simplified—list of the "solid" conclusions reached by philosophy in WISDOM's book *Philosophy and Its Place in Our Culture*:

> Since there is a widespread sense among non-philosophers and also among philosophers that in the course of its two-and-a-half-thousand years of existence it has made no progress, I venture to give the following list of achievements (even if strikingly negative):
>
> Knowledge in general is not obtainable by Reason alone (though it is in certain narrowly defined fields). ...
>
> No knowledge is based on pure experience or pure observation.
>
> There are no synthetic a priori truths.
>
> No well-formed statements (other than self-contradictions) are false or meaningless a priori. ...
>
> All empirical knowledge is fallible.
>
> [pp. 105–106]

4.4 From Effluvium to the Electromagnetic Field

4.4.1 *Peter of Maricourt and Gilbert*

In the previous discussions, we have repeatedly encountered a pattern of development in the various branches of physics, primarily in mechanics and astronomy: The Greeks reached a relatively high level, the Middle Ages gradually rediscovered the ancient legacy, and building on this foundation, later centuries surpassed those achievements. This schema fails to hold for electromagnetic phenomena, for here, European science received nothing more from the Greeks than nomenclature. Thus "electricity" and "magnetism" are derived from the Greek words for amber (ἤλεκτρον) and magnetite (ἠλίδος Μαγνήτις); see Quotation 4.18. Medieval scholars found nothing in ARISTOTLE about magnets and were therefore compelled to develop new methods on their own. In 1269, PETER OF MARICOURT (PETRUS PEREGRINUS) carried out a number of investigations into the properties of magnets and even made use of experimental methods, which was unusual at that time. Using a small iron needle, he determined the forces on the surface of a spherical magnet, drawing the needle's direction point by point and thereby obtaining—as we would put it today—the directions of the magnetic field lines. These experiments revealed that the magnetic field lines run like the meridians on the globe, meeting at two opposite poles (Figure 4.56). Indeed, it is MARICOURT who gave us the term "pole."

Figure 4.57 reproduces one of MARICOURT's sketches, which shows an idea for a perpetual motion machine based on magnetism.

The extent to which interest in magnetic phenomena was outside the mainstream is shown by the fact that neither THOMAS AQUINAS nor ALBERTUS MAGNUS, both of whom were contemporaries of MARICOURT, mentions him or his work.

More than three centuries passed before WILLIAM GILBERT (1544–1603), court physician to the English queen, ELIZABETH I, continued the work of PETER OF MARICOURT in a remarkably similar spirit and with similar methods. GILBERT's book *De magnete, magneticisque corporibus et de magno magnete tellure* appeared in 1600, the same year in which GIORDANO BRUNO was burned at the stake. This work is also significant from the point of view of natural philosophy in that it stresses the importance of direct experimentation before GALILEO (Quotation 4.19). GILBERT's most important contribution can already be seen in the book's title: The Earth is a large magnet. This is how GILBERT presents the theory of the compass. He recog-

Figure 4.54 *continued*

> hydra of [religious] orders; And what have I seen? Oh, enough. I have seen PETER I and CATHERINE and FRIEDRICH and JOSEPH and LEIBNIZ and NEWTON and EULER and WINCKELMANN and MENGS and HARRISON and COOK and GARRICK. Are you satisfied with that? Good. But look here at a few trifles: Here I have a new enormous country, here a fifth part of the world, there a new planet, and a small convincing little proof that our Sun is a satellite, and look here, finally I have made, in my eighty-third year, an airship.
>
> —LICHTENBERG, *Gelehrte und gemeinnützige Aufsätze*, 1783

◀ **Figure 4.55** IMMANUEL KANT (1724–1804): Studies in mathematics, philosophy, and theology at the University of Königsberg; there he became a lecturer in 1755 and professor in 1770.

Even in his youth, KANT made notable contributions to the development of the natural sciences. For example, he interpreted the forces acting among particles of matter in the Newtonian sense and postulated the existence of both attractive and repulsive forces, which, however, did not balance each other because the first was to vary with distance as $1/r^2$, whereas the second varied as $1/r^3$. His name is enshrined in physics with the Kant–Laplace hypothesis on the origin of the solar system, which was presented in the 1755 treatise *Allgemeine Naturgeschichte und Theorie des Himmels*. (LAPLACE's treatise appeared in 1796 with the title *Exposition du système du monde*.) The presentations of the two authors do not agree on every point; however, they have in common the bold intention of explaining the present state of the solar system as the result of development from a primordial cloud, thereby opposing the dogma that the state of the cosmos had remained unchanged since its creation. With LAPLACE, the planets break off from the already condensed rotating Sun, while KANT has the planets condensing on their own, which is closer to today's consensus.

KANT's most important philosophical works are *Kritik der reinen Vernunft* (*Critique of Pure Reason,* 1781), *Kritik der praktischen Vernunft* (*Critique of Practical Reason,* 1788), and *Kritik der Urteilskraft* (*Critique of Judgment,* 1790).

> *The Copernican Revolution in Metaphysics*
>
> Hitherto it has been assumed that all our knowledge must conform to objects. But all attempts to extend our knowledge of objects by establishing something in regard to them a priori, by means of concepts, have, on this assumption, ended in failure. We must therefore make trial whether we may not have more success in the tasks of

continued on next page

Figure 4.55 *continued*

metaphysics, if we suppose that objects must conform to our knowledge. This would agree better with what is desired, namely, that it should be possible to have knowledge of objects a priori, determining something in regard to them prior to their being given. We should then be proceeding precisely on the lines of Copernicus' primary hypothesis. Failing of satisfactory progress in explaining the movements of the heavenly bodies on the supposition that they all revolved round the spectator, he tried whether he might not have better success if he made the spectator revolve and the stars to remain at rest. A similar experiment can be tried in metaphysics, as regards the *intuition* of objects. If intuition must conform to the constitution of the objects, I do not see how we could know anything of the latter a priori; but if the object (as object of the senses) must conform to the constitutions of our faculty of intuition, I have no difficulty in conceiving such a possibility.

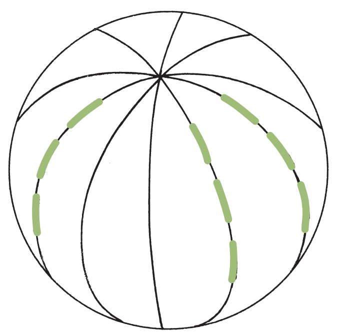
—Kant, *Critique of Pure Reason*, preface to the second edition [p. 22]

Quotation 4.18

To pass on, I will begin to discuss by what law of nature it comes about that iron can be attracted by that stone which the Greeks call magnet from the name of its home, because it is found within the national boundaries of the Magnetes. This stone astonishes men, because it often makes a chain out of little rings hanging from it. For you may sometimes see five or more hanging in a string and swayed by a light breeze, where one hangs from another beneath it, and one from another learns the stone's power and attraction: to such a distance does its power hold force, oozing through and through.

In matters of this sort many principles have to be established before you can give a reason for the thing itself, and you must approach by exceedingly long and roundabout ways; accordingly I crave all the greater attention of ears and mind.

—Lucretius, *On the Nature of Things*, Book VI, ll.905–920



◀ **Figure 4.56** Peter of Maricourt had already maintained in 1269 that the "magnetic lines of force" of a spherical magnet, like the Earth's meridians, meet at the poles.

nized the character of the forces acting between magnetic poles, discovered both attractive and repulsive forces, and determined, moreover, that breaking a magnet into two cannot separate the two poles because each fragment again becomes a magnet with two poles. He observes the magnetic inclination, that is, the deviation of the compass needle from the horizontal, and surmises that it must therefore be possible to use this information in determining geographic latitude without the need for astronomical observation.

Gilbert also made detailed investigations of electrical phenomena. He knew, for example, that not only amber, but also a whole host of other materials (glass, wax, sulfur, and some gemstones, for example) could be electrified through friction. He described some significant differences between electrical and magnetic phenomena, his most important contribution relating perhaps to the character of the forces: Magnets elicit a turning response (*verticitas*), whereas electrical force is expressed through attraction (*attractio*). Gilbert did not yet mention the two kinds of electrical charges and did not yet know that among electrical phenomena, repulsion might also occur (Figure 4.58).

4.4.2 *The Chronology of Progress*

With Gilbert's work, the seventeenth century made its contribution to the investigation of electromagnetic phenomena in its first year, and essentially made no progress thereafter. Descartes, of course, included all known natural phenomena in his philosophy, so in this uniform cosmology there was a place for magnetic phenomena as well (Figure 4.59). We also have to bring up for special mention the multifaceted and ingenious experimentalist, Otto von Guericke, the mayor of Magdeburg, who constructed the first "triboelectric" machine, which produced static electricity by friction (Figure 4.60), thereby laying the experimental foundations for intensive electrostatic investigation. However, his scientifically minded contemporaries were busy with and to some extent blinded by the success in mechanics, in the formulation of natural-philosophical foundations, and in the derivations of quantitative answers using the most advanced mathematical methods of the era. As a result, interest in electricity diminished to such an extent that von Guericke's electricity-producing machine was forgotten for two generations.

As we saw in Section 3.2, Kepler attributed an important role to magnetism. He surmised that the motion of the planets is due to the magnetic attraction of the Sun.

In Newton, we find statements about magnetism and electricity in his *Opticks*, though always in the form of questions (Quotation 4.20). In the two "questions" related to these topics, Newton supports two different conceptions concerning the nature of electric and magnetic forces. In one, he essentially adopts Gilbert's idea that magnetic and electric substances fill the surrounding space with a fluid (*effluvium*). This fluid was assumed to be capable of penetrating ordinary heavy bodies. In the other question, Newton interprets magnetic and electric interactions as action at a distance. Here we meet the brilliant conjecture that material particles interact at very small distances via their electrical forces, with this interaction being independent of whether they are put into an electrified state with the aid of friction.

The first half of the eighteenth century witnessed a renewed interest in electricity. By the middle of the century, it had become as fashionable to perform electrical experiments in the salons of the nobility as it was to read the *Encyclopédie* (Figure 4.61). The craze for experimentation increased even more after the invention of the Leyden jar; now one could perform experiments that were so visually impressive that they not only amused salon guests in polite society, but could serve as attractions at public shows and fairs.

Table 4.2 offers a schematic representation of the problems in electricity and magnetism that were the focus of investigation from the beginning of the eighteenth century together with the names of the researchers who played a significant role in this work and what they achieved.

We see that in the first three-quarters of the eighteenth century, it was essentially qualitative research in electrostatics that was carried out. This work served, of course, as a necessary precondition for the quantitative investigations that would begin in the last quarter of the century. In the first decades of the nineteenth century, then, the fundamental laws of electrostatics and—we should add—magnetostatics were brought into the form in which they are used today.

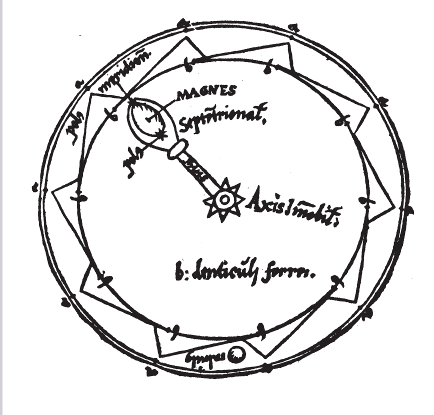
Experimental aids to the discovery of relationships between electrical and magnetic phenomena became possible with the invention of a source of strong and steady electrical current: the galvanic cell. Interestingly, its discovery by Volta occurred precisely in the year 1800.

It is somewhat surprising that it was not until 1820 that it came to be noticed that a magnetic needle could be influenced by an electrical current, but then, in only a few years, all the laws relating to the connection between electrical current and magnetism were written down in the form to which we are now accustomed.

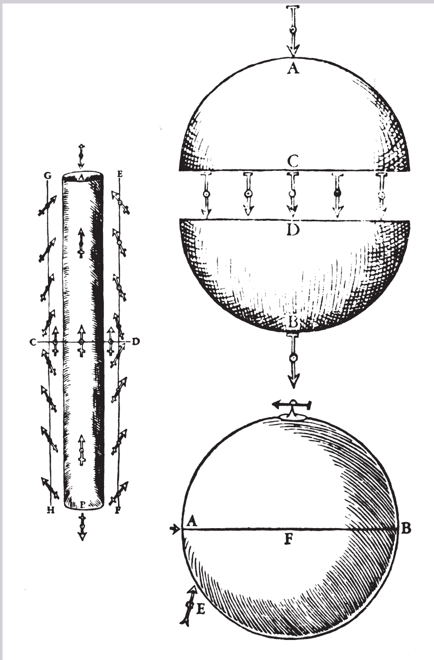
◀ ▶ **Figure 4.58**
In Gilbert's book *De magnete …*, we find many familiar pictures.

After this, only two more discoveries were needed in order to complete the entire edifice of classical electrodynamics. Both of these are of fundamental importance, and they are connected to the two greatest personalities of electrodynamics: Faraday, who found the relationship between the changes in a magnetic field over time and the associated electric field (in simpler terms, the law of induction), and Maxwell, who recognized the converse phenomenon, thereby bringing to completion the field of classical electrodynamics with the system of equations that bears his name.

Behind the increasing precision and refinement of the quantitative description of electromagnetic phenomena lurks a change of view of crucial significance; namely, by the end of the eighteenth century, the idea, going back to Newton, of action at a distance had become completely acceptable, not only in science but also as an intellectual concept. Thus not only did Coulomb's laws reflect this new attitude and



▲ **Figure 4.57** Peter of Maricourt hoped to use the magnetic force to construct a perpetual motion machine.



Quotation 4.19

As handed down by [Plato and Aristotle], the loadstone merely attracted iron, the rest of its virtues were all undiscovered. But that the story of the loadstone might not appear too bare and too brief, to this singular and sole known quality there

continued on next page

Quotation 4.19, ***continued***

were added certain known figments and falsehoods, which in the earliest times, no less than nowadays, used to be put forth by raw smatterers and copyists to be swallowed by men. As for instance, that if a loadstone be anointed with garlick, or if a diamond be near, it does not attract iron. Tales of this sort occur in Pliny, and in Ptolemy's *Quadripartitum*; and the errors have been sedulously propagated, and have gained ground (like ill weeds that grow apace) coming down even to our own day, through the writings of a host of men, who, to fill out their volumes to a proper bulk, write and copy out pages upon pages on this, that, and the other subject, of which they knew almost nothing for certain of their own experience. Such fables of the loadstone even Georgius Agricola himself, most distinguished in letters, relying on the writings of others, has embodied as actual history in his books *De Natura Fossilium*. Galen noted its medicinal power. ... But his commentator ... repeats the story of the garlick and the diamond, and moreover introduces Mahomet's shrine vaulted with loadstones, and writes that, by the exhibition of this (with the iron coffin hanging in the air) as a divine miracle, the public were imposed upon. But this is known by travelers to be false. Yet Pliny relates that Chinocrates the architect had commenced to roof over the temple of Arsinoe at Alexandria with a magnet-stone, that her statue of iron placed therein might appear to hang in space. His own death, however, intervened, and also that of Ptolemy, who had ordered it to be made in honour of his sister. ... And as yet we have not set ourselves to overthrow by argument those errors and impotent reasonings of theirs, nor many other fables told about the loadstone, nor the superstitions of impostors and fabulists: for instance ... that a white loadstone may be procured as a love potion: or as Hali Abbas thoughtlessly reports, that if held in the hand it will cure gout and spasms: Or that it makes one acceptable and in favour with princes, or eloquent, as Pictorio has sung ...: Or that by day it has a certain power of attracting iron, but by night the power is feeble, or rather null: Or that when weak and dulled the virtue is renewed by goats' blood, as Ruellius writes: Or that Goats' blood sets a loadstone free from the venom of a diamond: ... Or that it removed sorcery from women, and put to flight demons, as Arnaldus de Villanova dreams. ... With such idle tales and trumpery do plebeian philosophers delight themselves and satiate readers greedy for hidden things, and unlearned devourers of absurdities.

