Erwin Schrödinger and the Genesis of Wave Mechanics

Christian Joas
Niels Bohr Archive

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14 March 2018

Erwin Schrödinger (1887-1961)

1887	born in Vienna
1898–1906	*Akademisches Gymnasium*
1906–1910	Vienna University
1910	PhD
1914	Habilitation
1914-1918	military service
1921	Zürich appointment
1926	**wave mechanics**
1927	Max Planck's successor in Berlin
1933-1936	Oxford
1936-1938	Vienna
1938-1955	Dublin
1955	return to Vienna
1961	death in Vienna, buried in Alpbach/Tyrol


iħψ̇=Hψ
Erwin Schrödinger
*12.VIII.1887 + 4.1.1961
Annemarie Schrödinger
*31.XII.1896 + 3 X.1965
R I P

Schrödinger's wave equation

"I have been reading your communication like a curious child who eagerly listens to the solution of a riddle it has struggled with for a long time. And I rejoice over the beauties that my eyes discover, which I must, however, study in much greater detail in order to grasp them in their entirety."

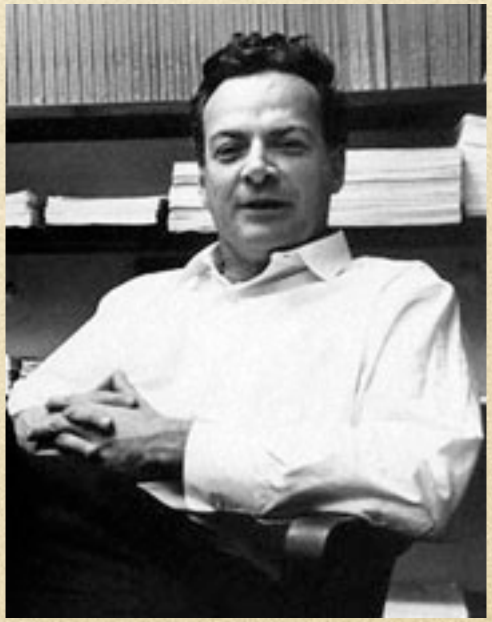
Max Planck to Schrödinger (1926)

"I just received a submission by Schrödinger to *Annalen [der Physik]*. Schr[ödinger] seems to find the exact same results as Heisenberg and you, but in a completely different, totally crazy way: no matrix algebra, but boundary-value problems."

Arnold Sommerfeld to Wolfgang Pauli (1926)

"Where did we get that from? Nowhere! It is not possible to derive from anything you know. It came out of the mind of Schrödinger, invented in his struggle to find an understanding of the experimental observation of the real world."

Richard P. Feynman (1965)



Part I:

Quantum theory and the crisis of the mechanical worldview

Quantum theory and the crisis of the mechanical worldview:

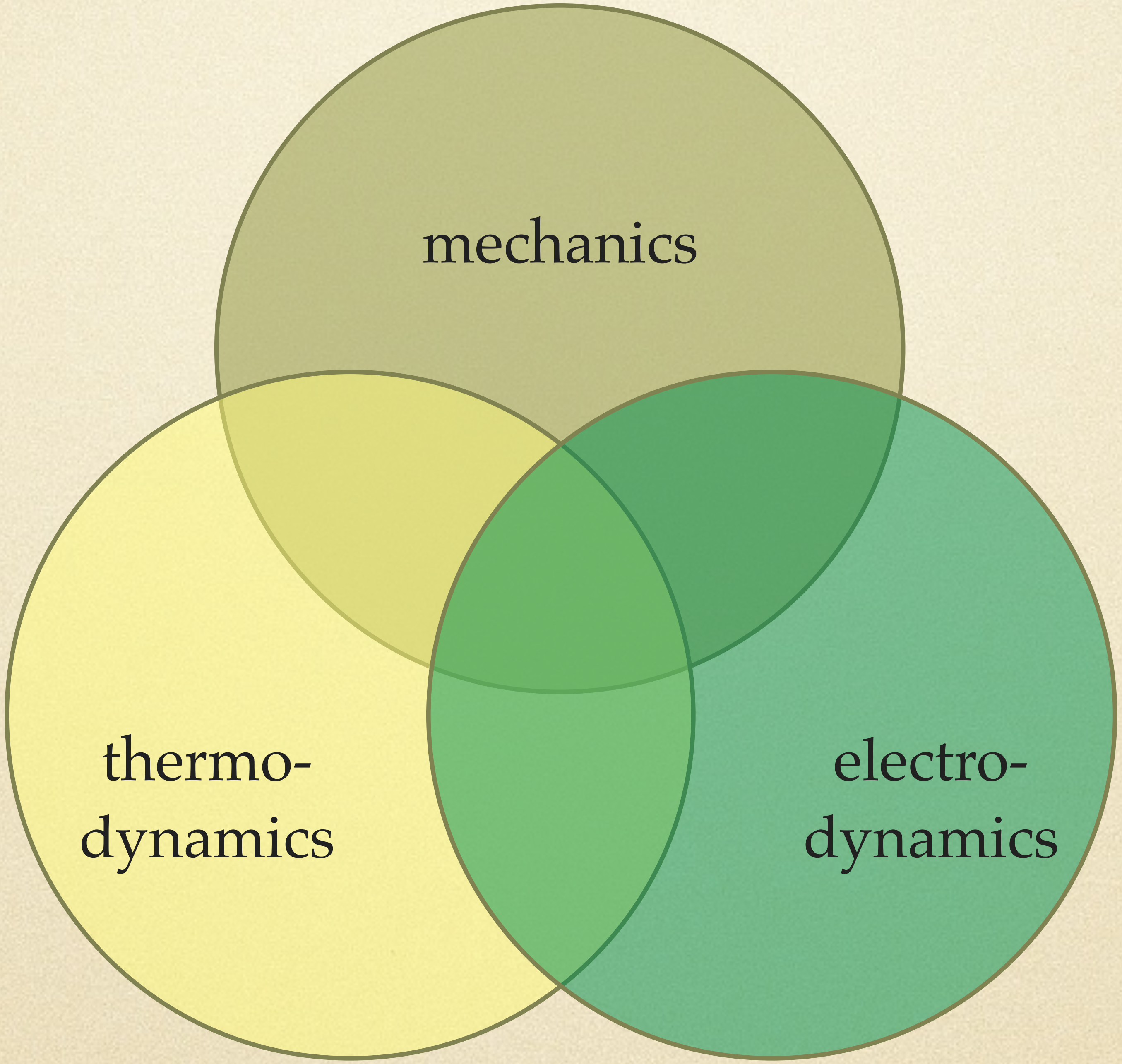
19th century physical theories

- **Mechanics**
- **Electrodynamics**
- **Thermodynamics**

Solvay 1911

Quantum theory and the crisis of the mechanical worldview:

Borderline Problems

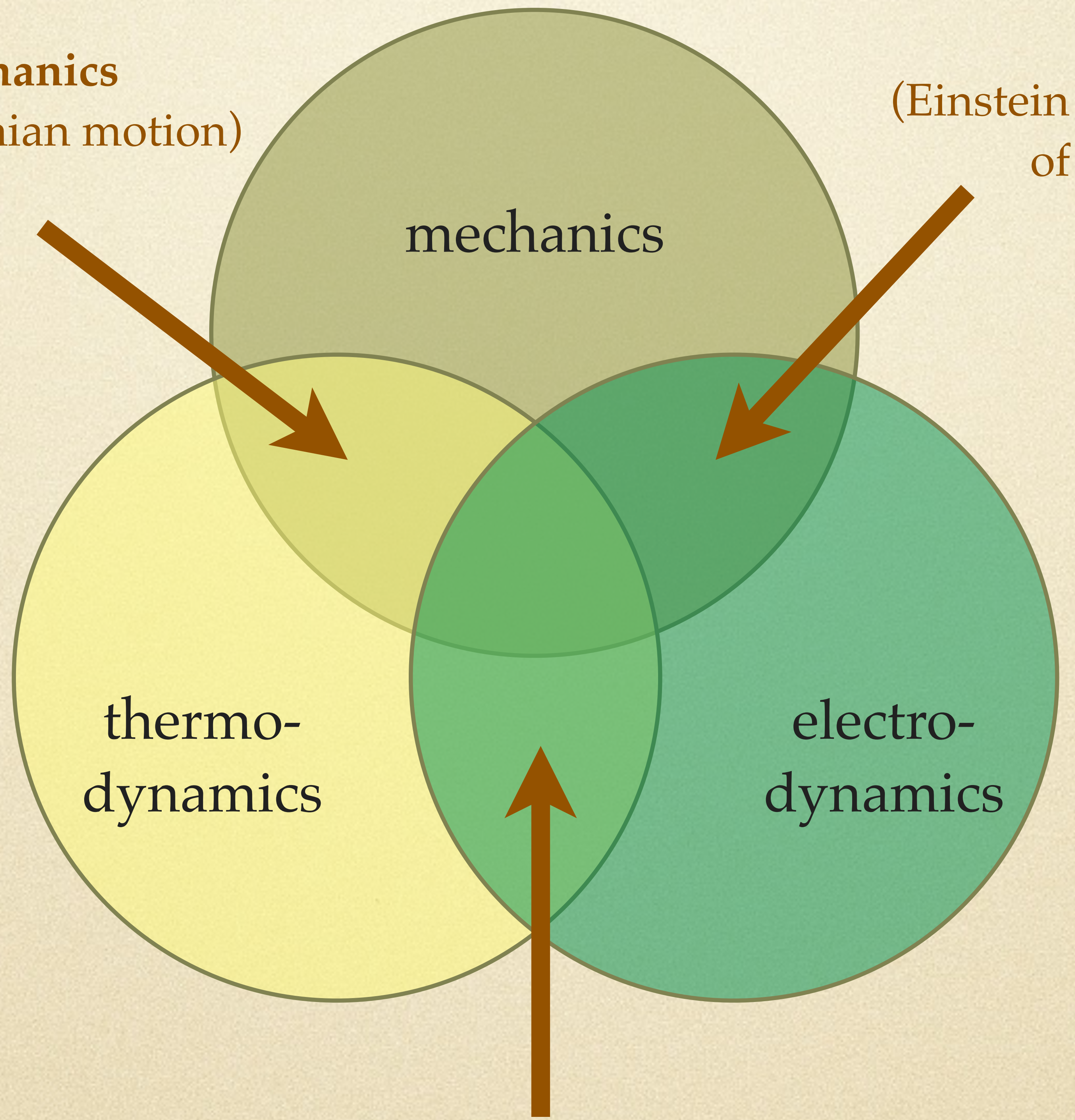


Quantum theory and the crisis of the mechanical worldview:

Borderline Problems

statistical mechanics
(Einstein 1905: Brownian motion)

relativity
(Einstein 1905: electrodynamics of moving bodies)



quantum theory
(Planck 1900: black-body radiation law)

Quantum theory and the crisis of the mechanical worldview:

