = -1$, which I interpreted as a rotation through two right angles in the xz, as $i^2 = -1$ had corresponded to such a rotation in the plane xy*

*Triplet*




Neves (2008)

$$x + iy + jz$$

# Analytical representation of direction in 3D – Hamilton (1843)

## Addition of triplets

$$(a + ib + jc) + (x + iy + jz) = (a + x) + i(b + y) + j(c + z) \quad ✓$$

## Multiplication of triplets

$$\begin{aligned}(a + ib + jc)(x + iy + jz) &= ax + aiy + ajz + ibx + ibiy + ibjz + jcx + jciy + jcjz \\ &= ax + i^2by + j^2cz + iay + ibx + jaz + jcx + ijbz + ijcy \\ &= (ax - by - cz) + i(ay + bx) + j(az + cx) + ij(bz + cy) \quad ?\end{aligned}$$

*But what are we to do with ij? Shall it be of the form $\alpha+i\beta+j\gamma$?*

1st attempt: $ij = \pm 1$

2nd attempt: $ij = 0$

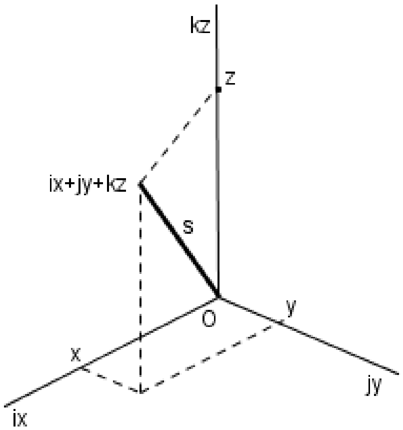
3rd attempt: $ij = -ji = k$

*Details in Hankins (1980, p. 295-300)*

# Analytical representation of direction in 3D – Hamilton (1843)

Thus in a very dramatic manner Hamilton discovered and announced the discovery of quaternions. These are hypercomplex numbers of the form $w + ix + jy + kz$, where $w$, $x$, $y$, and $z$ are real numbers, and $i$, $j$, and $k$ are unit vectors, directed along the $x$, $y$, and $z$ axes respectively. The $i$, $j$, and $k$ units obey the following laws:

$$ij = k \qquad jk = i \qquad ki = j$$
$$ji = -k \qquad kj = -i \qquad ik = -j$$
$$ii = jj = kk = -1$$




$$
\begin{aligned}
(ix + jy + kz)(ix' + jy' + kz') &= i^2xx' + ijxy' + ikxz' + jiyx' + j^2yy' + jkyz' + kizx' + kjzy' + k^2zz' \\
&= \ldots = (-xx' - yy' - zz') + i\underbrace{(yz' - zy')}_{x''} + j\underbrace{(zx' - xz')}_{y''} + k\underbrace{(xy' - yx')}_{z''} \\
&= (-xx' - yy' - zz') + ix'' + jy'' + kz''.
\end{aligned}
$$

| | i | j | k |
|---|---|---|---|
| i | -1 | k | -j |
| j | -k | -1 | i |
| k | j | -i | -1 |

# Modern Vector analysis – Gibbs

My first acquaintance with quaternions was in reading Maxwell's E. & M. where Quaternion notations are considerably used. I became convinced that to master those subjects, it was necessary for me to commence by mastering those methods. At the same time I saw, that although the methods were called quaternionic the idea of the quaternion was quite foreign to the subject. In regard to the products of vectors, I saw that there were two important functions (or products) called the vector part & the scalar part of the product, but that the union of the two to form what was called the (whole) product did not advance the theory as an instrument of geom. investigation.

Gibbs to Schlegel (1888)

*Quotation from Crowe (1967)*

# Modern Vector analysis – Heaviside

Maxwell exhibited his main results in quaternionic form. I went to Prof. Tait's treatise to get information, and to learn how to work them. [...] But on proceeding to apply quaternions to the development of electric theory, I found it very inconvenient. Quaternions were in their vectorial aspects antiphysical and unnatural [...]. So I dropped out the quaternion altogether, and kept to pure scalars and vectors, using a very simple vectorial algebra in my papers from 1883 onwards.

Heaviside (1893)

*Quotation from Crowe (1967)*

# Questions for discussion

- He writes and thinks in the notion of vectors and in particular vector calculus. And since it's the first time we see this notation in this course, I wonder if he was he himself was a contributor to these fields or did he simply just employ and use existing notation and mathematics?

- Was the interpretation of the inverse square law as a surface of sphere known to Maxwell? Because he includes the 4π in various equations, and he doesn't seem to care or take notice of this very nice and simple geometric interpretation that is sitting right there in front of him.

- Was he taken seriously as physicist on this subject at the time? Because most of the time he seems much more interested in the structure than explanations and physical reality. His methods and arguments seem to resemble more that of a Mathematician than a physicist.

# Weber's electrodynamics

Coulomb (1785) $$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}$$

Ampère (1822) $$\vec{F} = -\frac{\mu_0}{4\pi} I_1 I_2 \frac{\hat{r}}{r^2} f(\alpha, \beta, \gamma)$$

Faraday (1831) $$emf = -M\frac{dI}{dt}$$

Idea: $$Id\vec{\ell} \Leftrightarrow q\vec{v}$$

Weber's force $$\vec{F} \approx \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}\left(1 + k_1 v_1 v_2 + k_2 a_{12}\right)$$

*Credit: André Assis*

# Weber's electrodynamics

$$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\hat{r}}{r^2} \left( 1 - \frac{\dot{r}^2}{2c^2} + \frac{r\,\ddot{r}}{c^2} \right)$$

$$\dot{r} = \frac{dr}{dt}$$

$$\ddot{r} = \frac{d^2 r}{dt^2}$$

*Credit: André Assis*

# Weber's electrodynamics

$$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\hat{r}}{r^2} \left( 1 - \frac{\dot{r}^2}{2c^2} + \frac{r\,\ddot{r}}{c^2} \right)$$

In the static case (dr/dt = 0 and $d^2r/dt^2 = 0$) we return to the laws of Coulomb and Gauss.

Action and reaction. Conservation of linear momentum.

Force along the straight line connecting the particles. Conservation of angular momentum.

It can be derived from a velocity dependent potential energy:

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{1}{r} \left( 1 - \frac{\dot{r}^2}{2c^2} \right)$$

Conservation of energy: $$\frac{d(K+U)}{dt} = 0$$

*Credit: André Assis*

# Weber-Kohlrausch experiment (1856)

The goal was to compare two systems of electrical units. The principle was to discharge a Leyden jar (a capacitor) that had been storing a known amount of electric charge in electrostatic units, and then to see how long it took for a unit of electric current, as measured in electromagnetic units, to produce the same deflection in a galvanometer. The ratio turned out to be very close to the measured speed of light.

# Weber's electrodynamics

$$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\hat{r}}{r^2} \left( 1 - \frac{\dot{r}^2}{2c^2} + \frac{r\,\ddot{r}}{c^2} \right)$$

$$\dot{r} = \frac{dr}{dt}$$

$$\ddot{r} = \frac{d^2 r}{dt^2}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \frac{m}{s}$$

*Credit: André Assis*

# Last chapter of Maxwell´s treatise (1873)

## CHAPTER XXIII.

### THEORIES OF ACTION AT A DISTANCE.

# End of module feedback

- Please go to b.socrative.com (student login)
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