—William Gilbert, *On the Magnet* [pp. 1–2, 6, 7]

become new supports for Newton's ideas, but Ampère's laws of electrodynamic were also born in accord with the fundamental postulates of Newtonian mechanics. Slowly, step by step, to some extent under the coercion of experimental facts and based on the work of Faraday and Maxwell, the concept of the *electromagnetic field as physical reality* began to gain acceptance.

In the following sections we pursue the developments sketched above in greater detail.

4.4.3 *Qualitative Electrostatics*

Joseph Priestley's 1767 book *The History and Present State of Electricity, with Original Experiments* offers an interesting summary of the qualitative investigations into electrostatics that had been carried out during the first three-quarters of the eighteenth century and even previously. Priestley himself was actively involved in several different branches of science and is known above all for his work in chemistry, in particular for the discovery of oxygen. His book is of interest for a variety of reasons. As the title informs us, it is a summary, or a status report as we would call it today. However, what is also of interest is that Priestley went to the trouble to reproduce the experiments himself and was thus able to contribute a number of original insights into electrostatics. Quotation 0.5, taken from that book, should be read by all those who are under the impression that it is only a phenomenon of our times that the proliferation of research papers has become so great that it is impossible for any one individual to keep up with new developments, or that the rapid acceleration in the pace of scientific research began only recently.

Table 4.2 shows the principal participants in qualitative electrostatics: Stephen Gray, Dufay, Franklin, Aepinus, and Priestley. In Figure 4.62 can be seen two pages from the table of contents of Priestley's book, and we would now like to spend a bit of time discussing the researchers who are linked, according to Priestley, with the beginning and end of this phase of development.

Stephen Gray (1666–1736) worked with such simple equipment as a glass rod. His most important observation is that certain materials, then called nonelectric substances, conduct electricity. This nomenclature goes back to Gray himself and his pupil Jean Théophile Desaguliers, a Huguenot who had fled France. Gray had determined that a test component made of a "nonelectric" material, when hung from an isolating silken thread or laid on an isolating plate, could be brought into a state of electrification. Figure 4.63 presents the principle of the experiment that Gray carried out at his country estate with the help of a friend. He first determined that an electrical effect could be detected even on a metal nail driven into the stopper of a rubbed glass cylinder. Then he attached a string to this nail, and was successful in transmitting the electrical effect. He conducted the experiments first in the castle gallery and then continued them out of doors in the fine autumn weather. He succeeded in transmitting the electrical state over a distance of 886 feet. We must point out, however, that this remained an electrostatic experiment and not one involving the conduction of electric current. It is characteristic of Gray's inventiveness that he successfully employed this method to transmit signals. Gray here also made an observation that was to be later of great importance in treating the questions of the similarities and differences between electric and thermal phenomena: The electrical status of a body is a function of its surface area and not its volume.

Stephen Gray carried on a lively correspondence with the overseer of the French royal gardens, Charles François de Cisternay Dufay (1698–1739). Dufay's great accomplishment was the discovery that there exist two kinds of electricity. Previously, it had been known only that a body that had been electrified by means of friction first attracts light objects in its vicinity and then, after they are touched, repels them, and that electrified bodies repel one another. It then was observed, however, that in rubbing a glass rod and a piece of resin (Quotation 4.21), these two objects do not repel, but attract. Dufay therefore distinguished two types of electricity: glass electricity (*électricité vitreuse*) and resinous electricity (*électricité résineuse*). As a consequence, alongside the single-fluid theory of electricity there arose a two-fluid theory, which was worked out in detail by the official naturalist of the French royal court, Jean Antoine Nollet (1700–1770), who conceived of the two types of electricity in the form of an *effluvium* and an *affluvium* surrounding electrified bodies.

A new impetus to experimentation was provided by the Leyden jar, the invention of the pastor von Kleist, born in Vicewo (Vietzow), Poland, and by Musschenbroek, who was a professor in Leyden. Von Kleist came upon the invention by chance, Musschenbroek in the process of systematic experimentation, though also with the assistance of chance. Musschenbroek's goal was to prevent the well-known phenomenon that a conductor suspended in air by isolating threads, when brought into an electrified state, will sooner or later lose that state. To this end, he "electrified" water inside a glass flask by connecting a metallic rod leading into the flask through the stopper to an electrification machine utilizing friction. In one hand he held the flask, and when he touched the metal rod with the other (Figure 4.64), he experienced a powerful electric shock. Musschenbroek's panicked fright is colorfully described in Quotation 4.22.

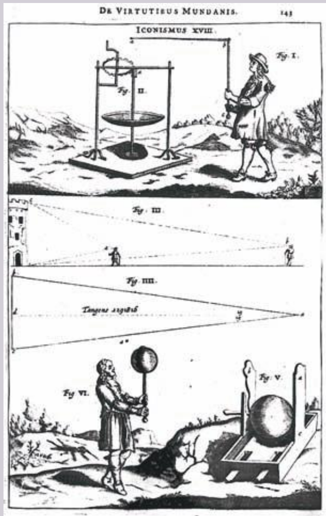
Incidentally, the second part of this quotation is worthy of note: Every scientist must display a certain *furor heroicus*, that is, a readiness to put his or her life on the line for the sake of scientific investigation, with renown as the reward for such sacrifice. Richman, the man mentioned in the quotation, lost his life in St. Petersburg through his experiments with electricity.

As we have mentioned, from this time on, it had become fashionable to experiment with electricity. Larger and ever more elaborate apparatuses were built and new experimental tools were employed. Figure 4.65 comes from a 1770 book of J. B. Horváth; Figure 4.66 shows some measuring instruments from Musschenbroek's book. Even physicians saw in electrical effects the promise of hidden possibilities and introduced electroshock therapy, although not with the purposes for which it has been used more recently.

Perhaps the most interesting personality and most successful experimentalist of this epoch was Benjamin Franklin (1706–1790, Plate XVIII). He was the first American to make significant contributions to the development of European science. Father Nollet, already mentioned, had completely different ideas from those of Franklin, and on becoming aware of Franklin's discoveries after the translation of his works into French, simply could not believe that a physicist by the name of Franklin existed and that moreover he lived in Philadelphia. He protested that the entire book must have been written by his detractors in order to besmirch his reputation.

Franklin learned about electricity from an itinerant showman. Fortunately for Franklin, he did not read the books of his European colleagues; his intellectual arsenal consisted of Newton's *Principia* and *Opticks*. Thus he was able to interpret his experimental findings more freely. Franklin is best known for his invention of the lightning rod. Even before Franklin, it had been apparent—because of the light and crackling sound of an electrical spark—that lightning is a similar

◀ **Figure 4.59** Descartes's interpretation of magnetism.



◀ **Figure 4.60** Guericke's triboelectric machine (Fig. V). (Library of the Hungarian Academy of Sciences.)

◀ **Figure 4.61** In the Age of Reason, electricity was a source of amusement in educated circles [Taton 1957].

1600	magnetic phenomena	Gilbert	(1540 – 1603)	Verticitas, Attractio Earth = magnet	Cartesian vortex theory
1672	triboelectric machine	Guericke	(1602 – 1686)	attraction, repulsion	
1700					
1705 – 1709		Hauksbee	(? – 1713)	gas discharge	
1729	qualitative electrostatics	Gray	(1666 – 1736)	induction, conduction conductor–insulator role of the surface	
1733		Dufay	(1698 – 1739)	two forms of electricity	vortex theory
1745		Musschenbroek	(1692 – 1761)	Leyden jar	Nollet 1746 affluvim–effluvium
		FRANKLIN	(1706 – 1790)	charge, + and –, peak effect, lightning rod, conservation of charge	one fluidum + atmosphere
1767	quantitative electrostatics	Priestley	(1733 – 1804)		
		Aepinus	(1724 – 1802)	explanation of induction	
		Cavendish	(1731 – 1810)		
1784		COULOMB	(1736 – 1806)	$F = k \frac{Q_1 Q_2}{r^2}$	two fluida, boreal and austral magnetic poles + action at a distance
		Galvani	(1737 – 1798)		
1800	direct current	VOLTA	(1745 – 1827)	voltaic pile	
		Davy	(1778 – 1829)		
1811	magnetic field of a current	Poisson	(1781 – 1840)	$\Delta V = -4\pi\rho$	
		Ørsted (1777–1851) Biot (1774–1862) Savart (1791–1842)		$d\vec{B} \sim \frac{i\, d\vec{l} \times \vec{r}_0}{r^2}$	
1820		AMPÈRE	(1775 – 1836)	$d\vec{F} \sim i_1 i_2 \frac{(d\vec{s}_1 \times (d\vec{s}_2 \times \vec{r}_0))}{r^2}$	
1826		Ohm	(1789 – 1854)	Ohm's law	
1831		FARADAY	(1791 – 1867)	$U_i = -\frac{d\phi}{dt}$	
1845	electromagnetic field			Faraday rotation	
		Weber	(1804 – 1890)	$L_{ik} = \frac{\mu_0}{4\pi} \oint\oint \frac{d\vec{s}_i\, d\vec{s}_k}{r}$	
		Neumann	(1798 – 1895)		
		Thomson	(1824 – 1907)		
1864		MAXWELL	(1831 – 1879)	$\omega = 1/\sqrt{LC}$ $\mathrm{rot}\, \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	
1873				$v = 1/\sqrt{\varepsilon\mu}$	
1886 – 1888		Hertz	(1857 – 1894)		
1900					

Table 4.2 Chronology of discoveries in the field of electrodynamics.

Quotation 4.20

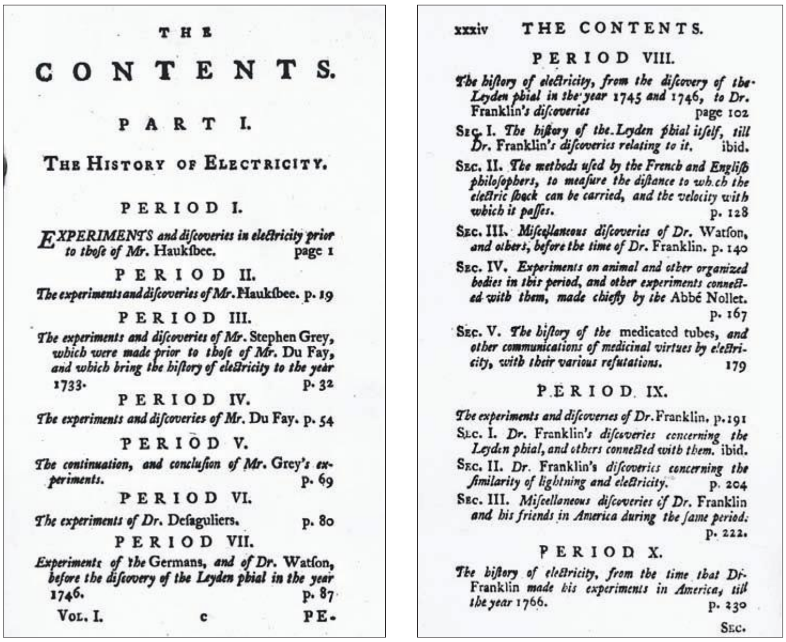
[If any one ask how a Medium can be so rare] ... Let him also tell me how an electrick Body can by Friction emit an Exhalation so rare and subtile, and yet so potent, as by its Emission to cause no sensible Diminution of the weight of the electrick Body, and to be expanded through a Sphere, whose Diameter is above two Feet, and yet to be able to agitate and carry up Leaf Copper or Leaf Gold, at the distance of above a Foot from the electrick Body? And how the

continued on next page

electrical phenomenon. However, FRANKLIN was the first to provide a complete proof of this assumption (Figure 4.67). He stretched an electrical conductor between a kite flying at a great height and a Leyden jar and showed that atmospheric electricity is just as capable of charging a Leyden jar as an electrification machine.

FRANKLIN further observed that a metallic point could conduct charge to a body and then conduct the charge away from it (Figure 4.68).

The reader will perhaps have noticed that we have been avoiding the word "charge," for this word is attached to a specific idea relating to the nature of electricity. The word "charge" was introduced by FRANKLIN, and so from now on, we will no longer require such circumlocutions as "bodies that have been brought into a state of electrification"; instead, we will speak of "electrically charged" bodies or bodies that have been given an electrical charge.



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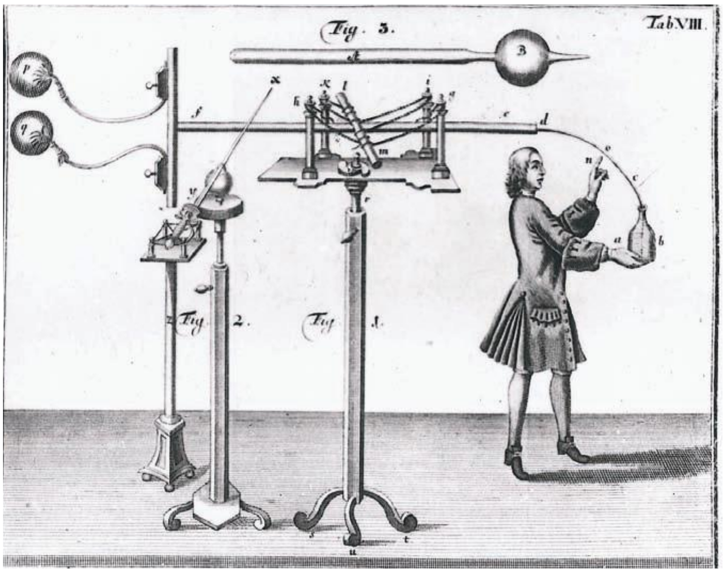
▲ **Figure 4.62** Two pages from the table of contents of PRIESTLEY's 1767 book *History and Present State of Electricity with Original Experiments*.

For FRANKLIN, there was only one type of charge, which he took to be glass electricity as described by DUFAY. The electrical state of a body depends on whether the body has more or less of these charges than normal. If the former, then we can say the body is charged with glass electricity, or—after EULER—that it is positively charged. If there is a shortage of electrical charges, then the body possesses—in the old terminology—resinous charge, or in the newer form, negative charge. Thus negative electric charge represents a shortage of charge; that is, the electrical charge has been removed from the material in question (Figure 4.69).



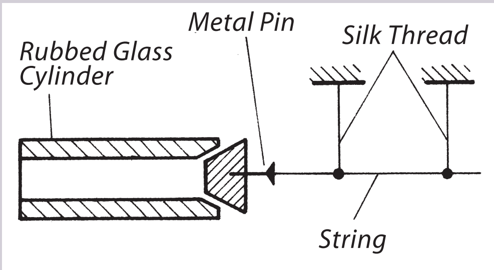
◀ **Figure 4.64** MUSSCHENBROEK discovered the "amplifying effect" of the Leyden jar in this manner [Winkler 1746]. (Library of the Hungarian Academy of Sciences.)