The two-fold crisis of mechanics

Mechanics

Conflicts with new empirical evidence:

black-body radiation
atomic spectra
specific heats
X-ray absorption

Borderline problems with:

electrodynamics
thermodynamics

Quantum revolution

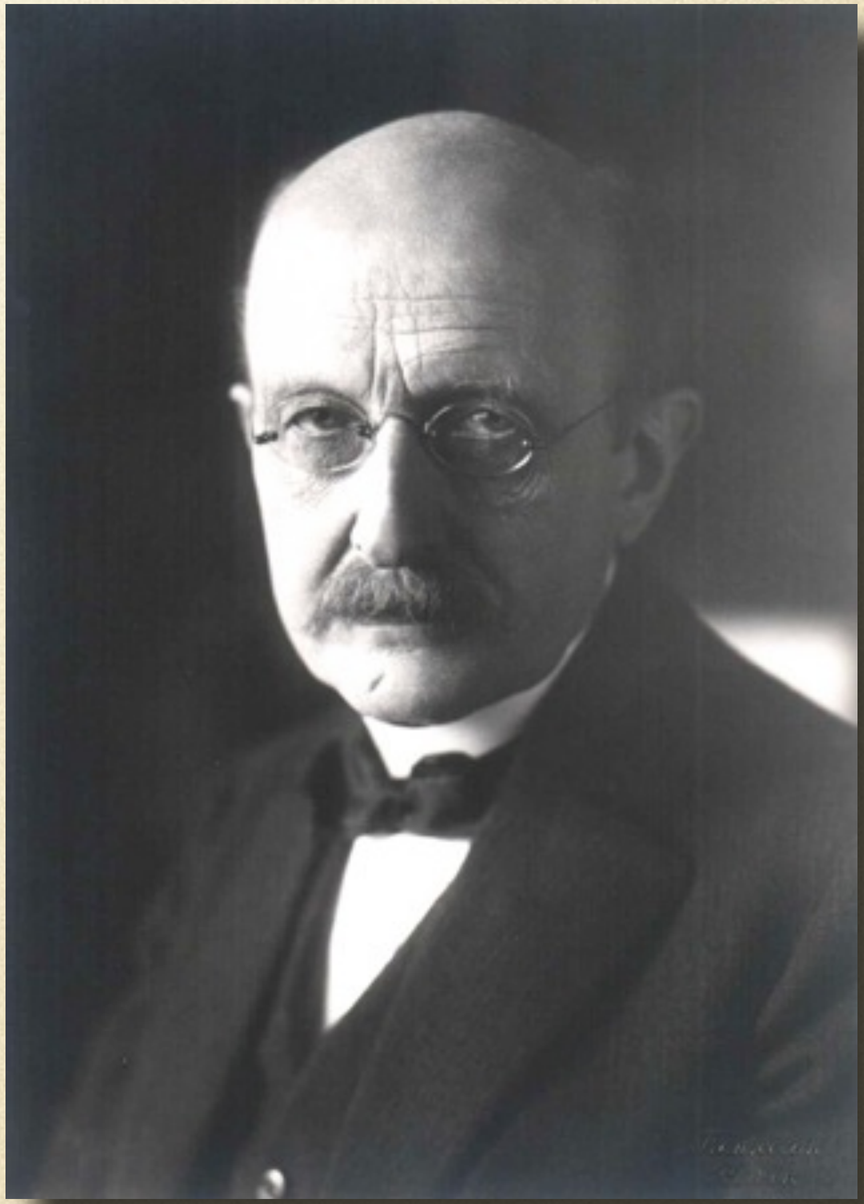
Quantum theory and the crisis of the mechanical worldview:

1900: Planck's quantum hypothesis

"act of desparation": Atoms absorb or emit energy only in elements (**quanta**) of finite size.

Leads to **black-body radiation law** describing recent empirical results:

$$\rho(\nu, T)d\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/kT}-1}d\nu$$



Max Planck (1858–1947)

Quantum theory and the crisis of the mechanical worldview:

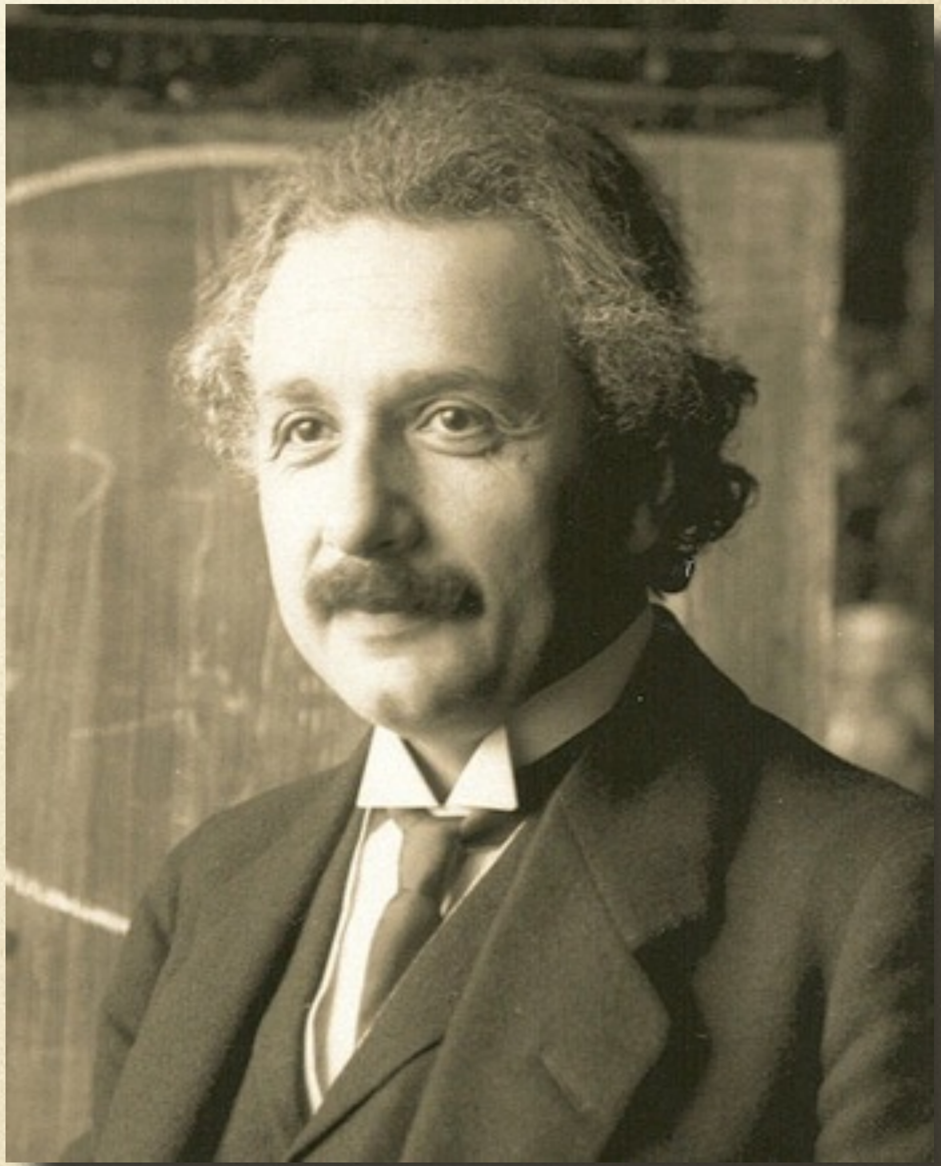
1905: Einstein's light quantum hypothesis

Light consists of **light quanta** whose energy is given by

$$E = h\nu$$

Highly controversial until the early 1920s.

Leads Einstein to the idea of **wave-particle dualism of light**.



Albert Einstein (1879–1955)

Quantum theory and the crisis of the mechanical worldview:

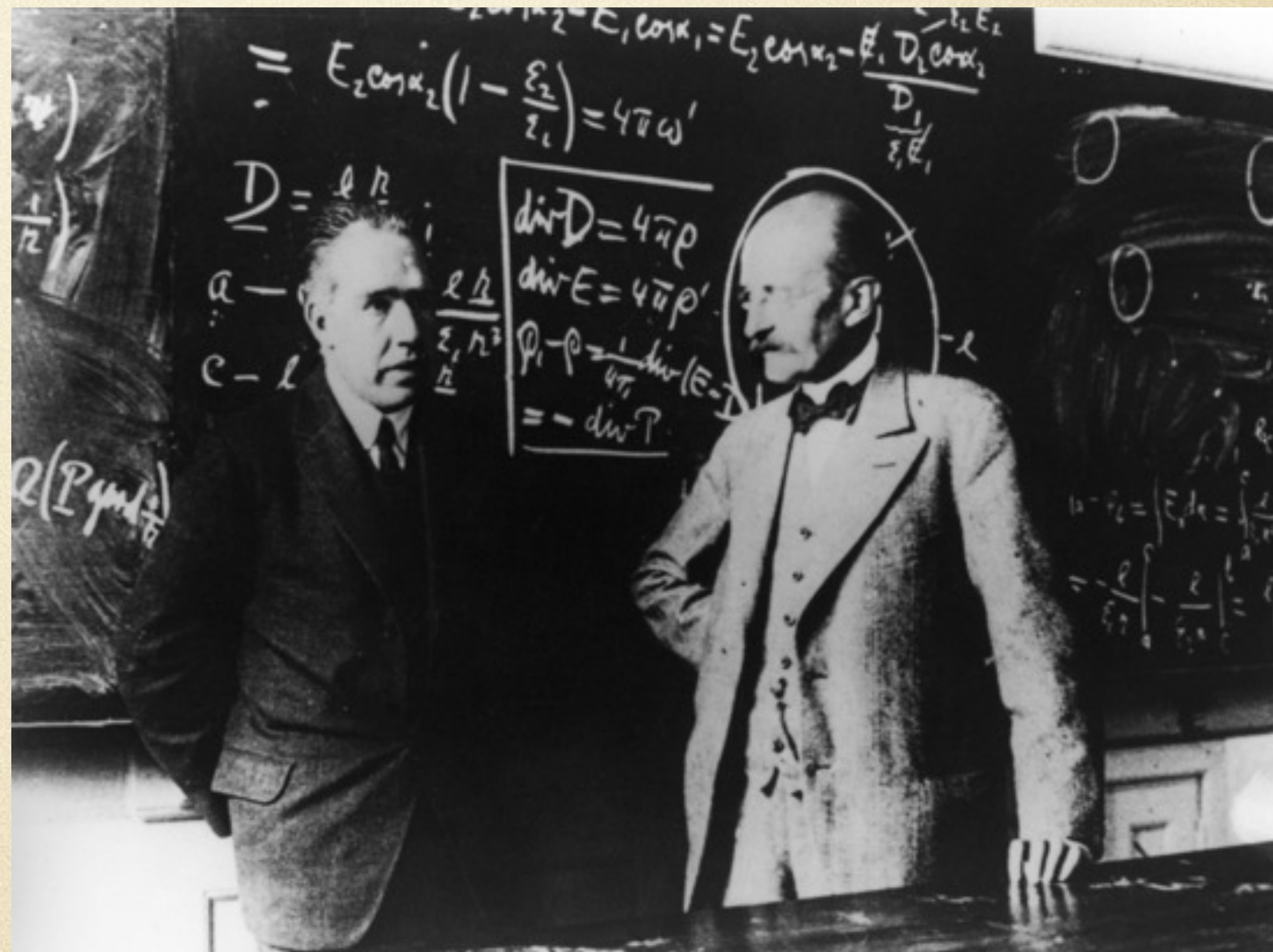
1913: Bohr’s model of the atom

Bohr’s planetary model explains hydrogen **spectrum**.

This leads to Bohr-Sommerfeld **quantum condition** and the so-called ”old” quantum theory.

$$\oint p dq = nh$$

Correspondence principle.