Quotation 4.20, ***continued***

Effluvia of a Magnet can be so rare and subtile, as to pass through a Plate of Glass without any Resistance or Diminution of their Force, and yet so potent as to turn a magnetick Needle beyond the Glass? ...

Have not the small Particles of Bodies certain Powers, Virtues, or Forces, by which they act at a distance, ... upon one another for producing a great Part of the Phaenomena of Nature? For it's well known, that Bodies act one upon another by the Attractions of Gravity, Magnetism, and Electricity; and these Instances shew the Tenor and Course of Nature, and make it not improbable but that there may be more attractive Powers than these. For Nature is very consonant and conformable to her self. How these Attractions may be perform'd, I do not here consider. What I call Attraction may be perform'd by impulse, or by some other means unknown to me. I use that Word here to signify only in general any Force by which Bodies tend towards one another, whatsoever be the Cause. For we must learn from the Phaenomena of Nature what Bodies attract one another, and what are the Laws and Properties of the Attraction, before we enquire the Cause by which the Attraction is perform'd. The Attractions of Gravity, Magnetism, and Electricity, reach to very sensible distances, and so have been observed by vulgar Eyes, and there may be others which reach to so small distances as hitherto escape Observation; and perhaps electrical Attraction may reach to such small distances, even without being excited by Friction.

—NEWTON, *Opticks*, Queries 22, 31



▲ **Figure 4.63** Schematic representation of GRAY's experiment.

Quotation 4.21

It is then certain ... that bodies which have become electric by contact are repelled by those which have rendered them electric; but are they repelled likewise by other electrified bodies of all kinds? And do electrified bodies differ from each other in no respect save their intensity of electrification? An

continued on next page

Quotation 4.21, ***continued***

examination of this matter has led me to a discovery which I should never have foreseen, and of which I believe no-one hitherto has had the least idea. ...

We see, then ... that there are two electricities of a totally different nature—namely, that of transparent solids, such as glass, crystal, &c. and that of bituminous or resinous bodies, such as amber, copal, sealing-wax, &c. Each of them repels bodies which have contracted an electricity of the same nature as its own, and attracts those whose electricity is of the contrary nature. We see even that bodies which are not themselves electrics can acquire either of these electricities, and that then their effects are similar to those of the bodies which have communicated it to them.

—Dufay, 1733 [Whitaker 1958, pp. 43–44]

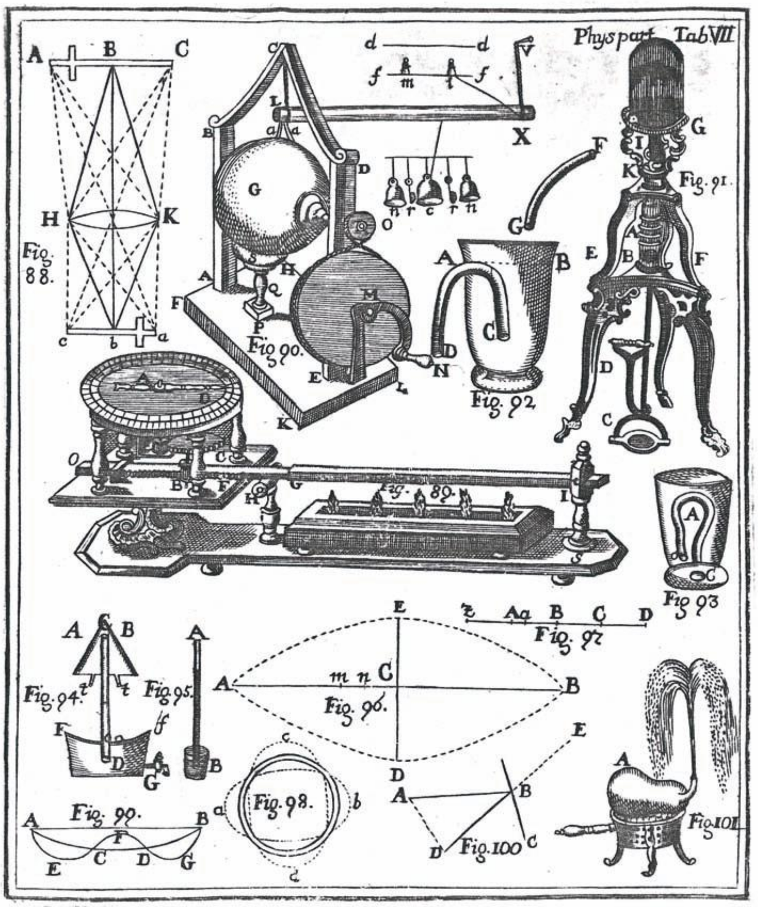
Quotation 4.22

Mr. Muschenbroek, who tried the experiment with a very thin glass bowl, says, in a letter to Mr. Réaumur, which he wrote soon after the experiment, that he felt himself struck in his arms, shoulder and breast, so that he lost his breath, and was two days before he recovered from the effects of the blow and the terror. He adds, that he would not take a second shock for the kingdom of France. ...

We are not, however, to infer from these instances, that all the electricians were struck with this panic. Few, I believe, would have joined with the cowardly professor, who said that he would not take a second for the kingdom of France. Far different from these were the sentiments of the magnanimous Mr. Boze, who with a truly philosophical heroism, worthy of the renowned Empedocles, said he wished he might die by the electric shock, that the account of his death might furnish an article for the memoirs of the French Academy of Sciences. But it is not given to every electrician to die in so glorious a manner as the justly envied Richman.

In France as well as in Germany experiments were made to try how many persons might feel the shock of the same phial. The Abbé Nollet, whose name is famous in electricity, gave it to one hundred and eighty of the guards, in the King's presence; and at the grand convent of the Carthusians in Paris, the whole community formed a line of nine hundred toises, by means of iron wires between every two persons (which far exceeded the line of one hundred and eighty of the guards) and the whole company upon the discharge of the phial, gave a sudden spring, at the same instant of time, and all felt the shock equally.

—J. B. Priestley, *The History and Present State of Electricity* [Vol. I, pp. 106–108, 125–126]

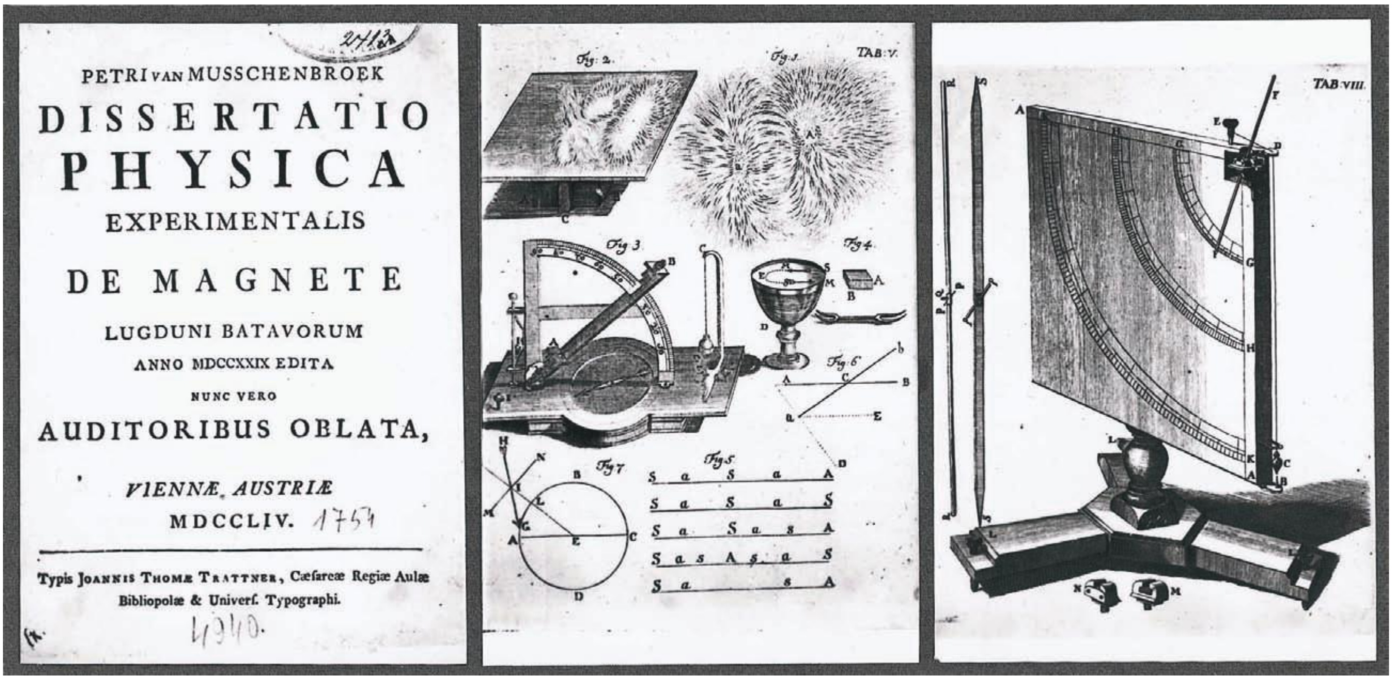


▶ **Figure 4.65** In the middle of the eighteenth century, experiments in electrostatics were carried out with instruments like these [Horvath 1770].

To explain electrical phenomena, Franklin assumed that electrical charges repel one another, whereas electrical charge and matter are mutually attractive. However, he could not thus explain why two bodies with a shortage of charge, consisting, so to speak, of "naked" material, repel each other. In our current terminology, the two bodies each carry a negative charge, and it seems obvious to us that there should be a repulsive force between them.

Franz Ulrich Theodor Aepinus (1724–1802), born in Rostock and active in St. Petersburg, attempted to find an answer to these open questions. According to his view, the particles of matter deprived of electricity repel one another just like particles of matter that constitute the electrical charge itself. One may interpret this statement, if we may use today's terminology, to say that the particles of matter remain after the removal of charges in a kind of ionized state.

We cannot determine exactly what motivated Franklin to view the electricity of glass as something present and to speak of a lack of something in regard to the electricity of resin. The terms "positive" and "negative" are logical consequences of such a viewpoint. Franklin apparently took the point of view that when a charged and uncharged body touched, the charges always flowed in one direction only. This is also the case, according to our present knowledge, when we consider

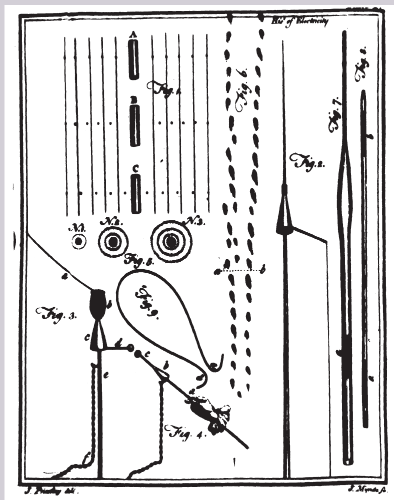


▲ **Figure 4.66** Title page and two typical pages of Musschenbroek's book. (Library of the University for Heavy Industry, Miskolc.)

the flow of electric charges in metallic bodies. We know that, contrary to Franklin's assumption, it is the negative charges that flow in metals while the positive charges remain in place. However, in Franklin's experiments in the transmission of charge to a body or the removal of charge from a body with the help of a metallic point, charge currents move in both directions in the air space between the metallic point and the body. It is quite likely that Franklin was led to the assumption that only glass electricity flows by the shape of the light phenomenon that faintly glows during such experiments, which is visible in the dark and indeed resembles a jet of water coming from a watering can.

Franklin's experiments and his interpretation of the results, which were then refined by Aepinus, finalized the charge conservation law. Charges were not produced by friction, but merely separated. By way of illustrating the separation of charges, Franklin compared bodies in an uncharged state to a wet sponge and the removal of charge by rubbing the body to the dripping of water when the sponge is compressed.

Alongside this monistic theory of electricity, however, the dualistic theory gained in importance. Even Coulomb embraced this model. Both models are able to explain the fact of conservation of charge, which follows automatically from the single-fluid model because a charge deficit necessarily agrees with the notion of removed charge. The action of force, however, can be more simply explained with the two-fluid model. To wit, if one assumes the existence of two forms of electricity, then it may simply be assumed that the force is proportional to both charges, that is to say, to their product, or as we would put it today, is proportional to $Q_1 \times Q_2$. With the single-fluid model, one must postulate a repulsive force between the excess charges, an attractive force between matter and charges, and finally, a repulsive force between matter and matter. The following argument shows how the two models lead to the same conclusion.



▲ **Figure 4.67** Franklin's experimental contraptions.

As every circumstance relating to so capital a discovery as this (the greatest, perhaps, that has been made in the whole compass of philosophy, since the time of Isaac Newton) cannot but give pleasure to all my readers, I shall endeavour to gratify them with the communication of a few particulars which I have from the best authority.

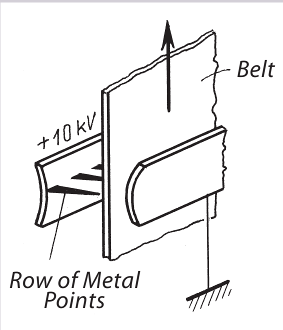
continued on next page

Figure 4.67 *continued*

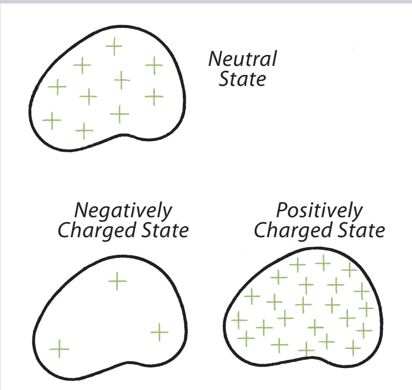
To demonstrate, in the completest manner possible, the sameness of the electric fluid with the matter of lightning, Dr. Franklin, astonishing as it must have appeared, contrived actually to bring lightning from the heavens, by means of an electrical kite, which he raised when a storm of thunder was perceived to be coming on. This kite had a pointed wire fixed upon it, by which it drew the lightning from the clouds. This lightning descended by a hempen string, and was received by a key tied to an extremity of it; that part of the string which was held in his hand being of silk, that the electric virtue might stop when it came to the key. He found that the string would conduct electricity even when nearly dry, but that when it was wet, it would conduct it quite freely; so that it would stream out plentifully from the key, at the approach of a person's finger.

At this key he charged phials, and from electric fire thus obtained, he kindled spirits [ignited alcohol], and performed all other electrical experiments which are usually exhibited by an excited globe or tube.

—J. B. Priestley, *The History and Present State of Electricity*, 1775 [pp. 216, 215]



◀ **Figure 4.68** The moving belt in a Van de Graff generator is charged with the "wonderful Effect of Points" discovered by Franklin. These generators have been used in accelerators for nuclear research to create several million volts of electric potential.



▲ **Figure 4.69** According to Franklin, there was only one type of charge.

We set $Q_1 = E_1 - M_1$ and $Q_2 = E_2 - M_2$, where M is the quantity of matter that in the uncharged state neutralizes the effect of the electrical charge E. The product $Q_1 \times Q_2$ then becomes

$$\begin{aligned} Q_1 \times Q_2 &= (E_1 - M_1) \times (E_2 - M_2) \\ &= E_1 \times E_2 - E_1 \times M_2 - E_2 \times M_1 + M_1 \times M_2, \end{aligned}$$

the right-hand side of which contains precisely the mathematical expressions for the above-mentioned interactions.

On the occasion of a farewell banquet for his scientifically minded friends, Franklin projected a vision of a future age of electricity. Before the dinner, a spirit stove on the far side of a river was lit with the aid of an electric spark, the turkey destined for the festive meal was killed with an electric shock, and the guests drank to the health of all the famous "electricians" in the world from electrified champagne glasses to the crackling of the discharge of electrified jars.

By the middle of the eighteenth century, additional phenomena had been described—even if only in qualitative form—whose significance would only later become clear (Quotations 4.23, 4.24 and Figure 4.70).

4.4.4 *Quantitative Electrostatics*

By now, the time had become ripe for establishing quantitative laws to describe electrical phenomena, and indeed, several researchers succeeded in quantifying the electrical attractive force. The "natural-philosophical" basis for this was the same for all of them: Following Newton, they assumed the existence of forces acting at a distance between charged bodies and looked for the laws that described those forces. The law was found independently by four researchers: Priestley, Cavendish, Robison, and Coulomb, after the latter of whom the law was named. Priestley formulated the law precisely in his 1767 book and provided a reasoned basis for it. It was known from experiments that charge accumulates on the surface of electrical conductors and that there is no perceptible electric charge in the interior of hollow bodies made of electrically conducting material (Quotation 4.25). The forces emanating from the surface charges could compensate for an arbitrary point in the interior of a closed surface only if the force were inversely proportional to the square of the distance from the charge (Figures 4.71, 4.72). Cavendish arrived at the law via a similar train of thought; moreover, he confirmed the law with the aid of a torsion balance. It is of historical interest that the idea of using a torsion balance to measure small forces was mentioned independently by several researchers. Cavendish credits a Reverend Michell for being the source of the idea of the torsion balance as well as for providing him with the first exemplar.

Robison made his measurements in 1769, earlier than Coulomb, but did not produce a consistent formulation of the law. He obtained different values for repulsive and attractive forces: in one case he got slightly higher values than the square law, and in the other, slightly lower ones.

As an engineer and military officer, Charles Coulomb (1736–1806, Figure 4.73) spent a number of years overseeing fortifications, and it was not until 1776 that he was able to devote all of his energies to scientific work. He made notable contributions to a number of scientific fields; for example, he was awarded a competition prize from the French Academy of Sciences for the construction of the best compass, and on the basis of that work was elected to the academy. In a prize-winning paper, Coulomb describes a torsion balance and shows that the torque necessary to rotate the torsion thread through a given angle is proportional to the fourth power of the thread's diameter and inversely proportional to its length, with the proportionality factor depending on the material of which the thread is constructed. We also have Coulomb to thank for an extensive study of sliding friction in the case of rigid bodies. He was the first to investigate in depth the question of the force required to hold a ship on an inclined plane before launch.

Figure 4.74 shows Coulomb's torsion balance as it appeared in his original article.

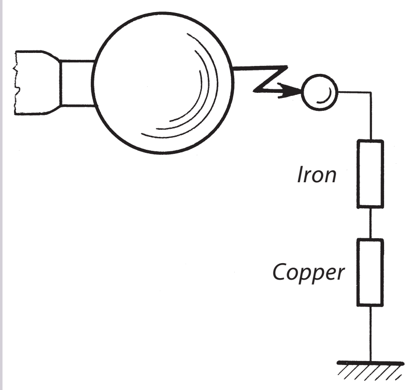
Cavendish would today occupy a higher place in the history of the study of electricity had he published his results promptly. As it turned out, they were only published 100 years after the fact, in 1879. The reason for publication then was that Lord Kelvin had discovered in Cavendish's manuscripts the formula for the relationship between the charge on a sphere and the charge on a flat circular plate at the same potential. While this was only an experimental result, it suggested that Cavendish must have surmised some concepts and their qualitative roles that were to become very important later on. A treatment of this problem is even today possible only with university-level knowledge and would be considered a difficult question on an examination in theoretical electrodynamics.

Of course, Cavendish did not write about potential but used the term "degree of electrification" instead. Thus for him, two metallic conductors have the same degree of electrification when they are joined by an electrical conductor. Today we would say that both conductors are on the same potential. Cavendish raised the question of the relationship between the charges of both bodies in this case. When determining this charge relationship, Cavendish in effect determined the ratios of the electric capacitances of the two metallic bodies. The carrying out of these measurements required a clear understanding of electrical phenomena. Cavendish did not report on the details of how these measurements were made, but it is likely that he touched the conductors many times with a small test sphere, each time discharging the test sphere by grounding it.

Cavendish also investigated the properties of dielectrics and determined that the ability of a conductor to store a charge is altered if various insulating materials are placed in the vicinity of the conductor. This fact was rediscovered by Faraday almost two generations later.

Cavendish also measured the conductivity of various substances and introduced the concept of resistance, preceding Ohm by half a century. What is astonishing here is that he determined the ratio of conductivities of seawater and iron to be $1 : 4 \times 10^6$, an excellent result. Cavendish remarked that he obtained this value by means of a simple measuring procedure. Today, one would measure electrical resistance also with very simple instruments, namely a voltmeter and an ammeter. In Cavendish's time, the creation of such devices was still far in the future, and one learns from his manuscript how he took these measurements: Cavendish compared the strengths of the electric shocks he felt when he touched—through the various materials—the electrodes of a charged Leyden jar. From the subjective sensation of the shock, he estimated numerical values for the conductivities. It is practically a miracle that he was able to obtain such good results with such a method.

Once the laws for the interactions of electrically charged bodies had been represented mathematically, there was nothing to stand in the way of the application of the mathematical apparatus that had been worked out for treating gravitation from being applied to electrostatic phenomena. This was done by Poisson in an 1811 publication. With this work, electrostatics had finally arrived at the same level of mathematical perfection as mechanics. In addition, Poisson also contributed to the coming of age of magnetostatics in 1824.



▲ **Figure 4.70** Priestley, on advice from Franklin, was the first to investigate the effect of the same current passing through different conductors.

Quotation 4.23

In a conversation I once had with Dr. Franklin, Mr. Canton, and Dr. Price, I remember asking whether it was probable that there was any difference in the conducting power of different metals. ... I have since endeavoured to carry into execution a scheme proposed by Dr. Franklin, viz. transmitting the same explosion of the battery through two wires at the same time, of two different metals, and of the same thickness. ... I first joined a piece of iron wire, and a piece of copper wire. The explosion totally dispersed the iron, and left the copper untouched. ... From these experiments it is easy to settle the order in which the metals above mentioned are to be ranked, with respect to the power of electricity to melt them. It is as follows. *Iron, brass, copper, silver, gold*. ... I make no doubt but that an explosion which melts a copper wire of any given diameter would disperse an iron wire of twice the diameter.

—J. B. Priestley, *The History and Present State of Electricity* [Vol. II, pp. 368–371]

Quotation 4.24

Magnets have been observed to lose their virtue, or to have their poles reversed by lightning. The same did Dr. Franklin by electricity. By electricity he frequently gave polarity to needles, and reversed them at pleasure.

—J. B. Priestley, *The History and Present State of Electricity* [Vol. I, p. 214]

Laplace (Figure 4.75) had earlier determined that for a system of point masses whose individual particles act on one another analogously to those in Newton's law of gravitation, the force on a given point mass can be represented by partial derivatives of a certain quantity. Laplace did not give this quantity the name "potential," by which we know it today, but he did specify the value

$$V = G\sum_i \frac{m_i}{r_i}.$$

Laplace also showed that this quantity satisfies the equation

$$\Delta V \equiv \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0,$$

today known as Poisson's equation. Poisson also determined that at locations where continuously distributed electrical charges are located, this equation must be modified to read

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -4\pi\rho.$$

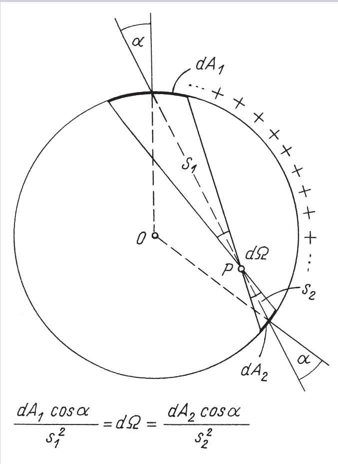
In this form, the equation is known as the Laplace–Poisson equation. Here ρ denotes the charge density.

Poisson was able to solve electrostatic problems that even by today's standards are quite complicated. For example, he calculated the charge distribution on the surface of two opposing metallic spheres.

Finally, electrostatics was brought into its present-day form by Green and Gauss (Figure 4.76).

4.4.5 *Flow of Electric Charge*

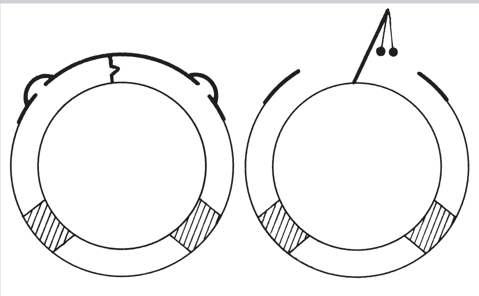
Even in the electrostatic experiments it was obvious that during charging or discharging, a flow of electrical charges takes place; in other words, one can talk about an *electrical current*. The laws of electrical currents and, in particular, their magnetic effects could be successfully investigated only when it had become possible to generate sustained flows of electric charge of a strength suitable for experimentation. Various effects of the flowing charges have been previously



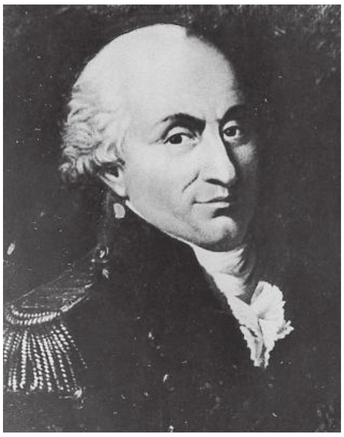
▲ **Figure 4.71** If electrical force falls off with distance as $1/r^2$, then the forces from dA_1 and dA_2 acting on the point P cancel each other out, since we have

$$F_1 \propto \frac{dA_1}{s_1^2} = \frac{d\Omega}{\cos\alpha} = \frac{dA_2}{s_2^2} \propto F_2.$$

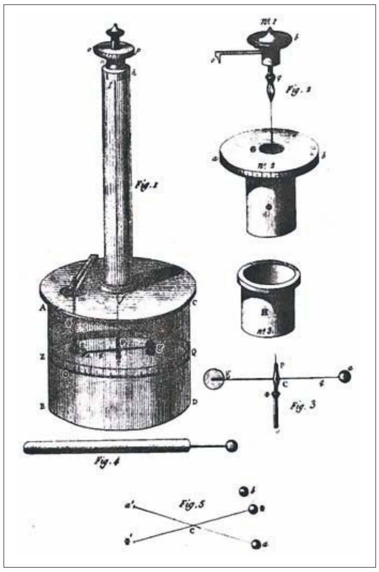
The entire surface of the sphere can be decomposed into pairs of surface elements whose effects balance each other out.



▲ **Figure 4.72** An experimental arrangement for proving that inside a metallic electrode there is no electrical effect. At the same time, this will also prove the $1/r^2$ law, at least to the extent of the accuracy of the measurement. Let us denote any deviation from the inverse-square law by the quantity *e* in the formula $1/r^{2+\varepsilon}$. The precision available to Cavendish allowed the limit $|\varepsilon| < 1/50$; Maxwell could improve this to $|\varepsilon| < 1/21{,}600$. Today, it has been further reduced to $|\varepsilon| < 3 \times 10^{-16}$ (Williams, Faller, Hill, 1971). These investigations have a fundamental importance for modern physics, being connected with the question of whether the photon has a nonzero rest mass. In the case of a nonzero mass, according to Yukawa (see Figure 5.140), the potential would have, instead of $1/r$, the following dependence on distance: $e^{-r/r_0}/r$, where $r_0 = h/2\pi mc$ and h is Planck's constant, m the rest mass, and c the velocity of light. Using this formula, $|\varepsilon| < 3 \times 10^{-16}$ implies an upper bound on the rest mass of the photon of $m < 2 \times 10^{-50}$ kg. This mass is many orders of magnitude less than the rest mass of the neutrino. Thus, at least according to our current knowledge, we may consider the rest mass of the photon to be equal to zero.



▲ **Figure 4.73** Charles Auguste de Coulomb (1736–1806): After nine years service as an engineering officer, began to work on scientific problems. He confirmed the law named for him with measurements carried out between 1784 and 1789. In his 1785 book *Théorie des machines*, he considered problems in friction and elasticity, including torsion. These investigations led to the construction of the torsion balance.



▲ **Figure 4.74** Coulomb's torsion balance. The balance and the measurements obtained with it were published beginning in 1784.

▲ **Figure 4.76** Carl Friedrich Gauss (1777–1855): Son of a mason; studied at the University of Göttingen, then instructor in Brunswick, and from 1807 until his death, director of the newly founded observatory and professor of mathematics and astronomy in Göttingen.

In his early years he worked on problems in number theory. The results are collected in his 1801 book *Disquisitiones Arithmeticae*. He shifted the focus of his work about every ten years: 1800–1820, astronomy; 1820–1830, geometry; 1830–1840, theoretical physics. His most important summary works are *Theoria motus corporum coelestium* (1809), *Disquisitiones circa superficies curvas* (1827), *Intensitas vis magneticae terrestris ad mensuram absolutam revocata* (1833), and *Dioptrische Untersuchungen* (1840).

Of his contributions to mathematics that immediately influenced the development of physics or have continued to have application in physics, we mention the following: the method of least squares, the Gaussian error function, the internal geometry of surfaces, the hypergeometric series, the Gaussian complex plane, and the gamma function (complex factorial).

His most important contributions to physics are these: the principle of least constraint, the general theory of optical mapping with lenses, Gauss's theorem on electrostatics, the Gaussian system of units, a measuring procedure for the magnetic moment using his method of "principal positions."

Gauss is rightly referred to as the prince of mathematicians. His mathematical talent was revealed in his earliest years. He was nine years old when his elementary school class was given the exercise of adding up the numbers from 1 to 60, which Gauss solved in a matter of seconds, having noted that 61 = 1 + 60 = 2 + 59 = 3 + 58, and so on, and that there are 30 such pairs, and therefore 30 × 61 = 1830.

In searching for solutions to the equation $x_n - 1 = 0$, Gauss realized which regular polygons could be constructed with straightedge and compass; he was thus able to construct a 17-sided regular polygon. He was so proud of this result that he ordered the figure to be carved on his tombstone. Although it can be seen from Gauss's diary that by 1818—thus before Bolyai and Lobachevsky—he had discovered non-Euclidean geometry, he did not dare publish his results, because he feared that the world was not yet mature enough to accept such ideas.

◀ **Figure 4.75** Pierre-Simon Laplace (1749–1827): Of peasant stock, he first considered a religious vocation. D'Alembert secured a position for the 22-year-old Laplace as mathematics teacher in Paris. Laplace actively took part in social and scientific-organizational life. Together with the chemist Claude-Louis Berthollet (1748–1822), founded the Society of Arcueil for the promotion of young scientists. Member of the *Académie*; named minister of the interior for a brief period by Napoleon during the period of the Consulate.