Niels Bohr (1885–1962) Max Planck (1858–1947)
(in Auditorium A at Copenhagen, 1930)

Quantum theory and the crisis of the mechanical worldview:

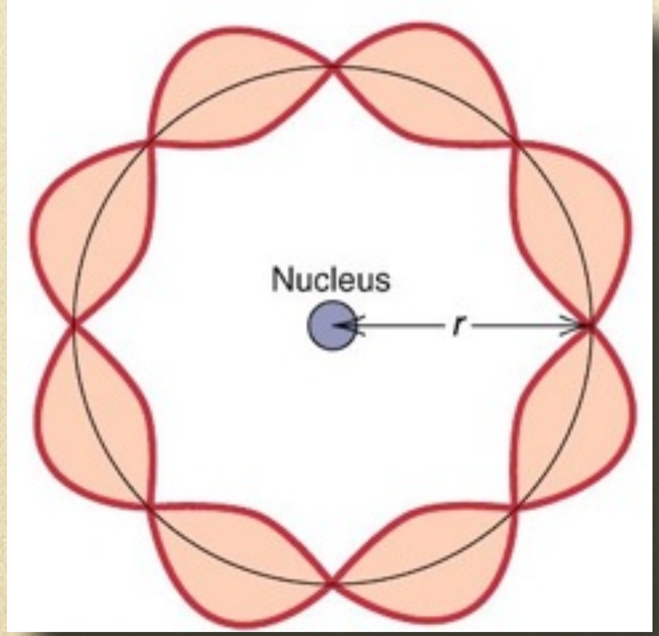
1924: De Broglie’s matter waves

Inspired by relativity, de Broglie postulates **wave-particle duality** for light **and** matter.

Matter waves: “a periodic phenomenon, of a nature that remains to be determined, which is associated with every piece of matter.“

Bohr’s quantum condition can be **explained** as the resonance condition for the “phase wave” along an electron orbit.

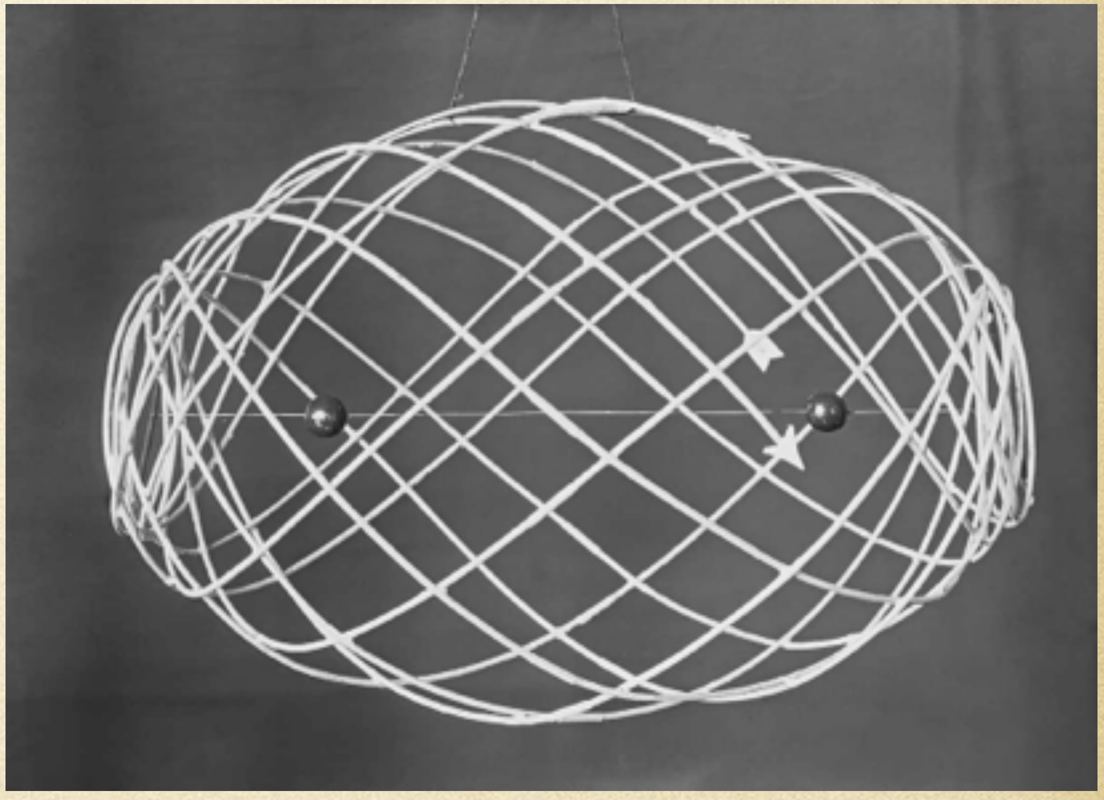
$$\lambda = \frac{h}{p}$$



Louis de Broglie (1892–1987)

Quantum theory and the crisis of the mechanical worldview:
„Old“ quantum theory (ca. 1913–1925)

- More **heuristic scheme** than full-fledged theory.
- Classical mechanics “patched up” by **auxiliary** quantum conditions.
- **Fails to explain** many phenomena: Helium spectrum, Zeeman effect, multiplet structure of atomic spectra, polarization of fluorescent light, degenerate systems, aperiodic phenomena (e.g., scattering).
- From ca. 1923 on, physicists search for **“sharpened” formulation** of Bohr’s correspondence principle.



Model of a hydrogen molecule ion according to the old quantum theory, built for Deutsches Museum, Munich, using calculations by Wolfgang Pauli, ca. 1923.

Quantum theory and the crisis of the mechanical worldview:

1925: Matrix mechanics

Heisenberg proposes a **new** mechanics: quantum mechanics.

"Umdeutung": classical quantities are **reinterpreted** in terms of observable quantities, i.e., transition frequencies and amplitudes.

This amounts to a **"sharpening"** of Bohr's correspondence principle.

Born and **Jordan** soon realize that Heisenberg's involved algebra is nothing but **matrix algebra** for infinite-dimensional matrices.

Copenhagen conference (1933).
Front row: Pauli, Jordan, Heisenberg, Born

Quantum theory and the crisis of the mechanical worldview:

1926: Wave Mechanics

Based on **de Broglie's ideas** about matter waves.

Behavior of matter governed by **wave equation**

$$\Delta\psi + \frac{2m}{K^2}\left(E + \frac{e^2}{r}\right)\psi = 0$$

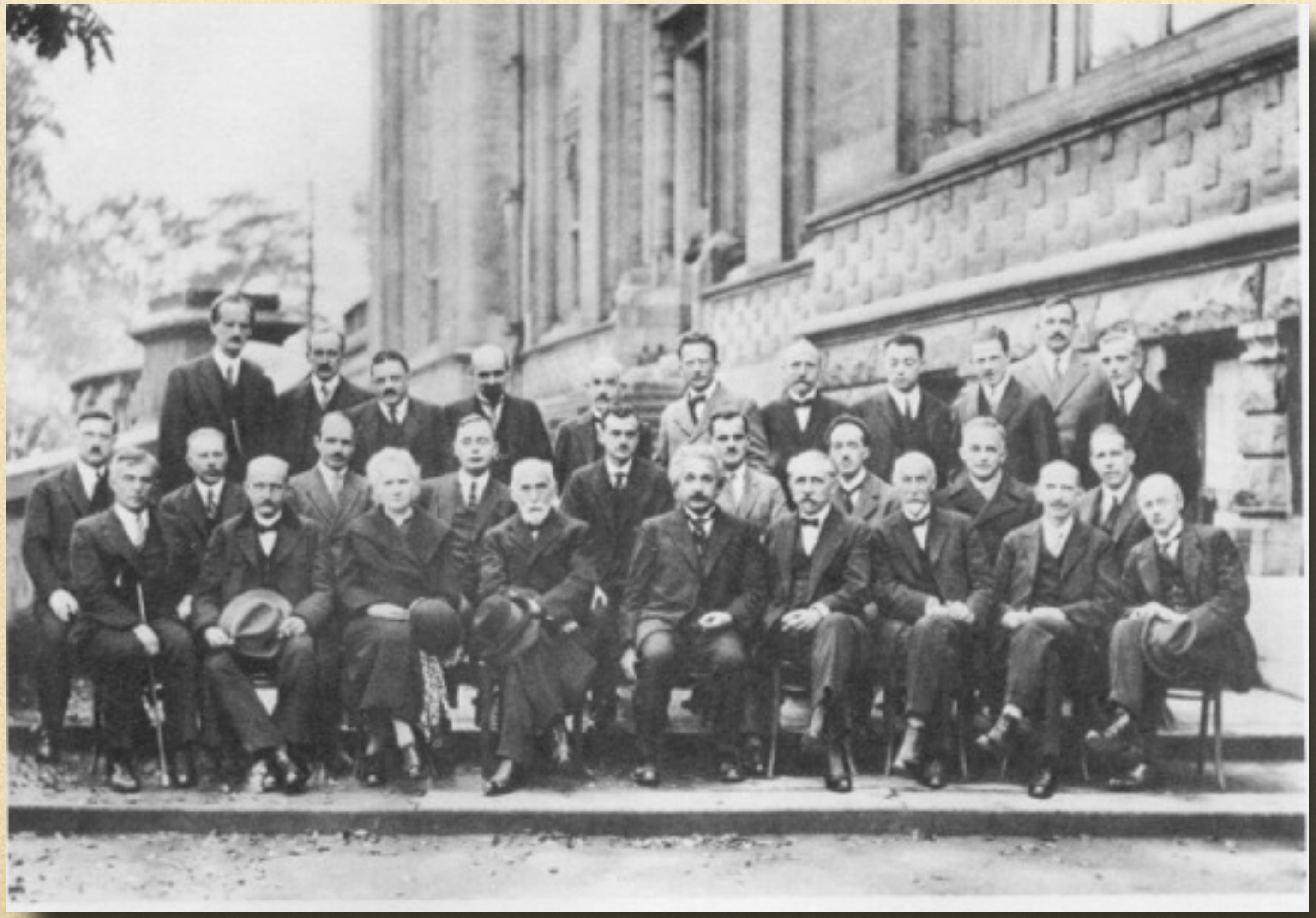
Astonishingly, **equivalence** with matrix mechanics is quickly established, despite the **vastly different routes** Schrödinger and Heisenberg took.

Erwin Schrödinger (1887–1961)

Quantum theory and the crisis of the mechanical worldview:

Further Development of Quantum Mechanics

- probability interpretation (Born) and transformation theory (Dirac, Jordan, von Neumann)
- many-body quantum mechanics (Heisenberg, Dirac, London,...)
- relativistic extension and spin (Pauli, Dirac)
- uncertainty relation (Heisenberg, Pauli)
- early quantum field theory (Jordan, Dirac)
- interpretation (Born, Schrödinger, Bohr, Heisenberg, Einstein ...)
- “applications” ...



Solvay 1927

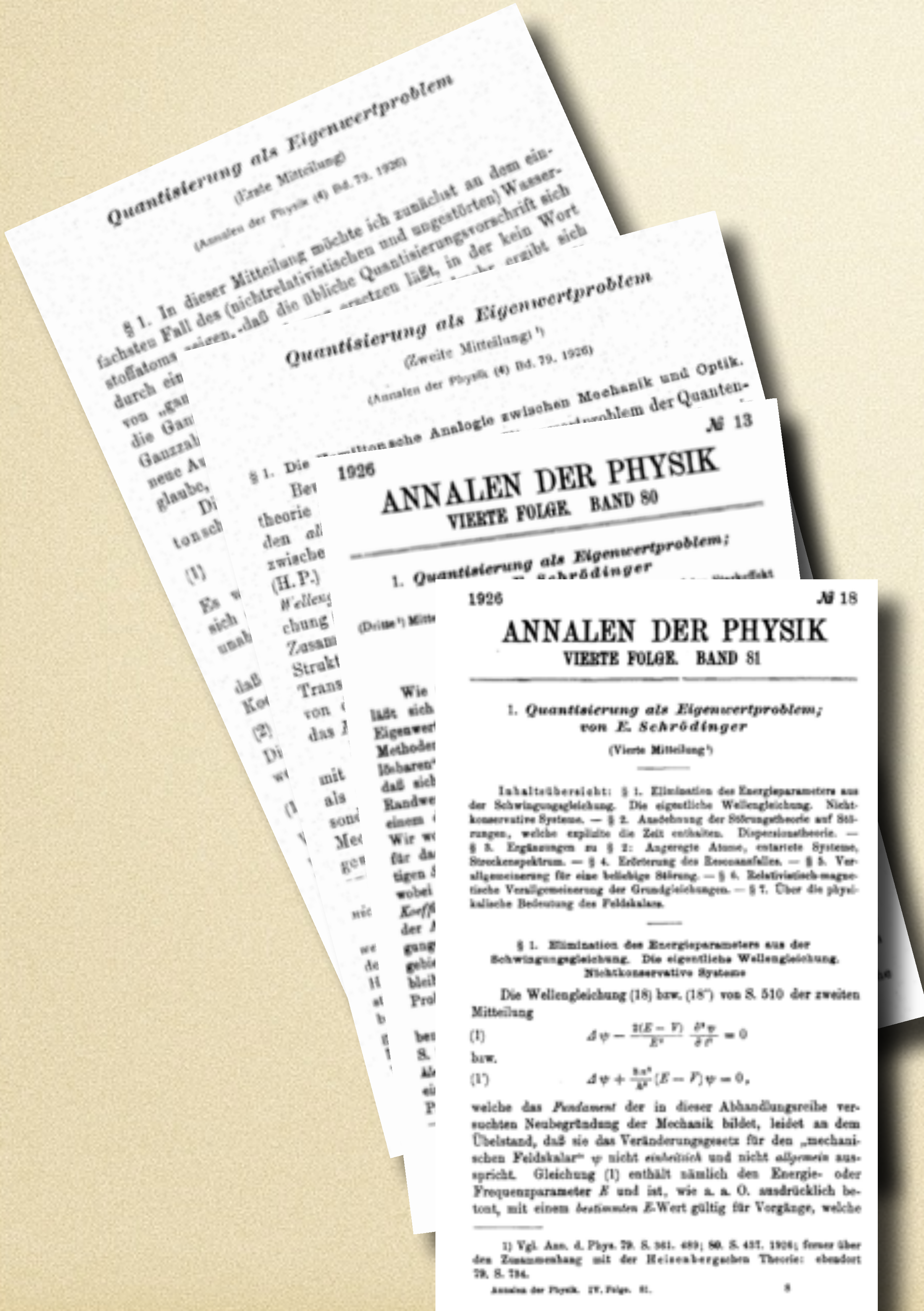
Quantum theory and the crisis of the mechanical worldview: **Revolution or Transformation?**

- Is the new mechanics a **wholesale replacement** or rather a **transformation** of classical physics?
- Both matrix and wave mechanics build upon **knowledge** embedded in classical physics and in the old quantum theory:
    - **Matrix mechanics**: Fourier transformations, secular perturbation theory, co-vibrations and virtual oscillators ,...
    - **Wave mechanics**: Hamilton-Jacobi theory, theory of differential equations, Hamilton's optical-mechanical analogy, ...
- Also Schrödinger's wave mechanics can be seen as a **"sharpening" of the correspondence principle**.

Part II:

The genesis of wave mechanics

The roots of wave mechanics



- 1926: Schrödinger publishes **series of four communications** in "Annalen der Physik": Quantization as an Eigenvalue Problem.
- Communications I and II present two very **different derivations** of the wave equation.
- How then **did** Schrödinger find his wave equation? What were the **roots** of wave mechanics?

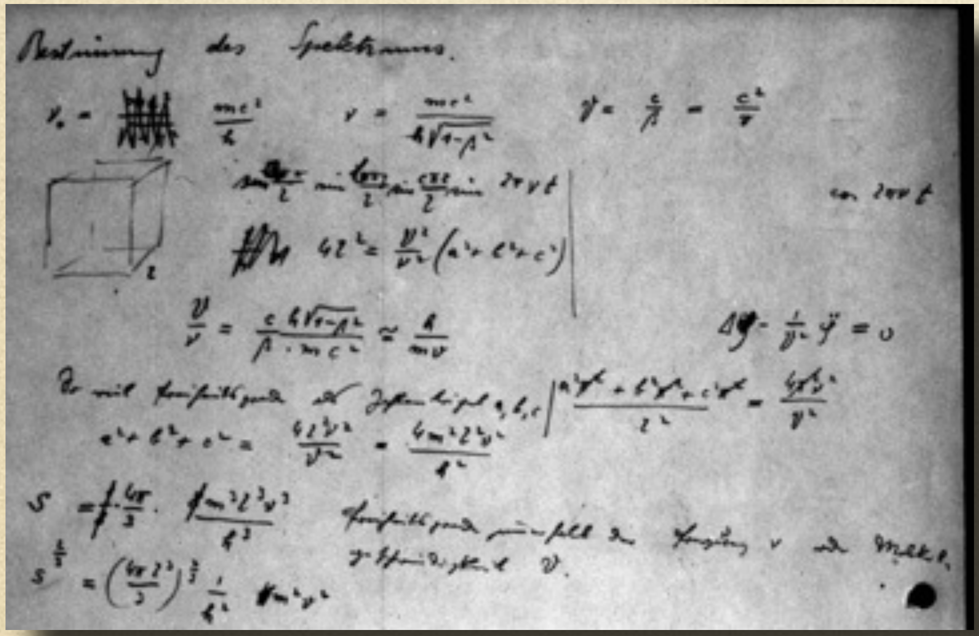
The roots of wave mechanics

Gas Statistics

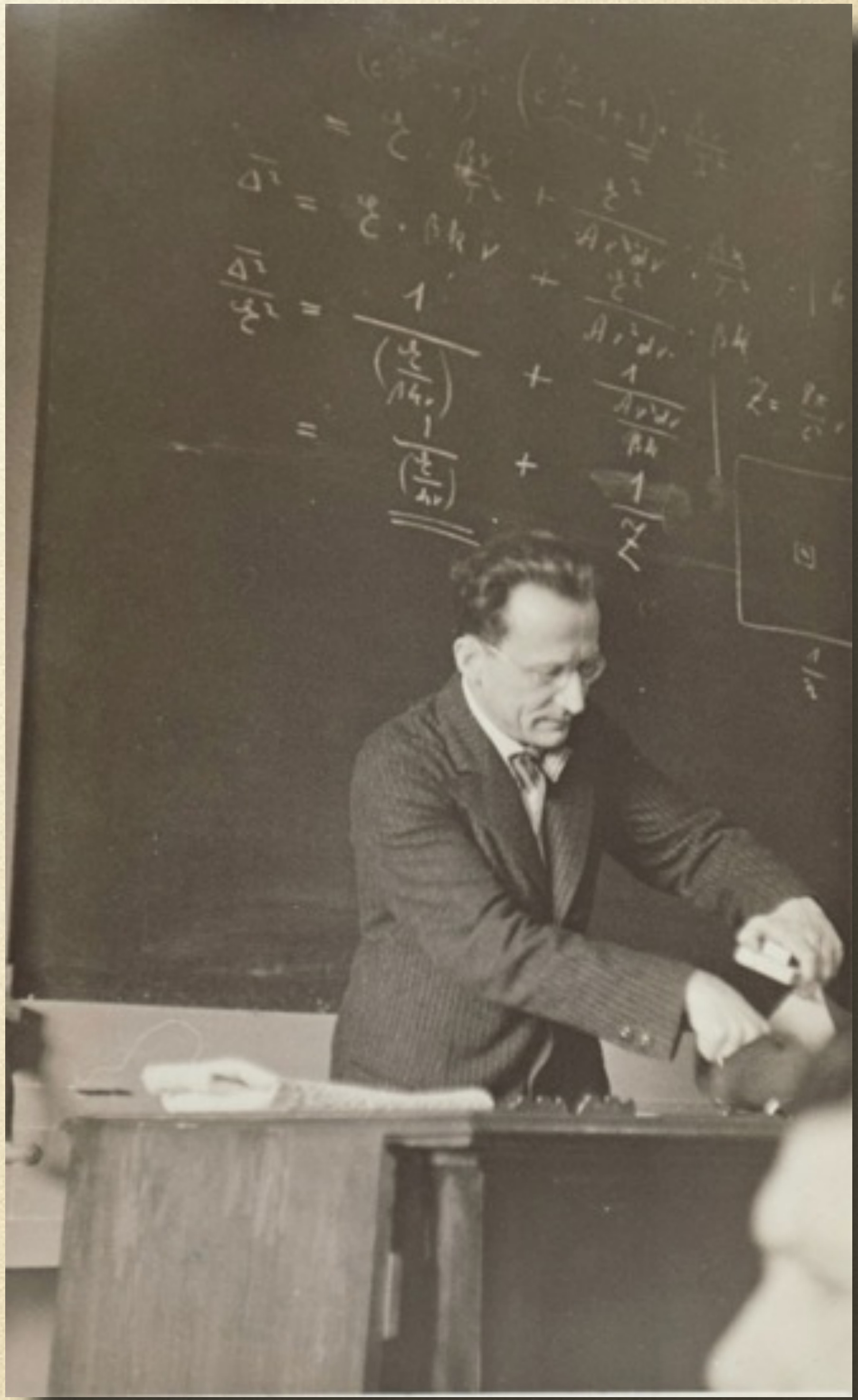
Schrödinger's central interest in 1924–1925: **Quantum statistics** of the ideal gas.

Schrödinger tries to make sense of the new **Bose-Einstein statistics**. Unlike his contemporaries, he is unwilling to accept the existence of a statistics **sui generis** for microscopic particles.

Through Einstein, he learns about **de Broglie** and discovers that Bose-Einstein statistics can be interpreted as a classical Boltzmann statistics of standing matter wave modes.



verso of AHQP 40-8-001 (ca. Nov. 1925)



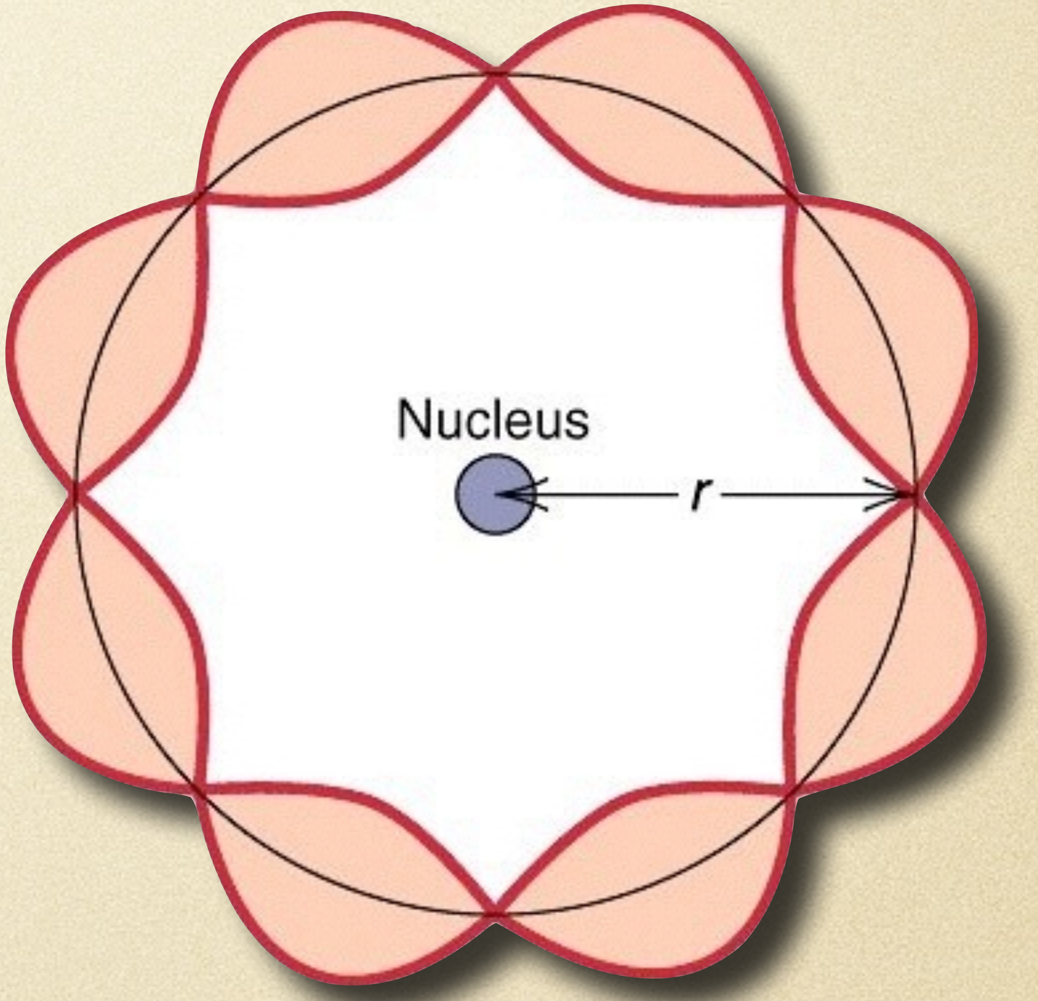
The roots of wave mechanics
Atomic Physics

Alternative Explanation:

Raman and Forman (1969): Schrödinger has early interest in "**theoretical spectroscopy**," as displayed in his 1922 paper "On a Remarkable Property of Quantum Orbits of a Single Electron."