His more important works are these: *Exposition du système du monde* (1796); in this work written in a high literary popular style, we find a theory of the origin of the solar system (Kant–Laplace hypothesis).

Mécanique céleste (1799–1825), five volumes; here, alongside a number of concrete astronomical problems, the Laplace equation $\Delta U = 0$ is considered. Laplace's contributions to probability theory and statistics are found in the two treatises *Théorie analytique des probabilités* (1812) and *Essai philosophique sur les probabilités* (1814). In the foreword to the last-named work, we meet the frequently cited Laplace's demon.

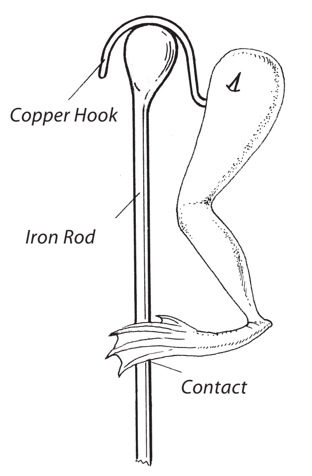
We shall have more to say in the text about Laplace's investigations in the theory of heat, still based on the *caloricum* theory. Here we would like to mention only the results on the theoretical determination of the speed of sound. Mersenne and Gassendi had measured it earlier, while Newton determined it theoretically ($v = \sqrt{p/\rho}$, where p is the pressure, ρ the density). The measurements carried out with greater precision by William Derham (1708) brought to light a discrepancy with the Newtonian theory. Laplace suggested that the adiabatic changes that occur as a result of compression and rarefaction of the air arising from wavelike propagation also need to be taken into account ($v = \sqrt{\kappa p/\rho}$, where $\kappa = c_p/c_v$, with c_p the specific heat at constant pressure, c_v the specific heat at constant volume).

After the publication of the first volume of the *Mécanique céleste*, Laplace was asked by Napoleon why the heavens are discussed over hundreds of pages, but God is never mentioned. Laplace is said to have answered, *Je n'ai pas eu besoin de cette hypothèse là, Sire* (I had no need of that hypothesis, your Majesty).

Quotation 4.25

May we not infer from this experiment, that the attraction of electricity is subject to the same laws with that of gravitation, and is therefore according to the squares of the distances; since it is easily demonstrated, that were the earth in the form of a shell, a body in the inside of it would not be attracted to one side more than another.

—J. B. Priestley, *The History and Present State of Electricity* [Vol. II, p. 374]



◀ **Figure 4.77** Galvani's most important observation: To induce a twitch in a frog's leg, the arrangement shown suffices, and neither a spark nor lightning nor atmospheric electricity is needed.

Quotation 4.26

But I took the animal into a closed room, and placed it on an iron plate; and when I pressed the hook which was fixed in the spinal marrow against the plate, behold! the same spasmodic contractions as before. I tried other metals at different hours on various days, in several places, and always with the same result, except that the contractions were more violent with some metals than with others. After this I tried various bodies which are not wood; but nothing happened. This was somewhat surprising, and led me to suspect that electricity is inherent in the animal itself. This suspicion was strengthened by the observation that a kind of circuit of subtle nervous fluid (resembling the electric circuit which is manifested in the Leyden jar experiment) is completed from the nerves to the muscles when the contractions are produced.

—Galvani [Whitaker 1958, pp. 68–69]

Quotation 4.27

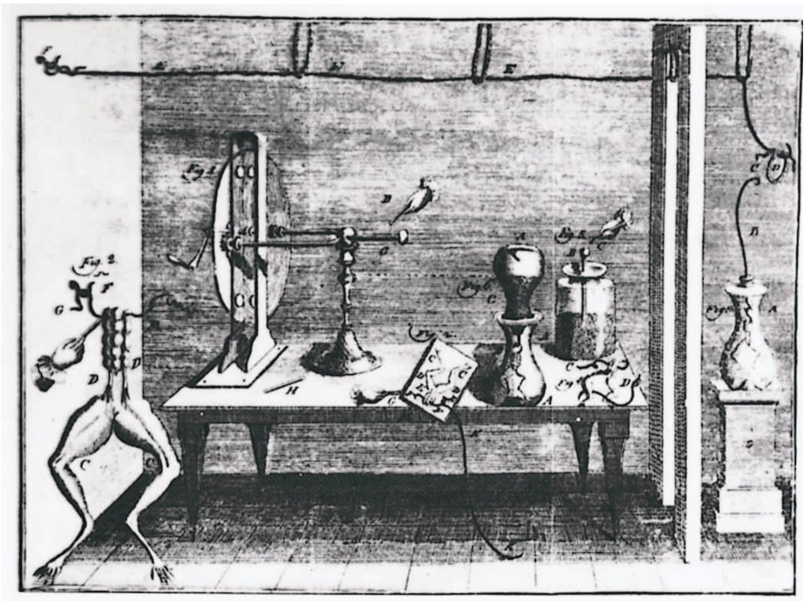
Yes, the apparatus of which I speak, and which will doubtless astonish you, is nothing but the assemblage of a number of good conductors of different types, arranged in a certain fashion. 30, 40, 60, or more pieces of copper—or, better, silver—each applied to a piece of tin—or, what is much better, zinc—and an equal number of levels of water, or some other liquid that is a better conductor than pure water, such as salt water, lye, &c., or pieces of cardboard, skin, &c., that have thoroughly absorbed these liquids; such layers interposed between each pair or combination of the two different metals, such an alternating sequence, and always in the same order, of these three types of conductors: this is all

continued on next page

observed. The heat and light from electrical sparks was evident; further, it was also known that in the proximity of a very strong discharge iron may become magnetized or its magnetic polarity can be reversed (Quotation 4.24).

A breakthrough in this field came in the year 1800. In that year, Alessandro Volta (1745–1827) wrote a letter to the president of the Royal Society in which he described a discovery involving constant direct current. The prehistory of this discovery goes back to the year 1780. Luigi Galvani (1737–1798), professor of anatomy at the University of Bologna, had learned from one of his assistants that while making an anatomical preparation of a nerve from a frog's leg, it had been observed that if one touched the nerve with a dissecting knife at a time when a discharge spark was being emitted from a—then ubiquitous—triboelectric machine elsewhere in the laboratory, the nerve twitched. It was only in 1791 that Galvani described how this phenomenon came to his attention and the various experiments that he conducted. From today's perspective, we can say that if Galvani's colleagues had actually observed a simultaneity between the electrical discharge and the twitching of the frog's leg, then they, in fact, could have been registering an electromagnetic wave emanating from the sparks. In the course of his own experiments, Galvani discovered that frogs' legs hung with copper hooks on an iron window grating would twitch if they accidentally touched the iron grating (Figure 4.77). This fact convinced Galvani that it was not something like a storm or atmospheric electricity that was responsible for the phenomenon. He finally carried out a variety of laboratory experiments (Figure 4.78) from which he concluded (Quotation 4.26) that this electrical phenomenon had its origin in the frog's leg itself, and with this he introduced the notion of animal electricity.

At about this time, Volta was professor of physics at the rival University of Pavia, where he had made a name for himself with a very effective electrostatic machine, called an electrophorus, and—more importantly—a very sensitive electroscope (Figure 4.79). Volta started out by reproducing one of Galvani's



▲ **Figure 4.78** Galvani's diagram of the frog's-leg experiment.

experiments, and he accepted GALVANI's interpretation that the electrical phenomenon originated in the frog's leg itself. But in the following year, in 1793, VOLTA performed very carefully a series of experiments already referenced in 1754 by the Swiss researcher SULZER: If we connect two different metal pieces together at one end then touch one of the pieces with our tongue, we perceive either a mild acid or a mild alkali taste, depending on the type of metal, just as when we touch with our tongue the positive or negative pole of a triboelectric machine. VOLTA immediately saw that the frog's leg in GALVANI's experiments served no function other than to make the presence of electricity visible and that the crucial factor was the contact between the two kinds of metal. He showed that no twitching occurs if only there is only one type of metal involved in the experiment. Finally, using an electroscope, he was able to prove directly that two connected pieces of different metals will each have a charge after being separated. VOLTA's great discovery was that this effect could be enhanced by stacking several zinc and copper plates alternately with a third material between them, such as damp cardboard, which VOLTA called a conductor of the second kind (Quotation 4.27 and Figures 4.80, 4.81).

VOLTA did not understand how his galvanic battery worked. He did, however, correctly state that the electricity thus obtained had the same properties as the electricity obtained through friction. He was of the opinion that the two metals played passive roles and thus the current flow that began when the circuit was closed would continue indefinitely without any change occurring in the battery. Although during VOLTA's lifetime the impossibility of a mechanical perpetual motion machine was already a well-accepted fact, with the mysterious and weightless electrical fluidums, everything still seemed possible.

Already in 1800, the same year in which VOLTA's discoveries were announced, intensive work with galvanic batteries was begun in England. A leading role here was played by HUMPHRY DAVY (1778–1829). He determined that chemical processes occurring in the galvanic batteries have a significant role in the electrical phenomena. His work can be seen as the starting point for the field of electrochemistry and indeed, as the starting point for all theories about the electrical nature of chemical processes.

The characteristic quantities in the description of the flow of electrical charges are the current intensity and the "electromotive force" that creates the flow. They are linked by the simplest formula in electrical engineering: Ohm's law.

"Let's take a galvanic battery and measure its voltage U; then we connect the poles of the battery with conductors of various lengths and various diameters made of the same material, and we measure the current intensity I in each case; finally, we repeat the experiment with various materials. We will immediately obtain Ohm's law in the following form:

$$U = I \times R.$$

Here, $R = R_{internal} + R_{external}$ and $R_{external} = \rho(l/A)$, where ρ is the specific resistance, l is length, and A is the cross-sectional area of the conductor."

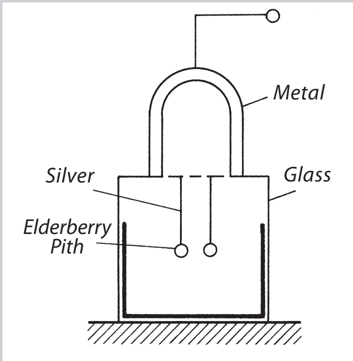
It is all so simple, and we do not understand why it was discovered only after the magnetic effect of electrical currents. But, of course, we are looking at this from today's point of view, which gives us a false picture.

GEORG SIMON OHM (Figures 4.82, 4.83) was the first to publish the relationship that now bears his name; he did so in two articles from the years 1826 and 1827. [The relationship in Figure 4.83 that reads $X = a/(b + x)$ would today be written $I = U/(R_{ext} + R_{int})$.] However, OHM's theory was not accepted until 1841, and it was not until a quarter century after his discovery that he attained his professorial status.

Quotation 4.27, ***continued***

that constitutes my new instrument, which imitates, as I have said, the effects of Leyden bottles, or electric batteries, by producing the same shocks as these; [and] whose activity, in truth, remains far below that of the aforesaid batteries, even when these have been charged to a high level, with respect to the force and noise of its explosions, to its spark, to the distance at which the discharge can occur, &c., equaling only the effects of a battery charged to a very weak degree, though a battery with an immense capacity; in other respects, however, it infinitely surpasses the virtue and power of these very same batteries, insofar as it does not need, as they do, to be charged in advance by means of foreign electricity; and insofar as it is capable of producing shocks every time one touches it in the correct manner, no matter how frequent these contacts are.

—ALESSANDRO VOLTA, "On the Electricity Excited by the Mere Contact of Conducting Substances of Different Kinds," 1800



◀ **Figure 4.79** VOLTA's electroscope.

▲ **Figure 4.80** VOLTA demonstrates the voltaic pile to General NAPOLEON BONAPARTE.

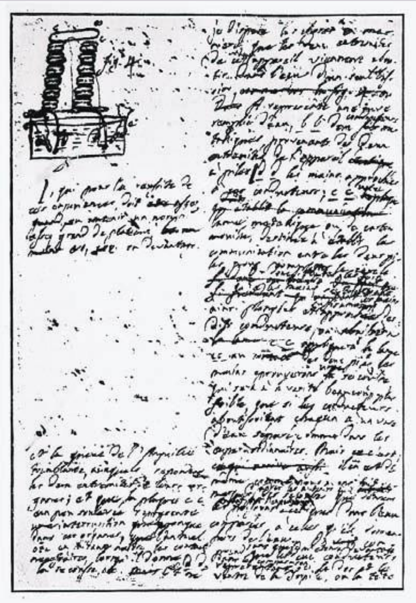
In establishing his law, Ohm could draw on two significant developments. Thomas Johann Seebeck (1770–1831) discovered thermoelectricity in 1821, and so Ohm was able to obtain a current source with constant voltage. On the theoretical front, Fourier's 1822 work on thermal conduction helped Ohm formulate analogous laws for electrical conduction. The problems that Ohm had to overcome were more conceptual than technical: for example, it was not clear whether a current along a conductor is constant or whether it may be "used up" in the process, and it was also not clear what was the relationship between the potential, as known from electrostatics, and the quantity that was measurable in the electric circuit and that was somehow analogous to the concept of temperature. It was also unknown whether current flows on the surface or through the interior of a conductor.

Ohm's simple law was extended by Gustav Kirchhoff to more complicated circuits. In 1845, Ohm worked out the two Kirchhoff laws for general circuits. Kirchhoff also made crucial progress in explaining the subject conceptually; for example, he pointed to the shared nature of the potential in the Poisson equation and the "electroscopic force" in Ohm's law.

Kirchhoff's first law, or current law, states that the sum of the currents meeting at a node in a circuit is equal to zero. In forming this sum, the currents flowing into the nodal point are considered to be negative, and those flowing out are considered positive.

Kirchhoff's second law concerns voltage: if one considers any closed loop within an electrical circuit, the sum of the "electromotive forces" (the internal voltages) is equal to the drop in voltage across the resistors. Again, the appropriate positive and negative signs must be used.

The introduction in 1894 of complex resistances, or impedances, in treating alternating-current circuits is the work of the American engineer Charles Steinmetz (1865–1923). For the quantitative treatment of phenomena in networks that are powered by generators with complex temporal voltage curves, a most original, almost magical, method was given by Oliver Heaviside (1850–1925), a method that was able to be justified mathematically only with difficulty via the methods of the Laplace transform and the theory of distributions.



▲ **Figure 4.81** The construction of the voltaic pile (from one of Volta's notebooks).

Quotation 4.28

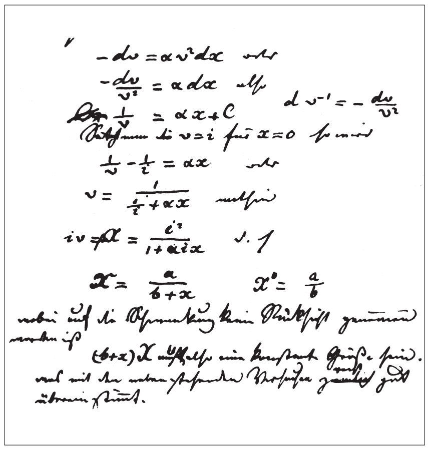
You certainly have a right to ask why it is inconceivable that no one tried the action of the voltaic pile on a magnet for twenty years. However, I believe that a cause of this is easily discovered: it simply existed in Coulomb's hypothesis on the nature of magnetic action; everyone believed this hypothesis as though it were a fact; it simply discarded every possibility of the action between electricity and so-called magnetic wires; the restriction was such that when M. Arago spoke of these new phenomena [of electromagnetism] at the Institute his remarks were rejected just as the ideas of stones falling from the heavens were rejected when M. Pictet read a memoir to the Institute on these stones. Everyone had decided that all this was impossible. ... Everyone resists changing ideas to which he is accustomed.