Therefore, **de Broglie**'s explanation of quantum orbits as resonance phenomenon gets picked up enthusiastically by Schrödinger.

Problem: Unlike in gas theory, **no evidence** for coherent research program of Schrödinger in atomic physics.



The roots of wave mechanics

Hamilton's Optical-Mechanical Analogy

Third Explanation:

Helge Kragh (1982): De Broglie's use of the **optical-mechanical analogy** appeals to Schrödinger because of his own explorations of Hamiltonian mechanics around 1920. This leads him to wave mechanics.

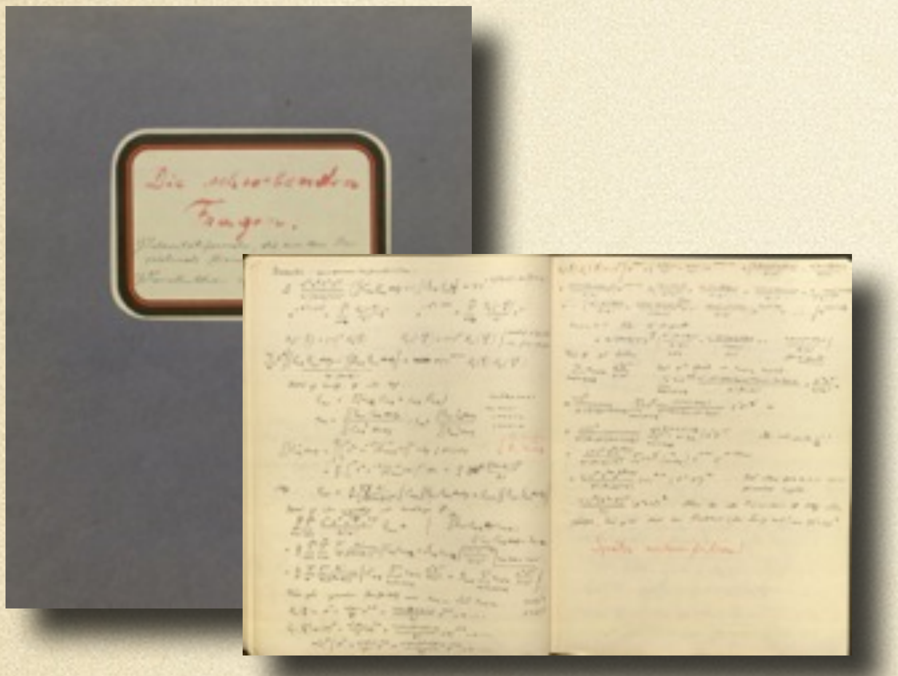
Problem:

Schrödinger's first communication on wave mechanics does not even mention the analogy. **It only appears in his second communication**.

This made historians like **Wessels** (1979,1983) doubt that Hamilton's analogy played a constructive role.

Sir William R. Hamilton
(1805–1865)

The roots of wave mechanics
Schrödinger's research notebooks



"Schrödinger has left **few traces** of how his ideas evolved as he worked towards wave mechanics"

Wessels (1979)

"A considerable **mystery** now obscures the historical record. (...) The **only surviving records** from this time (...) are a three-page set of rough notes titled 'H-Atom—Characteristic Vibrations,' and a 72-page research notebook titled 'Eigenvalue problem of the Atom I.'"

Moore (1989)

- **Both statements are wrong**: Schrödinger's notebooks provide a virtually **complete** account of the creation of wave mechanics.
- The notebooks lead to a **major revision** of previous accounts:
  - All three **roots** (gas statistics, theoretical spectroscopy, optical-mechanical analogy) have their logical place in an **ambitious research program** pursued by Schrödinger.
  - Hamilton's optical-mechanical analogy plays a **pivotal role**.

four-slide interlude:

Hamilton’s optical-mechanical analogy

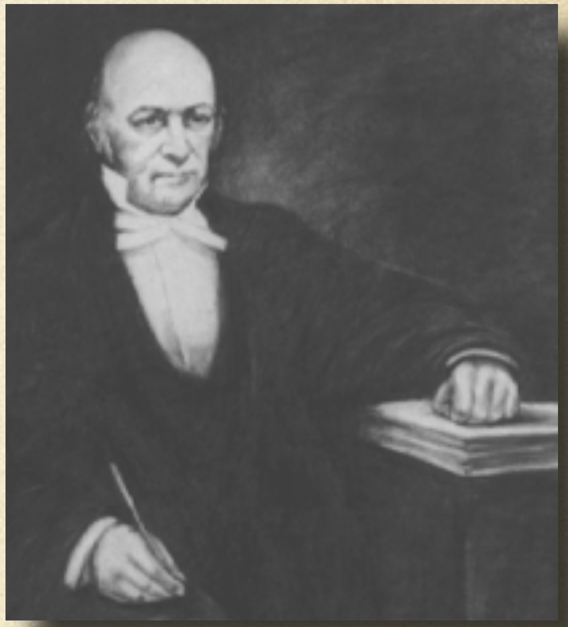
Hamilton’s optical-mechanical analogy

Step 1: Hamiltonian optics

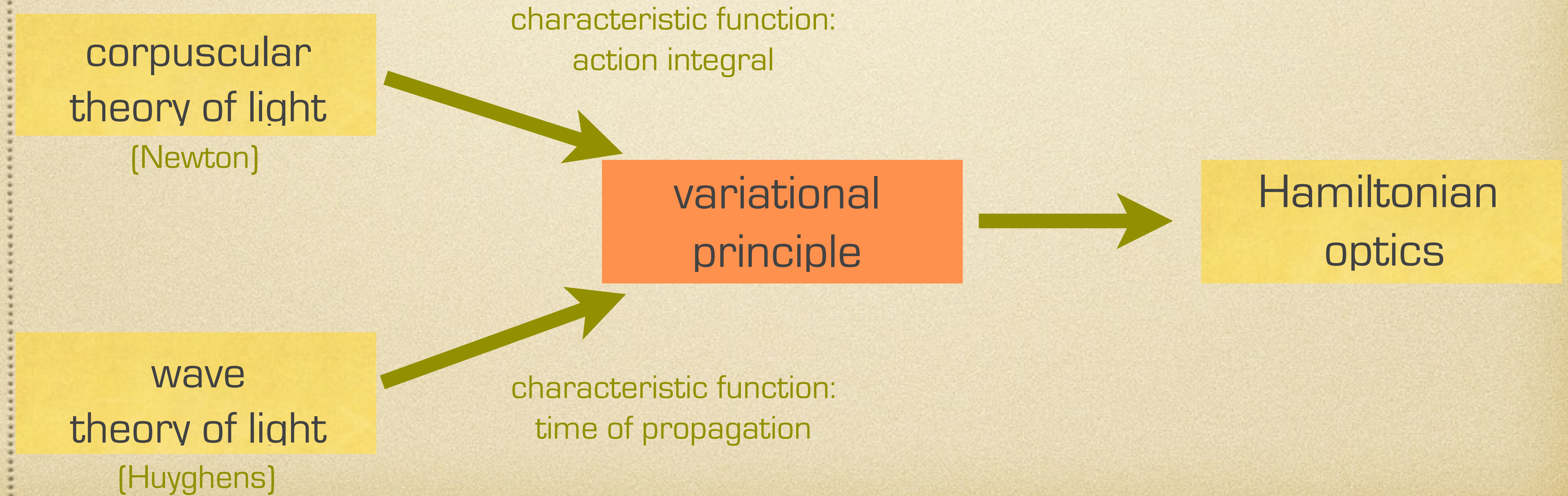
Hamilton‘s early work devoted to optics.

In1833, Hamilton casts ray optics into a **general scheme** comparable to Lagrangian mechanics:

“Those who have meditated on the beauty and utility, in theoretical mechanics, of the general method of **Lagrange** [...] must feel that mathematical optics can only then attain a coordinate rank with mathematical mechanics, or with dynamical astronomy, in beauty, power, and harmony, when it shall possess an appropriate method, and become the unfolding of **a central idea**.”



Hamilton (1805–1865)