—Ampère, Letter to a friend, 1820 [Williams 1966, p. 60]

▶ **Figure 4.82** Georg Simon Ohm (1789–1854): Was senior teacher of mathematics and physics at the gymnasium in Cologne (1817–1828); from 1833 directed the polytechnic school in Nuremburg, and was appointed professor at the University of Munich in 1849.

In addition to his well-known discoveries in basic electrical circuits, Ohm did significant work in acoustics. He investigated the role of overtones in human hearing (1843). Helmholtz seized on Ohm's work on hearing in working out his resonance theory.

▼ **Figure 4.83** Ohm's law in its original form.



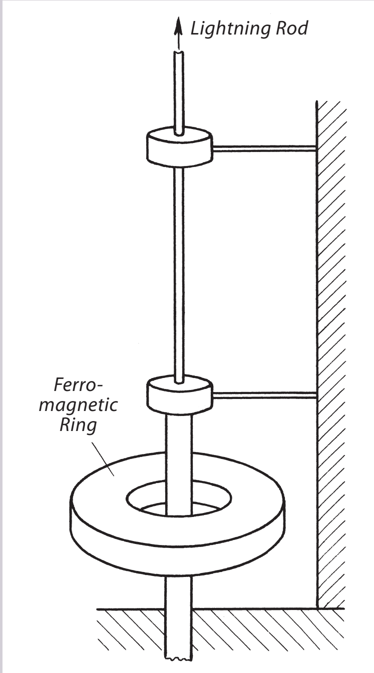
4.4.6 *The Magnetic Field of Electric Currents: Cross-Fertilization from Natural Philosophy*

Throughout the first two decades of the nineteenth century, experimenters were able to make use of equipment that could produce constant currents of suitable strength to bring conductors to incandescence and to carry out electrochemical investigations. Therefore, it seems surprising that the magnetic effects of current were only discovered in 1820.

At the beginning of the nineteenth century, a number of observations should have suggested to investigators that there is some kind of connection between magnetic fields and electrical current or, as one would have said at the time, the flow of the electric *fluidum* produces a magnetic effect in the surroundings. For instance, it was already known that in a house stuck by lightning, steel objects—knives, for example—that were close to the lightning strike became magnetized. Even today it is still a common practice to measure the very high currents (on the order of 100,000 amperes) that occur in lightning by using the magnetic effect (Figure 4.84). At that time, however, this was given little attention because no one was looking for a connection between electricity and magnetism. COULOMB's writings, as AMPÈRE remarked in a letter (Quotation 4.28), excluded all such possibilities. Strangely, the impulse to seek such a relationship came from philosophy: The extreme mechanical materialism emanating from the rationalism of the eighteenth century was protested against by Romanticism in art, literature, and philosophy. This movement emphasized a more unified and dynamic description of nature and mankind. In SCHELLING's natural philosophy, all natural phenomena are represented as diverse manifestations of a single fundamental principle, in constant battle with one another, but eventually reaching equilibrium.

ØRSTED embraced this philosophy; he spent years searching for a connection between electricity and magnetism. In this regard, natural philosophy exerted an immediate positive influence on the development of physics, and we have to rate this influence even more highly when we consider that FARADAY was also thinking along such lines. As is clear from ØRSTED's memoirs (Quotation 4.29), such a unifying natural-philosophical point of view can have disadvantages as well. ØRSTED assumed at the outset that the magnetic effect should emanate from an electrical conductor like light or heat and together with light and heat. For this reason, he started by looking for the magnetic effect around conductors that glowed with current. He chose a very thin platinum wire as the conductor because that could be made to glow readily. In fact, a weak current was sufficient for heating the thin wire, but this worked against the success of the experiment.

ØRSTED's discovery was of a purely qualitative character (Figure 4.85), and the theory that he proposed contributed neither an explanation of the phenomenon nor useful suggestions for further experimentation. Nevertheless, it was so completely unexpected that it received great attention in Europe. ØRSTED sent his article, which was written in Latin, to all the relevant scientific societies in Europe. It is already clear from the letter of AMPÈRE cited above (Quotation 4.28) that there was a general reluctance to believe in the correctness of the observation. However, the speed with which further theoretical and experimental results were achieved in this area proves that the leading intellects of the time had soon completely accepted this idea. Both the necessary experimental equipment and the requisite mathematical apparatus were at hand, so that within a few years, the associated theoretical description as we know it today had been completed.



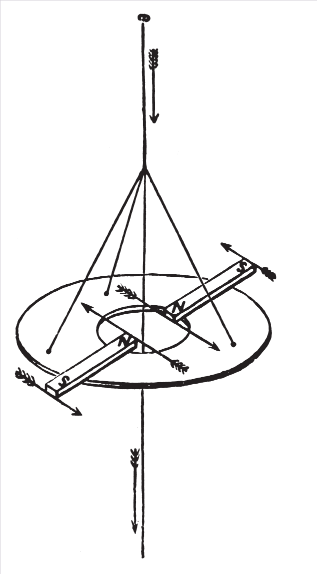
▲ **Figure 4.84** Even today, the magnitude of the current in a lightning strike (up to hundreds of thousands amperes) is determined from the magnetization of a ring made of ferrous material slipped onto the grounding stake.

Quotation 4.29

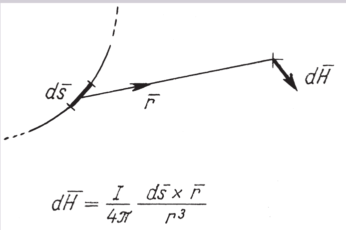
Electromagnetism itself was discovered in the year 1820, by Professor HANS CHRISTIAN OERSTED, of the University of Copenhagen. Throughout his literary career, he adhered to the opinion, that the magnetic effects are produced by the same powers as electrical. He was not so much led to this, by the reason commonly alleged for this opinion, as by the philosophical principle, that all phenomena are produced by the same original power. ...

In the month of July 1820, he again resumed the experiment, making use of a much more considerable galvanical apparatus. The success was now evident, yet effects were still feeble in the first repetitions of the experiment, because he employed only very thin wires, supposing that the magnetical effect would not take place, when heat and light were not produced by the galvanical current; but he soon found that conductors of a greater diameter give much more effect; and he then discovered, by continued experiments during a few days, the fundamental law of electromagnetism, viz. *that the magnetical effect of the electrical current has a circular motion round it*.

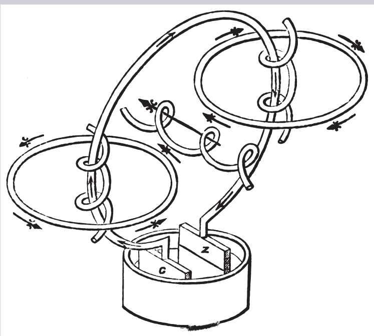
—ØRSTED, Article about his own discovery in *The Edinburgh Encyclopaedia* [Williams 1966, pp. 56, 58]



◀ **Figure 4.85** Ørsted's experiment as illustrated by Maxwell half a century later.



◀ **Figure 4.86** The Biot–Savart law as expressed today.



◀ **Figure 4.87** Maxwell explained the relationships between a current and a magnetic field with such drawings.

In 1820, the year of Ørsted's discovery, Biot and Savart gave a quantitative description of the magnetic effect generated by a current flowing through a conductor at any point in space. To measure the force, they employed a magnetized needle and used its period of oscillation as a measure of the intensity of the magnetic effect. Biot and Savart were primarily experimentalists, and their measurements were limited to two simple setups. Laplace then helped them to work out the precise expression of the law that followed. He pointed out that the measured effect could be understood as the sum of the effects of short segments of the conductor; the effect of each segment—a differential *current element*—being inversely proportional to the square of the distance to the point of observation and proportional to the current, to the length of the segment, and to a trigonometric factor expressing the apparent shortening of the segment due to its angle as "seen" from the observation point (Figures 4.86, 4.87).

Biot investigated the dependence of Earth's magnetic field on altitude. To this end, he ascended in a balloon with Gay-Lussac, reaching an altitude of more than 6000 feet (2000 meters) after overcoming great difficulties (Figure 4.88). (We note in passing that Gay-Lussac made a lone ascent in a balloon up to an altitude of 23,000 feet.) Through their measurements, they determined that the strength of Earth's magnetic field remains essentially unchanged up to the altitudes that they reached.

4.4.7 *The Interaction of Currents: An Extension of Newton's Ideas*

Also in the year 1820, André Marie Ampère (Figure 4.89) investigated the interaction of currents experimentally and proposed a mathematical theory of this interaction (Figure 4.90). Maxwell considered Ampère's work in this area to be among the most significant contributions ever in the history of science and called its author the "Newton of electrodynamics." He viewed Ampère's work as exemplary in the rigor of its logic, although it was clear to him that Ampère did not arrive at his results in the way suggested by his publications (Quotation 4.30).

On the basis of experimental observations and the application of Newton's natural philosophy, Ampère sought to derive a law for determining the strength of the interaction between two current elements. He based his derivation on the following experimental observations (Figures 4.91, 4.92):

1. The force is proportional to the product of the two currents.
2. The force is unchanged if, in the case of constant current intensities, we multiply all lengths—that is, the distances of the current elements from one another and the lengths of the current elements themselves—by a constant factor.
3. The net force exerted by a circuit on a given current element of another circuit is always perpendicular to that current element.
4. The forces between two current elements satisfy Newton's third law: they are equal to each other and are directed along the line joining them.

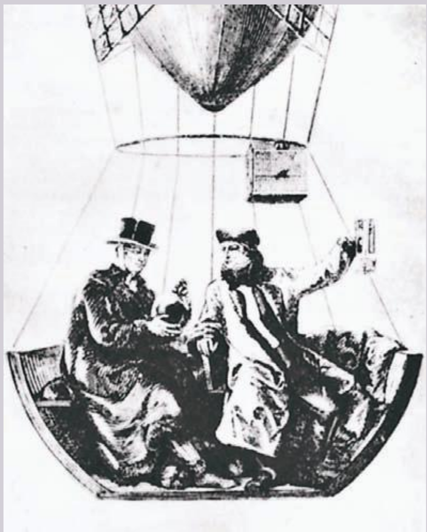
Beginning with these axioms, Ampère arrived at the following law of forces:

$$d^2 F_A = \frac{ii' \, ds \, ds'}{r^2}\left(\cos\varepsilon - \frac{3}{2}\cos\theta\cos\theta'\right).$$

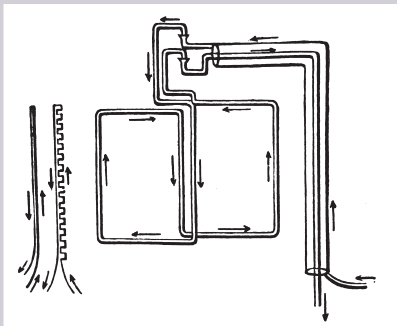
The meaning of the symbols in the formula is clarified in Figures 4.91 and 4.92. Using today's vector notation, this relationship can be expressed as

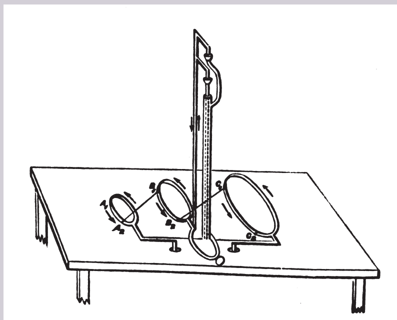
$$d^2 \mathbf{F}_A = ii'\mathbf{r}\left(\frac{d\mathbf{s} \, d\mathbf{s}'}{r^3} - \frac{3}{2}\frac{(d\mathbf{s}\cdot\mathbf{r})(d\mathbf{s}'\cdot\mathbf{r})}{r^5}\right). \qquad (1)$$

▲ **Figure 4.89** André Marie Ampère (1775–1836): Taught physics in Bourg and Lyon, then professor at the École Polytechnique; in 1820, he derived the law describing the electrodynamic interaction of currents. Today, the influence of substances on a magnetic field can still be best understood with the model of Ampère's molecular current.



▲ **Figure 4.88** Biot and Gay-Lussac ascending in a balloon to measure the magnetic field of Earth [Millikan 1965].





▲ **Figure 4.90** Ampère's experiment as presented by Maxwell. The illustrations in Ampère's original publications are filled with so many details that they tend to obscure the essential ideas.

In today's physics texts, one finds a different relationship for the interaction of two current elements, because in the form given above, the force law cannot easily be brought into agreement with the Biot–Savart law for the magnetic effect of a current element. The reason is that today we do not require that the principle *action* = *reaction* be satisfied for every current element. However, because Ampère's formula does a good job of describing all measurements relating to a closed circuit, the discrepancy that we mentioned indicates only that the decomposition of the resultant force into a sum of forces that can be associated with the individual current elements is not unique. In fact, the relationship for the resultant force does not change if on the right-hand side of Ampère's formula (1) a term is added that disappears upon integration over a closed circuit and therefore has no effect on the result. A simple expression of this sort is

$$(\mathrm{d}\mathbf{s}\cdot\mathbf{r})\,\mathrm{d}\mathbf{s}'.$$

This expression may be summed, for fixed $\mathrm{d}\mathbf{s}'$, over all $\mathrm{d}\mathbf{s}$ (that is, over a closed circuit S with elements $\mathrm{d}\mathbf{s}$). We see, then, from Figure 4.92 that $\mathrm{d}\mathbf{s} = -\mathrm{d}\mathbf{r}$, and consequently,

$$\mathrm{d}\mathbf{s}\cdot\mathbf{r} = -(\mathbf{r}\cdot\mathrm{d}\mathbf{r}) = -\frac{1}{2}\mathrm{d}\mathbf{r}^2.$$

Quotation 4.30

The experimental investigation by which Ampère established the laws of the mechanical action between electric currents is one of the most brilliant achievements in science.

The whole, theory and experiment, seems as if it had leaped, full grown and full armed, from the brain of the "Newton of electricity." It is perfect in form, and unassailable in accuracy, and it is summed up in a formula from which all the phenomena may be deduced, and which must always remain the cardinal formula of electrodynamics.

The method of Ampère, however, though cast into an inductive form, does not allow us to trace the formation of the ideas which guided it. We can scarcely believe that Ampère really discovered the law of action by means of the experiments which he describes. We are led to suspect, what, indeed, he tells us himself, that he discovered the law by some process which he has not shewn us, and that when he had afterwards built up a perfect demonstration, he removed all traces of the scaffolding by which he had raised it.

Faraday, on the other hand, shews us his unsuccessful as well as his successful experiments, and his crude ideas as well as his developed ones, and the reader, however inferior to him in inductive power, feels sympathy even more than admiration, and is tempted to believe that, if he had the opportunity, he too would be a discoverer. Every student should therefore read Ampère's research as a splendid example of scientific style in the statement of a discovery, but he should also study Faraday for the cultivation of a scientific spirit, by means of the action and reaction which will take place between the newly discovered facts as introduced to him by Faraday and the nascent ideas in his own mind.