Hamilton's optical-mechanical analogy

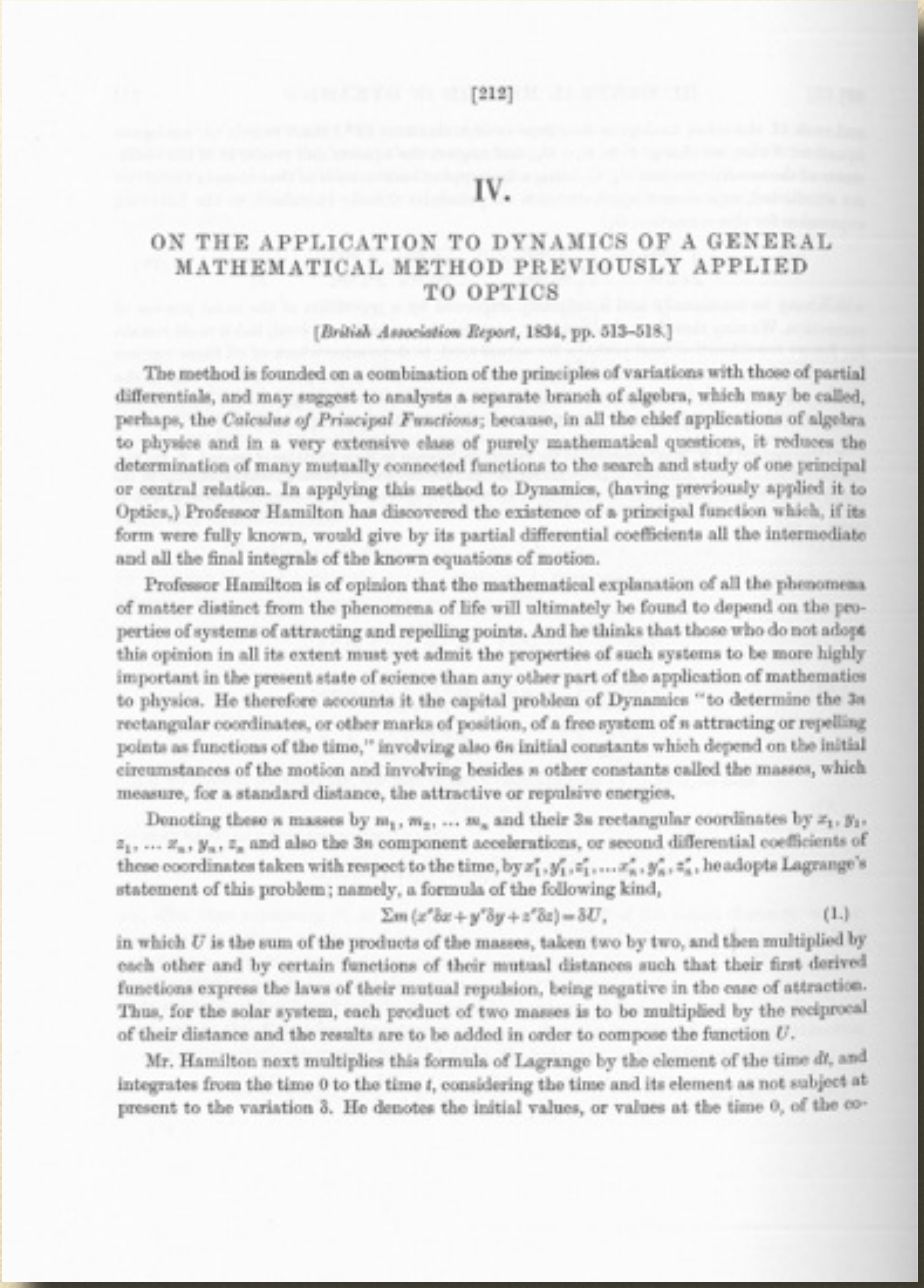
Step 2: Unifying Optics and Mechanics

In 1834, Hamilton attempts to **unify optics and mechanics through formal analogy.**

He applies the general method he had developed for optics also to mechanics (today known as **Hamilton-Jacobi** method).

Both optics and mechanics obey a principle of least "action."

Hamilton (1833): "By this view the research of the most complicated orbits, in lunar, planetary, and sidereal astronomy, is reduced to the study of the properties of a single function [S]; which is **analogous to my optical function**, and represents the **action** of the system from one position to another."



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IV.

ON THE APPLICATION TO DYNAMICS OF A GENERAL MATHEMATICAL METHOD PREVIOUSLY APPLIED TO OPTICS

[*British Association Report*, 1834, pp. 513–518.]

The method is founded on a combination of the principles of variations with those of partial differentials, and may suggest to analysts a separate branch of algebra, which may be called, perhaps, the *Calculus of Principal Functions*; because, in all the chief applications of algebra to physics and in a very extensive class of purely mathematical questions, it reduces the determination of many mutually connected functions to the search and study of one principal or central relation. In applying this method to Dynamics, (having previously applied it to Optics,) Professor Hamilton has discovered the existence of a principal function which, if its form were fully known, would give by its partial differential coefficients all the intermediate and all the final integrals of the known equations of motion.

Professor Hamilton is of opinion that the mathematical explanation of all the phenomena of matter distinct from the phenomena of life will ultimately be found to depend on the properties of systems of attracting and repelling points. And he thinks that those who do not adopt this opinion in all its extent must yet admit the properties of such systems to be more highly important in the present state of science than any other part of the application of mathematics to physics. He therefore accounts it the capital problem of Dynamics "to determine the $3n$ rectangular coordinates, or other marks of position, of a free system of n attracting or repelling points as functions of the time," involving also $6n$ initial constants which depend on the initial circumstances of the motion and involving besides n other constants called the masses, which measure, for a standard distance, the attractive or repulsive energies.

Denoting these n masses by $m_1, m_2, \dots m_n$ and their $3n$ rectangular coordinates by $x_1, y_1, z_1, \dots x_n, y_n, z_n$ and also the $3n$ component accelerations, or second differential coefficients of these coordinates taken with respect to the time, by $x_1'', y_1'', z_1'', \dots x_n'', y_n'', z_n''$, he adopts Lagrange's statement of this problem; namely, a formula of the following kind,

$$\Sigma m\,(x''\delta x + y''\delta y + z''\delta z) = \delta U, \qquad (1.)$$

in which U is the sum of the products of the masses, taken two by two, and then multiplied by each other and by certain functions of their mutual distances such that their first derived functions express the laws of their mutual repulsion, being negative in the case of attraction. Thus, for the solar system, each product of two masses is to be multiplied by the reciprocal of their distance and the results are to be added in order to compose the function U.

Mr. Hamilton next multiplies this formula of Lagrange by the element of the time dt, and integrates from the time 0 to the time t, considering the time and its element as not subject at present to the variation δ. He denotes the initial values, or values at the time 0, of the co-

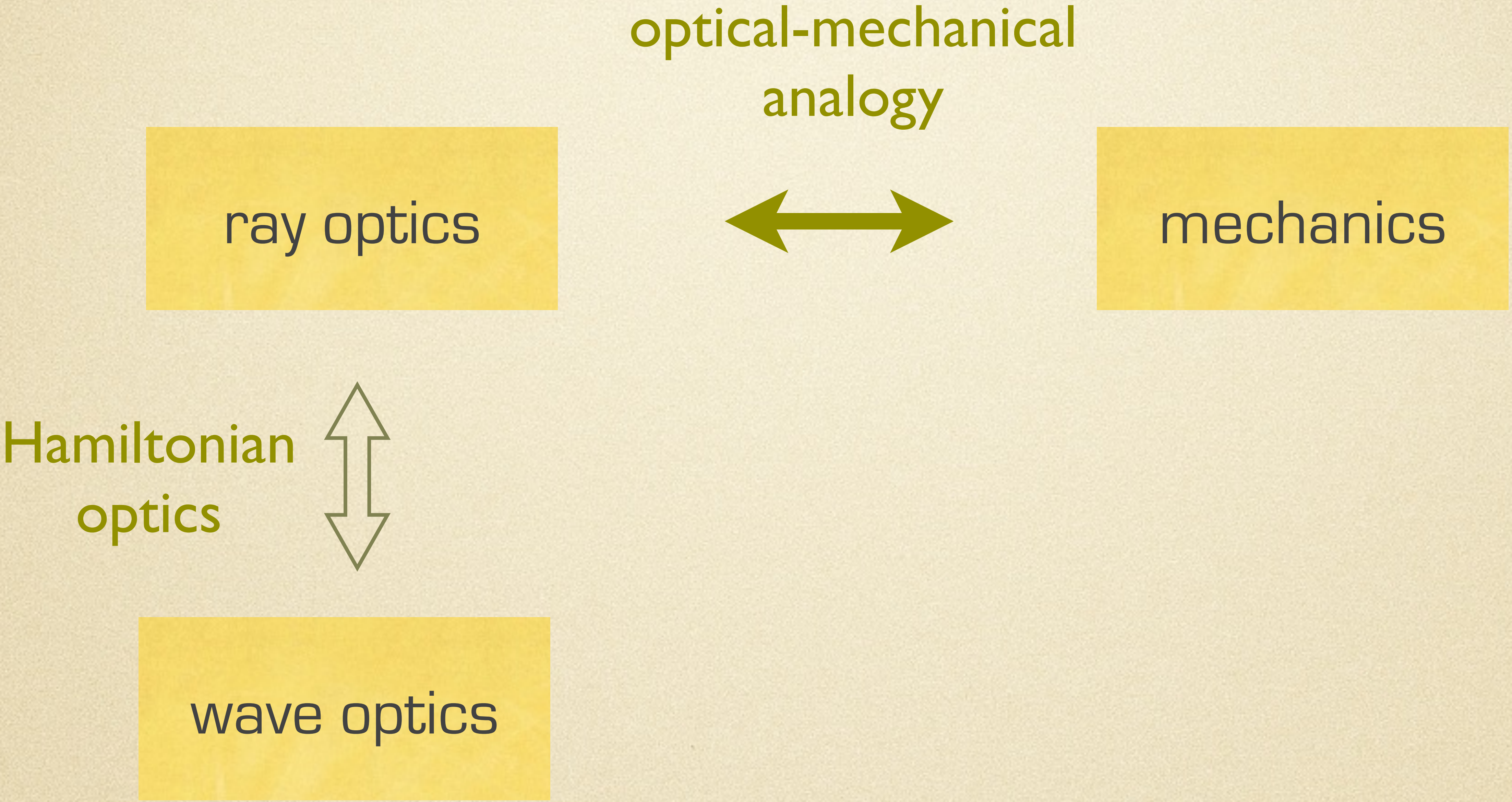
Brit. Assoc. Report, 1834, pp. 513–518

Hamilton's optical-mechanical analogy

Step 2: Unifying Optics and Mechanics

Optics:	**Mechanics:**
Characteristic function is time of propagation T: $$T = \int \frac{n}{c} ds$$ n refractive index, c light velocity	Characteristic function is action intergral S: $$S = \int \sqrt{2m(E-U)} ds$$ m mass, $E-U$ kinetic energy
Integrand is inverse phase velocity $1/u$: $$\frac{1}{u} = \frac{n}{c}$$	Integrand is particle momentum p: $$p = \sqrt{2m(E-U)}$$
Fermat's principle: $$\delta T = 0$$	**Maupertuis's principle:** $$\delta S = 0$$
This implies: Light rays are orthogonal to surfaces of equal time T (wave fronts).	This implies: Particle trajectories are orthogonal to surfaces of equal action S.

Hamilton's optical-mechanical analogy
Step 2: Unifying Optics and Mechanics



Until the 1920s, Hamilton's optical-mechanical analogy is nothing more than a little-known formal peculiarity.

Part II (contd.):

The genesis of wave mechanics

The genesis of wave mechanics

The changing roles of Hamilton's analogy

Erwin Schrödinger (1887–1961)

- **ca. 1918–1922**: Schrödinger **encounters** the analogy when trying to generalize classical mechanics.
- **Oct–Nov 1925**: Schrödinger reads **de Broglie** in the context of gas theory and uses matter waves to explain Bose-Einstein statistics.
- **early 1926**: Analogy provides **heuristic guidance** in attempts to establish the wave equation.
- **Feb. 1926**: Schrödinger **"completes"** Hamilton's analogy; analogy turns from heuristic tool into **formal constraint** on possible theories.
- **mid-1926**: Analogy turns into an **interpretational device.**