—James Clerk Maxwell, *A Treatise on Electricity and Magnetism* [pp. 175–176]

We have thus to integrate a total differential over a closed curve, and therefore, as is well known, the integral vanishes.

Of course, we may also investigate total differentials of a more complicated form. For example, if we add the total differential

$$\mathrm{d}\left[\mathbf{r}(\mathrm{d}\mathbf{s}'\cdot\mathbf{r})\frac{ii'}{r^3}\right], \quad -\mathrm{d}\mathbf{r}=\mathrm{d}\mathbf{s},$$

to the force $\mathrm{d}^2\mathbf{F}_A$, we obtain

$$\mathrm{d}^2\mathbf{F}=\mathrm{d}^2\mathbf{F}_A+\mathrm{d}\left[\mathbf{r}(\mathrm{d}\mathbf{s}'\cdot\mathbf{r})\frac{ii'}{r^3}\right]=\frac{ii'}{r^3}\left[(\mathrm{d}\mathbf{s}\cdot\mathbf{r})\mathrm{d}\mathbf{s}'+(\mathrm{d}\mathbf{s}'\cdot\mathbf{r})\mathrm{d}\mathbf{s}-(\mathrm{d}\mathbf{s}\cdot\mathrm{d}\mathbf{s}')\mathbf{r}\right].$$

This expression is completely symmetric with respect to the current elements, so both forces are equal. However, they are no longer parallel to the line joining the current elements. If we integrate $\mathrm{d}^2\mathbf{F}$ over the circuit S and take into account the vectorial relationship

$$(\mathrm{d}\mathbf{s}'\cdot\mathbf{r})\mathrm{d}\mathbf{s}-(\mathrm{d}\mathbf{s}\cdot\mathrm{d}\mathbf{s}')\mathbf{r}=\mathrm{d}\mathbf{s}'\times(\mathrm{d}\mathbf{s}\times\mathbf{r})$$

as well as the fact that the integral of the quantity $(\mathrm{d}\mathbf{s}\cdot\mathbf{r})$ over a closed path is zero, we then obtain

$$\mathrm{d}\mathbf{F}=ii'\oint\left[\mathrm{d}\mathbf{s}'\times\frac{\mathrm{d}\mathbf{s}\times\mathbf{r}}{r^3}\right]=i'\mathrm{d}\mathbf{s}'\times\oint_S i\frac{\mathrm{d}\mathbf{s}\times\mathbf{r}}{r^3}=i'\mathrm{d}\mathbf{s}'\times\mathbf{B}$$

for the force exerted by the circuit S on the current element $i'\mathrm{d}\mathbf{s}'$. This completely corresponds to the notion that the circuit S produces a magnetic field $\mathbf{B}$ at the location of the current element $i'\mathrm{d}\mathbf{s}'$ according to the Biot-Savart law:

$$\mathbf{B}=\oint_S i\frac{\mathrm{d}\mathbf{s}\times\mathbf{r}}{r^3}$$

and that this field, in turn, exerts the following force to the current element $i'\mathrm{d}\mathbf{s}'$ placed there:

$$\mathrm{d}\mathbf{F}=i'\mathrm{d}\mathbf{s}'\times\mathbf{B}.$$

Looking at it in this manner, it is no longer the case that the principle *action = reaction* is satisfied for the force between two current elements.

Ampère also gave a microphysical interpretation of the magnetic properties of matter—one that is still a useful model for understanding such properties today.

Ampère noticed and then proved theoretically that a current loop and a very flat magnet (magnetic double layer) generate exactly the same fields in the surrounding space (Figure 4.93). On the basis of this observation, he then associated small currents—the Ampère molecular circular currents—with the volume element of a larger body; the fields produced by these currents will then combine in the surrounding space. The total outward effect will depend on the orientation of these circuits with respect to one another and with respect to the external magnetic field (Figure 4.94).

Ampère's law of interaction was revised 20 years later by Wilhelm Eduard Weber (1804–1891). Although this revision led to a dead end, it is still worth mentioning here because in it we can see an approach to the later classical theory of electrons. Furthermore, the limits placed on a precise description of electromagnetic phenomena in the framework of Newton's theory of action at a distance become clear. Weber hypothesized that in an electrical current within a conductor, equal quantities of charge flow in each direction at the same velocity. According to Weber's theory, one therefore has

$$i=2\lambda u, \quad i'=2\lambda' u',$$

where λ is the charge per unit length and u the charge velocity. If we now consider Ampère's law, equation (1), together with the relations

$$\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{\mathrm{d}r}{\mathrm{d}s}\frac{\mathrm{d}s}{\mathrm{d}t}=\frac{\mathrm{d}r}{\mathrm{d}s}u, \quad \mathrm{d}\mathbf{s}\cdot\mathbf{r}=-r\frac{\mathrm{d}r}{\mathrm{d}s}\mathrm{d}s,$$

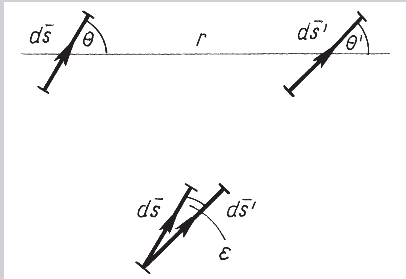
then after some rearrangement, we may express the force between the moving charges as

$$F = \frac{Q_1 Q_2}{r^2}\left[1 - \frac{1}{c^2}\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 + \frac{2r}{c^2}\frac{\mathrm{d}^2 r}{\mathrm{d}t^2}\right]. \tag{2}$$

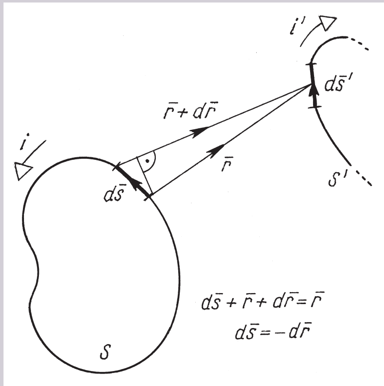
The first term in this formula clearly describes the electrostatic force, while the other two terms describe electrodynamic forces. There is no need to criticize this formula in detail; that was already done by WEBER's contemporaries. They noted, for example, that in a very simple arrangement, one could show that a moving charge would have to exert a magnetic field even on a stationary charge. Nevertheless, it is worth pointing out a conclusion that is not so obvious: WEBER's theory relates magnetic interactions exclusively to the motion of the electrical charge, with the charge velocity and acceleration appearing as characteristic quantities in the formula. (In this respect, the theory agrees with the later theory of the electron.) However, if we recall the analogy between gravitational and electrostatic forces, which was the model for the derivation of Coulomb's law, then perhaps we should not be surprised that the analogy may now be considered in the opposite direction (Figure 4.95). Thus, astronomers attempted to take the velocities of the heavenly bodies into account in their calculations of the gravitational force, and they obtained, on the basis of WEBER's formula (2), a relationship of the form

$$F \sim \frac{m_1 m_2}{r^2}\left[1 - \frac{1}{h^2}\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 + \frac{2r}{h^2}\frac{\mathrm{d}^2 r}{\mathrm{d}t^2}\right],$$

where h denotes the speed of propagation of the gravitational effect (TISSERAND, 1872). We meet here for the first time the perihelion motion of Mercury as an effect that would later become the touchstone for the validity of new theories of gravitation. Even if one takes into account the perturbing actions of the other planets on Mercury's orbit, when using the Newtonian theory of gravitation, the degree of agreement between the observed and calculated values of the advance of the perihelion is less than satisfactory. A remainder of 38^2 cannot be accounted for. The above formula provides an additional contribution of 14^2 if the speed of propagation of gravity is set at the velocity of light. It was RIEMANN who later succeeded in finding a force law that could explain the complete 38^2. The perihelion motion of Mercury will appear again in this book as an experimental proof of the correctness of Einstein's general theory of relativity (Section 5.2).

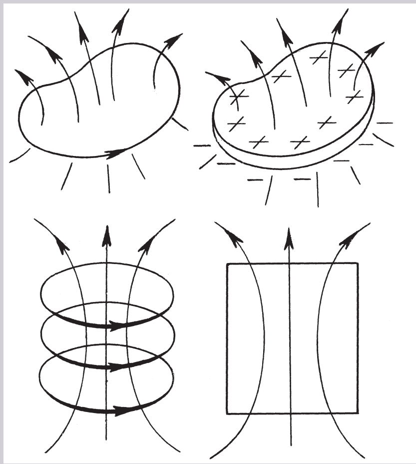


▲ **Figure 4.91** Definitions of the notation used in Ampère's force law.



▲ **Figure 4.92** If we fix the path element d**s'**, then for the differential path element d**s** on the circuit S, we have d**s** = –d**r**, so that –(**r** · d**r**) can be rewritten as follows:

$$-(\mathbf{r}\cdot\mathrm{d}\mathbf{r}) = -\mathbf{r}\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}s}\mathrm{d}\mathbf{s}.$$



▲ **Figure 4.93** AMPÈRE realized that the magnetic field of a current is equal to the field of a magnetic double layer. It follows that the magnetic field of a coil with current flowing through it and the magnetic field of a bar magnet (**B**-field) have the same form.

4.4.8 *Faraday: The Greatest of the Experimentalists*

Let us now return to the problems in electrodynamics that were being investigated during the first years of the 1820s. At this time, FARADAY (Figures 4.96, 4.98), who is often cited as the most important experimental physicist ever, began his investigations of electromagnetic phenomena. In 1821, he was asked to prepare a paper summarizing what was known in the field up to then and, like PRIESTLEY before him, he reproduced all the earlier experiments he was reporting on. In this report, FARADAY already superseded his predecessors; for example (cf. Figure 4.97), he proved that a current flowing through a conductor exerts a force on a pole of a bar magnet and showed that this force acts along a circle, incidentally giving the principle of the first electric motor.

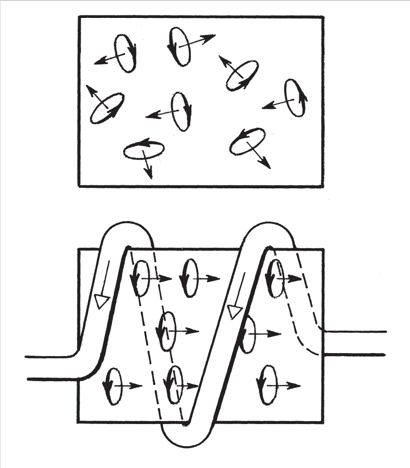
Figure 4.98 collects all the areas in which FARADAY made significant contributions over the course of his life. Here we review briefly the most important concrete results:

1. The induction theorem (now named after FARADAY) is the first of these. Influenced by Romantic natural philosophy, a number of researchers expressed the opinion that, analogously to the phenomenon of the influence of electrostatics on the current in a circuit, there should be some sort of influence of the current in one circuit on the current in a second circuit (Figure 4.99). Based on this analogy, an effect, for example, should appear whereby a current would generate a current in a neighboring conductor simply because such a current is flowing in the first circuit. Even AMPÈRE conjectured something of this kind, although in a letter in 1822, he mentioned that such an effect does not exist and there is merely something to be observed when the current is switched on or off. However, AMPÈRE did not investigate this

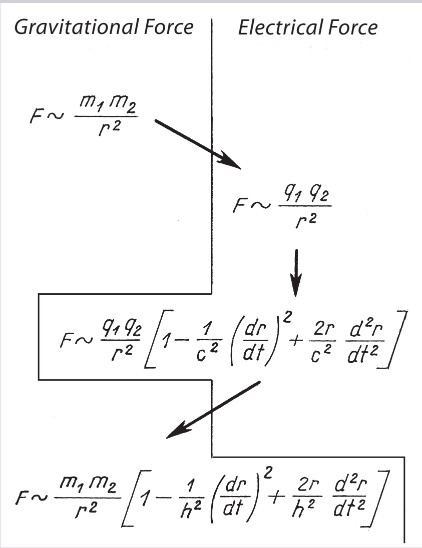
latter effect, thereby missing out on—as he himself later bitterly remarked—a very significant discovery. Even in the lists of experiments that FARADAY kept, we find over the course of many years the entry "no effect." Eventually, FARADAY noticed the same phenomenon that AMPÈRE had seen but ignored: To obtain an effect, something in the circuit had to change or—to put it more generally—the magnetic state of the entire system had to change. FARADAY also found that the current arising in the second circuit is inversely proportional to the total resistance of this circuit, in other words, that the voltage created by a change in the magnetic state is independent of the type of conductor, for only then can the current be exactly proportional to the reciprocal of the resistance of the circuit. FARADAY provided a quantitative formulation of this law using the simple and expressive expedient of the lines of force (Figure 4.100):

In a resting circuit loop, the induced voltage is proportional to the change in the number of lines of force per unit time ($U = -\mathrm{d}\Phi/\mathrm{d}t$).

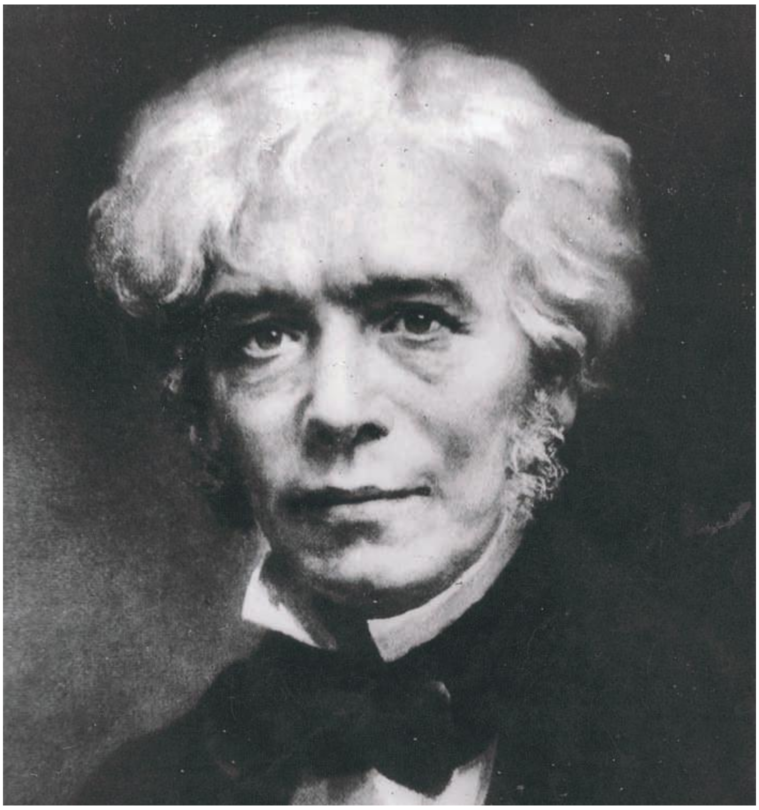
For a moving conductor, the induced voltage is proportional to the number of force lines intersected per unit time [$U = \mathbf{I}(\mathbf{v} \times \mathbf{B})$].



▲ **Figure 4.94** In a magnetic material, the AMPÈRE circular currents align under the influence of an external magnetic field.



▲ **Figure 4.95** Coulomb's law was formulated in analogy to Newton's law of gravitation, and Weber's law served as the starting point for a new law of gravitation that was supposed to be more precise, but turned out to be incorrect.