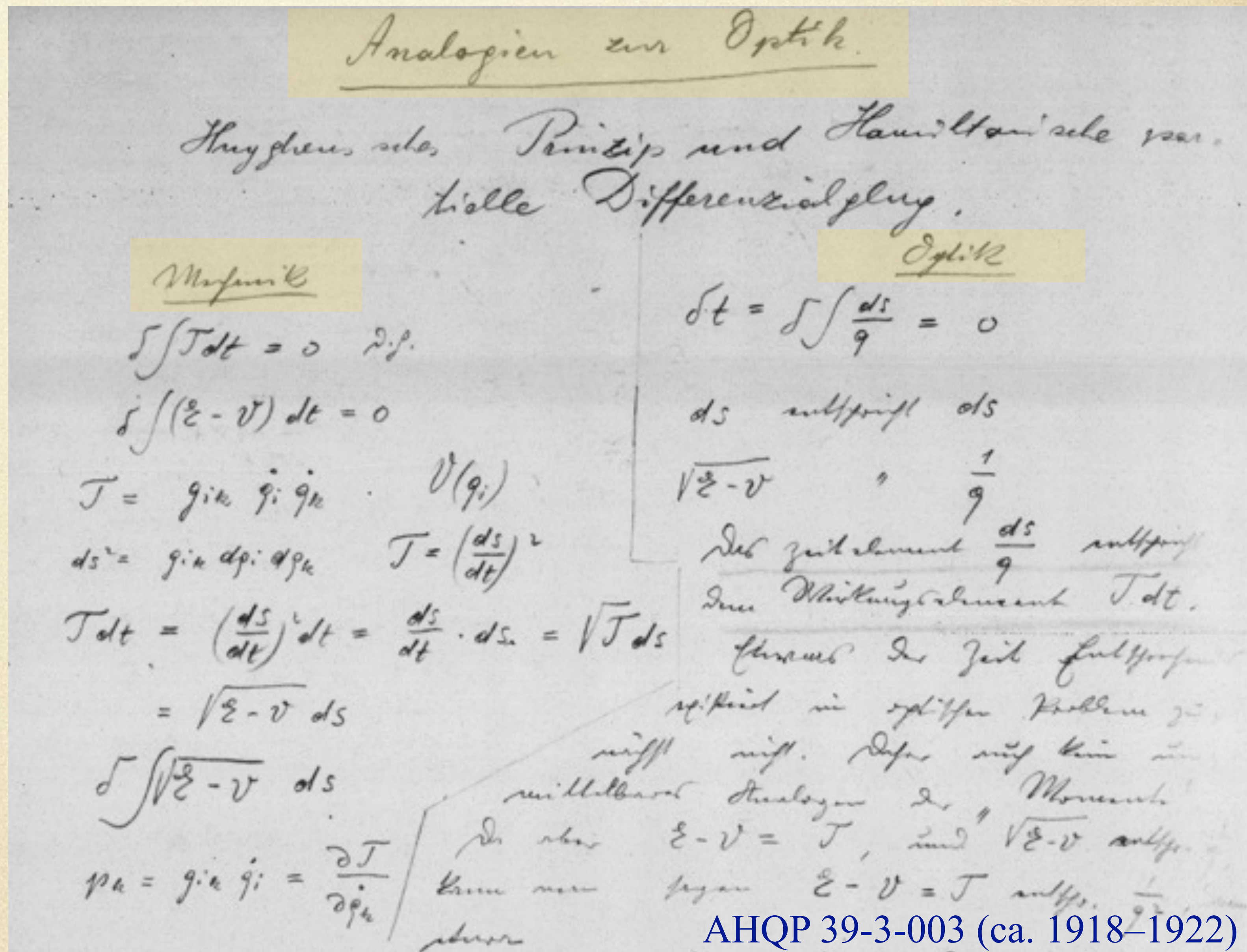
The genesis of wave mechanics

Schrödinger’s early research program

ca. 1920: Schrödinger pursues **extensive research program** to generalize classical mechanics.

When trying to **connect** Hertzian analytical mechanics to General Relativity, he encounters the **optical-mechanical analogy**.

Schrödinger hopes that this could lead to **explanation of quantum conditions** as Hertzian constraints.



AHQP 39-3-003 (ca. 1918–1922)

Schrödinger’s c. 1920 notebook on Tensor-Analytic Mechanics: working out Hamilton‘s analogy between mechanics (left) and optics (right).

The genesis of wave mechanics

Gas statistics

Physikalische Zeitschrift. 27. Jahrgang. 1926. Seite 95—101.

Zur Einsteinschen Gastheorie.

Von E. Schrödinger.

§ 1: Grundgedanke.

Als der wesentliche Punkt in der kürzlich von A. Einstein ausgearbeiteten neuen Gastheorie[1]) gilt wohl allgemein dieser, daß eine ganz neuartige Statistik, die sogenannte Bosesche Statistik[2]), auf die Bewegungen der Gasmoleküle anzuwenden sei. Diese neue Statistik als etwas Primäres, nicht weiter Erklärbares anzusehen, sträubt sich das natürliche Gefühl mit Recht[3]). Vielmehr scheint sich in ihr die Annahme einer gewissen Abhängigkeit voneinander oder einer Wechselwirkung der Gasmoleküle zu verhüllen, die jedoch in dieser Form nur schwer zu analysieren ist.

Man wird erwarten dürfen, einen tieferen Einblick in das eigentliche Wesen der neuen Theorie zu gewinnen, wenn es gelingt, die alten, an der Erfahrung erprobten und logisch wohlbegründeten statistischen Methoden in ihrem Recht zu belassen und die Änderung in den Grundlagen an einer Stelle vorzunehmen, wo sie ohne sacrificium intellectus möglich ist. Dazu führt folgender einfacher Gedanke: die Einsteinsche Gastheorie wird erhalten, indem man auf die Gasmoleküle die Form der Statistik anwendet, die, auf die „Lichtatome" angewendet, zum Planckschen Strahlungsgesetz führt. Aber man kann das Plancksche Strahlungsgesetz auch durch „natürliche" Statistik gewinnen, indem man

1) A. Einstein, Berl. Ber. 1924, S. 261; 1925, S. 3.
2) Bose, Zeitschr. f. Phys. 26, 178, 1924.
3) Vgl. A. Landé, Zeitschr. f. Phys. 33, 571, 1925. Im einzelnen kann ich allerdings diesen Ausführungen nicht zustimmen.

sie auf die sog. „Ätherresonatoren", d. i. auf die Freiheitsgrade der Strahlung anwendet[1]). Die Lichtatome treten dann nur als die Energiestufen der Ätherresonatoren auf. Der Übergang von der natürlichen zur Boseschen Statistik kann stets ersetzt werden dadurch, daß man die Begriffe „Mannigfaltigkeit der energetischen Zustände" und „Mannigfaltigkeit der Träger dieser Zustände" die Rollen tauschen läßt. Man muß also einfach das Bild des Gases nach demjenigen Bilde der Hohlraumstrahlung formen, das noch nicht der extremen Lichtquantenvorstellung entspricht; dann wird die natürliche Statistik — etwa die bequeme Plancksche Methode der Zustandssumme — zur Einsteinschen Gastheorie führen. Das heißt nichts anderes, als Ernst machen mit der de Broglie[2])-Einsteinschen[3]) Undulationstheorie der bewegten Korpuskel, nach welcher dieselbe nichts weiter als eine Art „Schaumkamm" auf einer den Weltgrund bildenden Wellenstrahlung ist.

Die Durchführung dieses Gedankens scheint mir von genügendem Interesse, um sie hier auseinanderzusetzen.

§ 2. Das Gas als Oszillatorensystem. Bestimmung der freien Energie.

Wir gehen aus von der geläufigen Vorstellung, daß jedes von den n in einem Volumen V eingeschlossenen Molekülen eines einatomigen idealen Gases eine diskrete Folge von genau

1) J. H. Jeans, Phil. Mag. 10, 91, 1905; P. Debye, Ann. d. Phys. 33, 1427, 1910. Vgl. den letzten Abschnitt von Planck, „Wärmestrahlung". — Ferner M. v. Laue, Ann. d. Phys. (4) 44, 1197, 1914.
2) L. de Broglie, Thèses. Paris (Edit. Masson & Cie.), 1924. Gleichlautend Ann. de Physique (10), 3, 22, 1925.
3) A. Einstein, l. c. § 8.

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- **Oct./Nov. 1925**: Schrödinger reads **de Broglie's** thesis (letter to Einstein, 3 Nov 1925).
- His **reinterpretation** of Bose-Einstein statistics as a "natural" Boltzmann statistics of matter waves becomes a strong argument for the correctness of the **wave picture** of matter.

"This amounts to getting serious about the de Broglie-Einstein undulatory theory of the moving particle, according to which the latter is nothing but a kind of 'crest' on a wave radiation forming the substratum of the world."

The genesis of wave mechanics

The relativistic wave equation

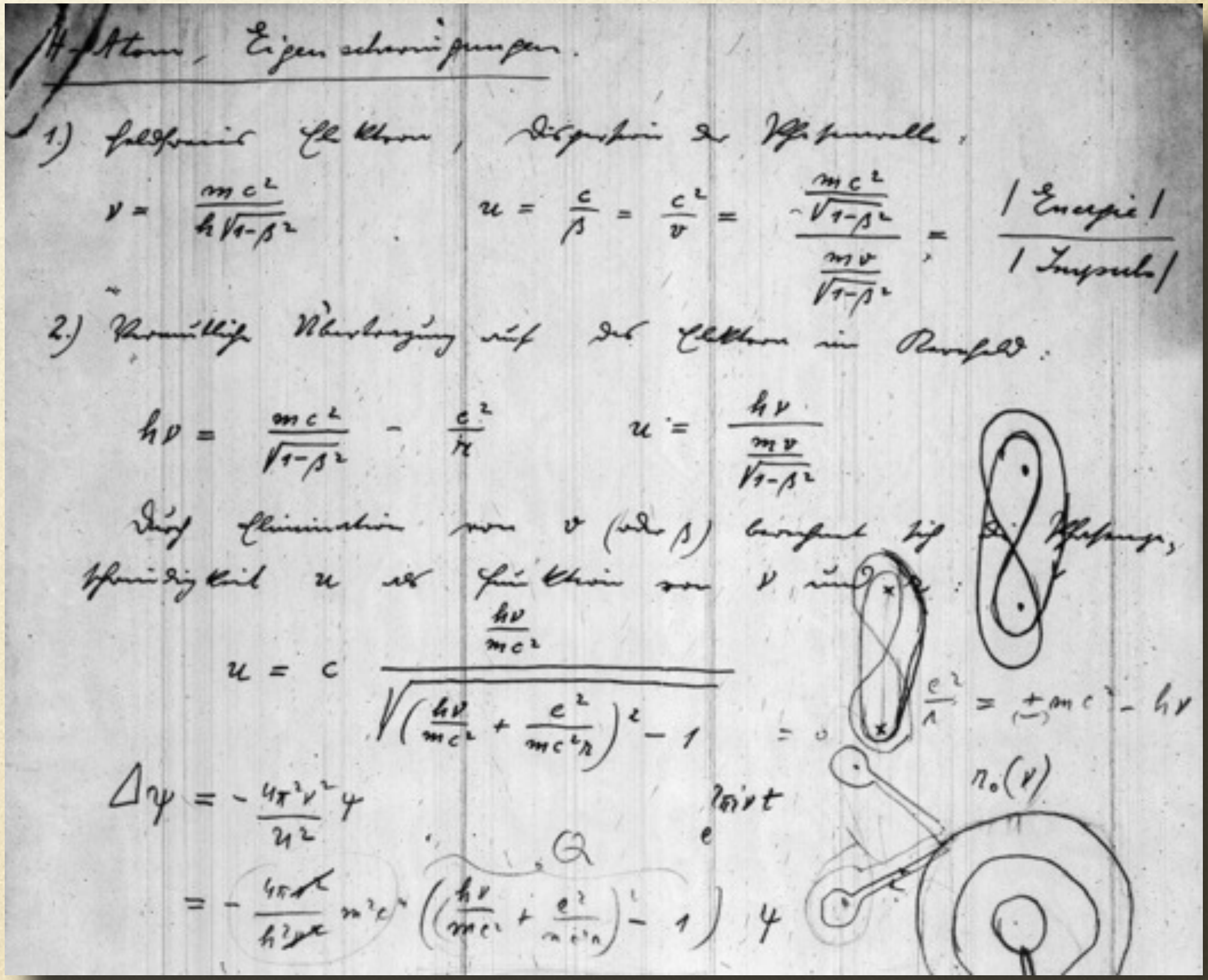
In his thesis, de Broglie refers to an analogy between **Fermat**'s principle and **Maupertuis**'s principle as an argument for matter waves.

Schrödinger recognizes this as an expression of the **Hamiltonian analogy**.

He introduces a tentative **relativistic wave equation** to describe de Broglie's matter waves.

Quantization rules explained as **eigenvalue problem** of a partial differential equation.