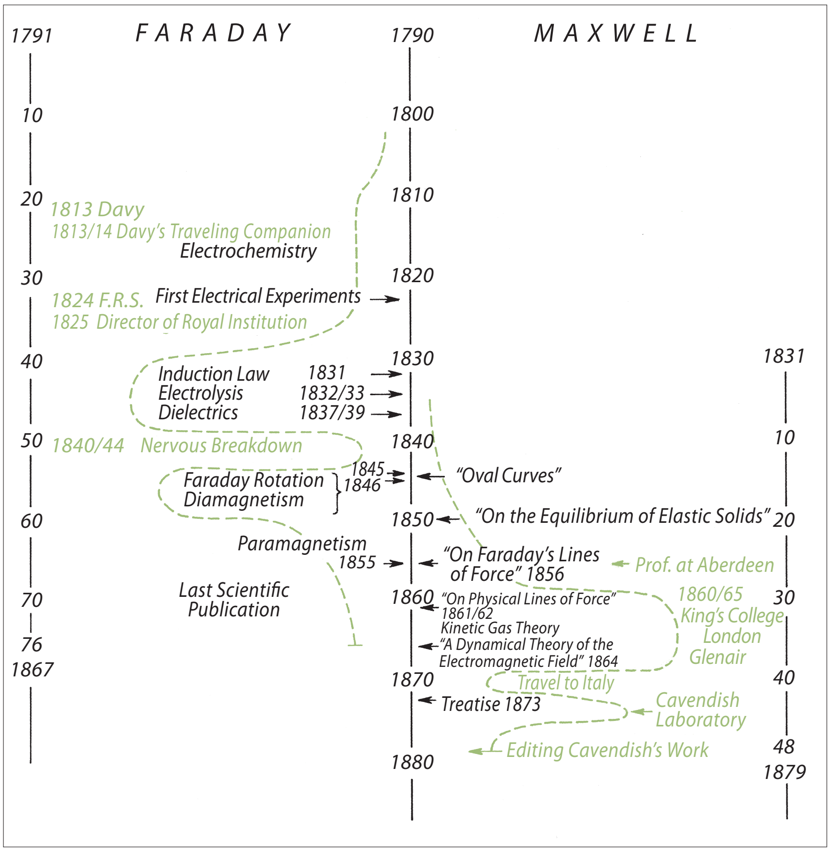
▲ **Figure 4.96** MICHAEL FARADAY (1791–1867): Apprenticed to a bookbinder, then laboratory assistant, and finally secretary to DAVY. 1824, member of the Royal Society; from 1825, director of the Royal Institution. His first works are devoted to problems in chemistry (1823: *On Fluid Chlorine*). He also worked on technological problems: production of stainless steel and glass with particular optical properties. After 1820, he began to study electricity. 1821: construction of the "rotation apparatus" (Figure 4.97); August 29, 1831: law of induction; 1833: laws of electrolysis; 1845: Faraday effect. In the years 1832–1856, his work was published as *Experimental Researches in Electricity* in successive paragraphs numbered 1 to 3340.

FARADAY is seen as the most important experimental physicist of all time, with whom, at most, RUTHERFORD can be compared.

Today, it is not really necessary to mention the practical significance of the induction law: It serves as the theoretical basis for the operation and the engineering design of all electrical generators and transformers.

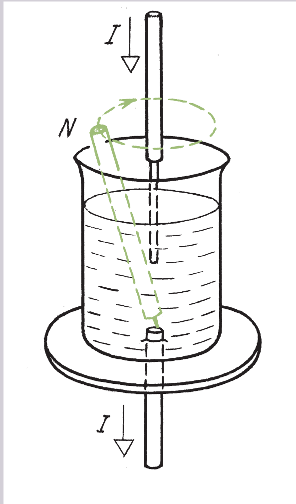
2. The laws of electrolysis derived by Faraday in the next two years are also of great significance. These are now known as Faraday's laws.

The first law states that the quantity of separated substances produced during electrolysis is proportional to the charge flow. By the second law, well-defined chemically meaningful values are assigned to the proportionality

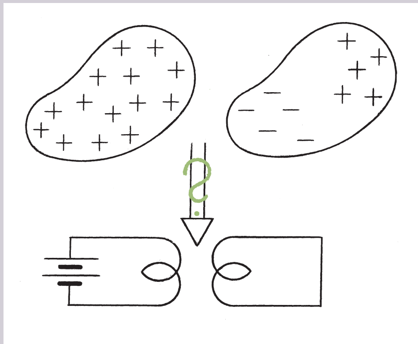


▲ **Figure 4.98** Life and work of the two greatest figures in the field of electrodynamics who also maintained scientific contact with each other and greatly respected each other.

> … before I began the study of electricity I resolved to read no mathematics on the subject till I had first read through Faraday's *Experimental Researches on Electricity.* …
>
> As I proceeded with the study of Faraday, I perceived that his method of conceiving the phenomena was also a mathematical one, though not exhibited in the conventional form of mathematical symbols. I also found that these methods were capable of being expressed in the ordinary mathematical forms, and thus compared with those of the professed mathematicians. …
>
> It is of great advantage to the student of any subject to read the original memoirs on that subject, for science is always most completely assimilated when it is in the nascent state, and in the case of Faraday's *Researches* this is comparatively easy, as they are published in a separate form, and may be read consecutively. If by anything I have here written I may assist any student in understanding Faraday's modes of thought and expression, I shall regard it as the accomplishment of one of my principal aims—to communicate to others the same delight which I have found myself in reading Faraday's *Researches*.
>
> —James Clerk Maxwell, Preface to *A Treatise on Electricity and Magnetism* [Vol. I, pp. viii, ix, xi]



◀ **Figure 4.97** With this setup, Faraday clearly demonstrated the circular magnetic lines of force and gave us the first electric motor as well.



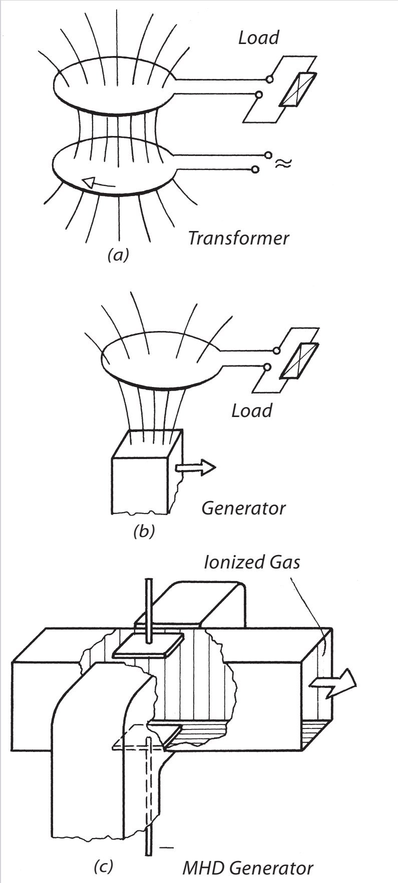
▲ **Figure 4.99** An electrically charged conductor causes the charges to separate in a nearby conductor (electrostatic induction). Could there be a similar phenomenon between two circuits?

factor that appears in the first law; to wit, it is stated that with equal quantities of charge flow, the quantities of separated material are directly proportional to the element's equivalent weight (Figure 4.101). In addition to its practical ramifications, this law has tremendous theoretical significance, because it makes it possible to descend into the world of atoms and demonstrate relationships between microphysical quantities: $M = (1/9.65 \times 10^7) \times (A/z)It$, giving the mass produced in terms of the current I, the time t, the atomic weight A, and the valence z.

3. Faraday also did important work on dielectrics; indeed, the very word "dielectric" came from Faraday. He introduced the notion of dielectric constant into physics and specified the methods for measuring it (Figure 4.102).
4. Faraday conjectured that a magnetic field should have an effect beyond that which it has on so-called magnetic materials. He thought that all matter without exception must exhibit some magnetic properties. He categorized materials as paramagnetic and diamagnetic, and then made a thorough investigation of diamagnetic materials.
5. The assumption that there must be some interrelation among various physical phenomena led Faraday to the discovery of yet another effect, having both practical and theoretical importance, which also bears his name. Faraday was searching for an interaction between magnetic fields and light, and following a number of fruitless experiments, he finally discovered that the plane of oscillation of linearly polarized light is rotated in certain media if they are located within a magnetic field (Figure 4.103). This rotation of the polarization plane is called the Faraday rotation.
6. We note as an aside that in his later years, Faraday searched without success for an interaction between gravitation and electromagnetism (Quotation 4.31). Even to this day, there has still been no successful demonstration of such a relationship.

We have yet to mention another extremely important contribution by Faraday. We will refrain from speaking about it in superlatives only because we would then have to do so for all of Faraday's work. This contribution has to do with the introduction of a completely new way of looking at electromagnetic phenomena. Moreover, here we encounter a strange phenomenon in the history of science, which calls for a brief explanation. The Cartesian vortex theory was abandoned in France toward the end of the seventeenth century and the beginning of the eighteenth century in favor of Newton's theory of action at a distance. The generation of mathematicians and theoretical physicists active at the end of the eighteenth century—whose leading representatives were Lagrange, Laplace, Poisson, and Ampère—had made Newtonian principles so much a part of their own thinking that they dealt with the new phenomenon of electromagnetism in terms of these principles. We have seen a concrete example of this in the case of Ampère, who in his derivation of the fundamental law of electrodynamics began with the Newtonian worldview. Because of the understandably great influence of the French mathematicians, no one dared to take a different approach. Faraday, however, was for the most part self-taught and never received a formal education. Maxwell, who was an admirer of Faraday and who brought Faraday's discoveries into a mathematical form to develop them further, continues the story thus in *A Treatise on Electricity and Magnetism*:

> For instance, Faraday, in his mind's eye, saw lines of force traversing all space where the mathematicians saw centers of force attracting at a distance; Faraday saw a medium where they saw nothing but distance; Faraday saw the



▲ **Figure 4.100 (a)** Faraday's law of induction: If the number of field lines surrounded by a circuit changes, then a voltage is induced in it. The emphasis here is on change, on movement, on the dynamic aspects.

> Have had an iron ring made (soft iron), iron round and 7/8 inches thick and ring 6 inches in external diameter. Wound many coils of copper wire round one half, the coils being separated by twine and calico – there were three lengths of wire each about 24 feet long and they could be connected as one length or used as separate lengths. By trial with a trough each was insulated from the other. Will call this side of the ring *A*. On the other side but separated by an interval was wound wire in two pieces together amounting to about 60 feet in length, the direction being as with the former coils; this side call *B*.
>
> Charged a battery of 10 pr. plates 4 inches square. Made the coil on *B* side one coil and connected its extremities by a copper wire passing to a distance and just over a magnetic needle (3 feet from iron ring). Then connected the ends of one of the pieces on *A* side with battery; immediately a sensible effect on needle. It oscillated and settled as last in original position. On *breaking* connection of *A* side with battery again a disturbance of the needle.

continued on next page

seat of the phenomena in real actions going on in the medium, they were satisfied that they had found it in a power of action at a distance impressed on the electric fluids. ...

It was perhaps for the advantage of science that Faraday, though thoroughly conscious of the fundamental forms of space, time, and force, was not a professed mathematician. He was not tempted to enter into the many interesting researches in pure mathematics which his discoveries would have suggested if they had been exhibited in a mathematical form, and he did not feel called upon either to force his results into a shape acceptable to the mathematical taste of the time, or to express them in a form which mathematicians might attack. He was thus left at leisure to do his proper work, to coordinate his ideas with his facts, and to express them in natural, untechnical language. [Vol. I, p. ix; Vol. II, p. 176]

As already mentioned, Faraday was influenced by Romantic natural philosophy. He was also influenced by Boscovich, who, as we have seen in Section 4.2, viewed the particles of matter as centers of force with an effect that spreads out over all of space and assumed that this effect was somehow inherent in all matter. Faraday, who was educated in the sober outlook of the empirical tradition, now sought to make these qualitative ideas more realistic with the help of concrete experiments and then attempted to use the ideas made more realistic in this way to interpret the phenomena. He made tangible the imagined lines of force with the help of iron filings. He produced images (Figure 4.104) like those that can be found today in any introductory physics textbook. As we have seen with Ampère, as well as with Weber, who developed Ampère's theory further, the theory of action at a distance is capable of explaining the interaction of circuits without having to postulate a medium in the space between them. Faraday, in contrast, assumed that in the regions surrounding a conductor with current flowing through it an electromagnetic field emerges with its lines of force. Thus, at the place of the second conductor, completely independently of whether we placed a conductor there or if there is current flowing in that conductor, that is in *space* itself, even in a vacuum, there will be a particular state distinguished from the state that would have been there in the absence of the current-carrying conductor in the vicinity. The first circuit therefore does not act directly on the second circuit, but rather by way of this "state of readiness." What is the historical significance of such a separation of the interaction process into two steps? Can this particular state of space be viewed as something that actually exists? One can show mathematically that as long as the changes are not too rapid, both formulations lead to the same results. However, if we concede the experimental fact that the effect of one circuit needs some time to reach the other, then the hypothesis of a medium of transmission is necessary, and its function is obvious. When Faraday was working out his idea of the electromagnetic field with the aid of electric and magnetic lines of force, the existence of electromagnetic waves had not yet been either theoretically founded or experimentally proven. However, the picture that Faraday had worked out for himself suggested to him the idea that electromagnetic effects could be transmitted by the lines of force in the form of transversal oscillations.

But Faraday's view is more than just the starting point for the future field theory, for his analogies between electric current and the "current" of magnetic field lines have proven very useful in power engineering and pedagogically in the visualization of complex magnetic phenomena.

Figure 4.100 *continued*

Made all the wires on *A* side one coil and sent current from battery through the whole. Effect on needle much stronger than before.

—Michael Faraday, Diary entry, August 29, 1831 [Elliott 1966, p. 257]

(b) Emil Lenz (1804–1865): Member of the academy at St. Petersburg, in 1833 formulated Lenz's law, according to which the direction of the induced current is such that its effect will hinder the cause that created it, so when a conductor is moved in a magnetic field, the induced current will flow so that the force exerted on it by the magnetic field will oppose its movement.

Joseph Henry (1797–1878): American physicist who came close to discovering the induction law. In 1828 he produced a very strong electromagnet by winding isolated wire into a multilayered coil. In 1832 he observed self-induction, and in 1842 determined experimentally that oscillations can result from the discharge of a capacitor.

Although in Faraday's experiments the dynamo principle (in the form of a copper disk rotating between the poles of a strong permanent magnet) and the transformer principle (in two coils wound around the same iron core) already played a role, it was a long way to a practical implementation of these ideas. Taking part in the development of the electric generator were Pixii (primitive alternating current generator, 1832), Clarke (rectification using a commutator, 1836), Pacinotti (1860), Gramme (ring-shaped armature, 1868), Ányos Jedlik (self-exciting dynamo, 1861), Siemens (recognition of the significance and practicality of the autoexcitation principle, 1866).

A number of researchers also took part in the perfection of the transformer; here we mention only Yablochkov (1876) and the triumvirate Déri–Bláthy–Zipernowsky (1885).

In the further development of generators, motors, transformers, and distribution systems, Nikola Tesla (1856–1943), who began his career in the Austro-Hungarian Empire (Budapest, 1880–1882), played an outstanding role: magnetic rotational field, multiphase system, asynchronous motor, Tesla coil.

(c) A modern magnetohydrodynamic (MHD) generator, transforming heat energy or kinetic energy directly into electricity, also employs the induction principle.