Schrödinger solves relativistic wave equation for hydrogen atom, but **fails** to derive the results of Sommerfeld's theory (**incorrect** energy levels).



AHQP 40-5-002 (late 1925 or Jan. 1926)

The genesis of wave mechanics

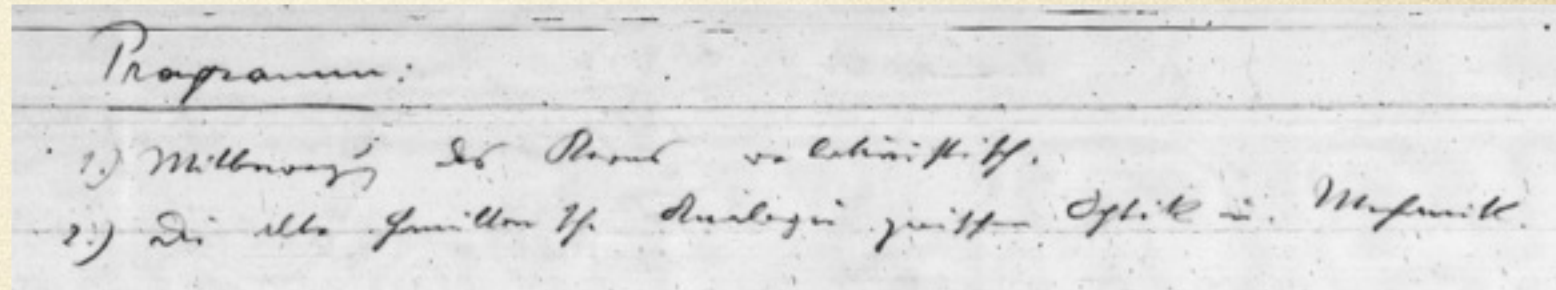
The nonrelativistic wave equation (1)

In early 1926, Schrödinger derives **nonrelativistic** wave equation yielding **correct** energy levels for hydrogen atom.

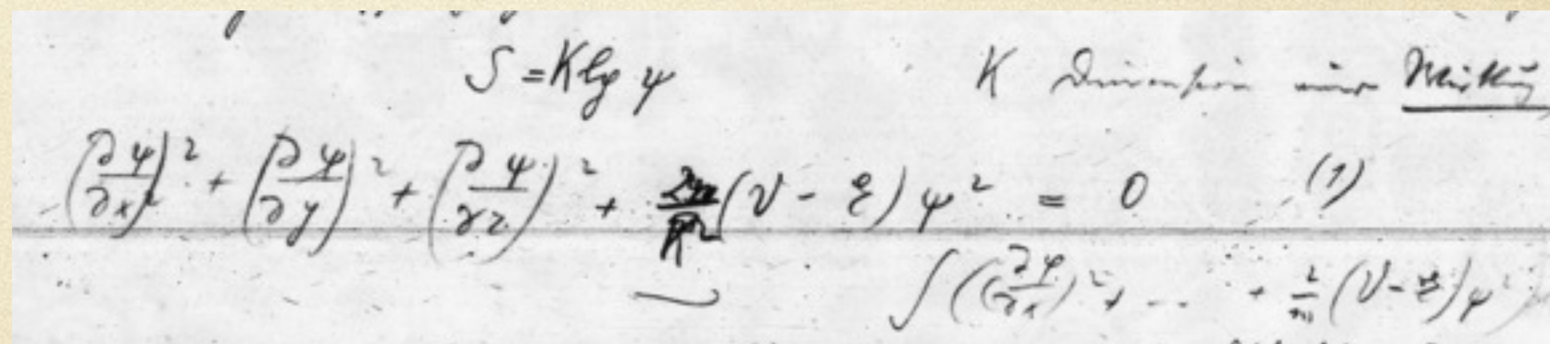
Schrödinger sends off first communication in late Jan 1926, deriving nonrelativistic wave equation from seemingly **ad-hoc** variational principle.

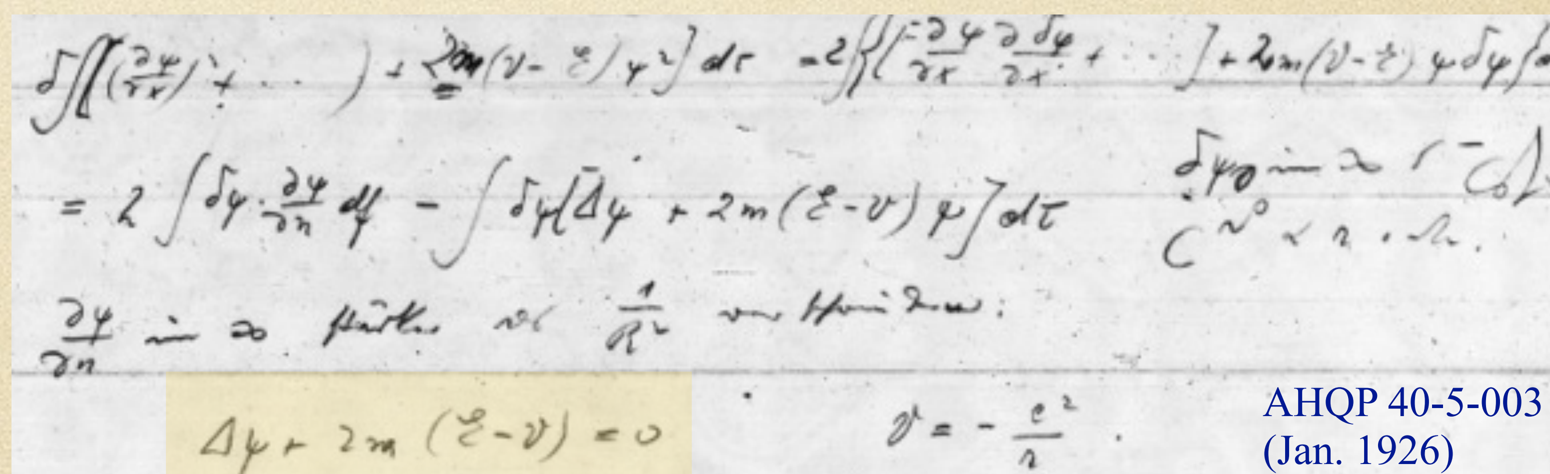
Notebooks show: starting point is Hamilton's **optical-mechanical analogy**.

Schrödinger **reinterprets** Hamilton's equation as a field equation for the "**action field**" and uses analogy as **heuristic prescription** to convert the classical action into a picture of wave fronts.



Program:
1.) relativistic treatment of the motion of the nucleus
2.) the old Hamiltonian analogy between optics and mechanics





AHQP 40-5-003
(Jan. 1926)

Schrödinger's wave equation (almost)

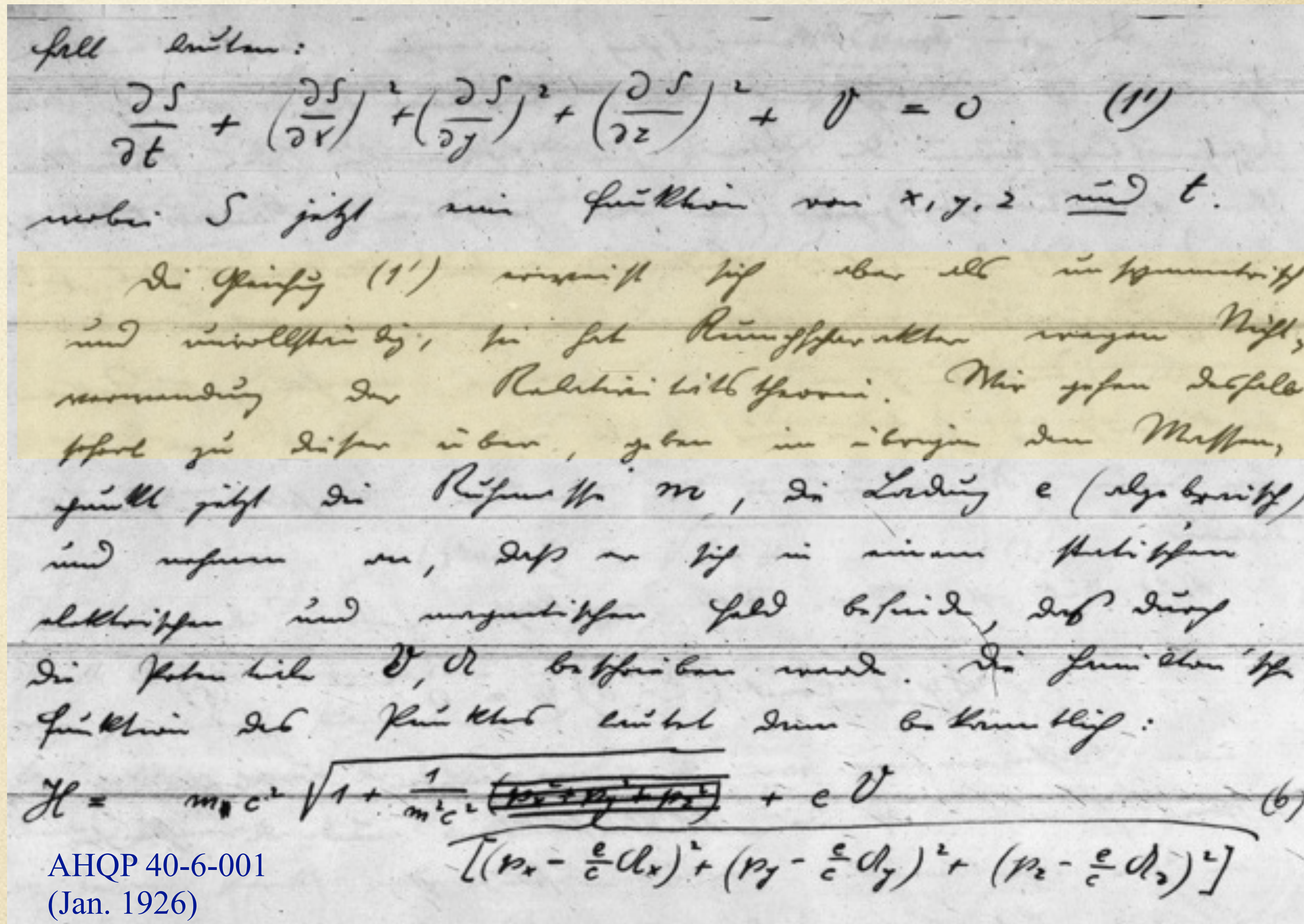
The genesis of wave mechanics

The nonrelativistic wave equation (2)

However, for Schrödinger, it is only a **limited success**.

He turns back to the **relativistic case**, trying to improve upon the nonrelativistic wave equation for a second communication.

Starting point again: **Hamilton's analogy!**



soll lauten:

$$\frac{\partial S}{\partial t} + \left(\frac{\partial S}{\partial x}\right)^2 + \left(\frac{\partial S}{\partial y}\right)^2 + \left(\frac{\partial S}{\partial z}\right)^2 + V = 0 \qquad (1')$$

wobei S jetzt eine Funktion von x, y, z und t.

Die Gleichung (1') erweist sich aber als unsymmetrisch und unvollständig, sie hat Rumpfcharakter wegen Nichtanwendung der Relativitätstheorie. Wir gehen deshalb sofort zu dieser über, geben im übrigen dem Massenpunkt jetzt die Ruhmasse m, die Ladung e (algebraisch) und nehmen an, daß er sich in einem statischen elektrischen und magnetischen Feld befinde, das durch die Potentiale V, $\mathfrak{A}$ beschrieben werde. Die Hamilton'sche Funktion des Punktes lautet dann bekanntlich:

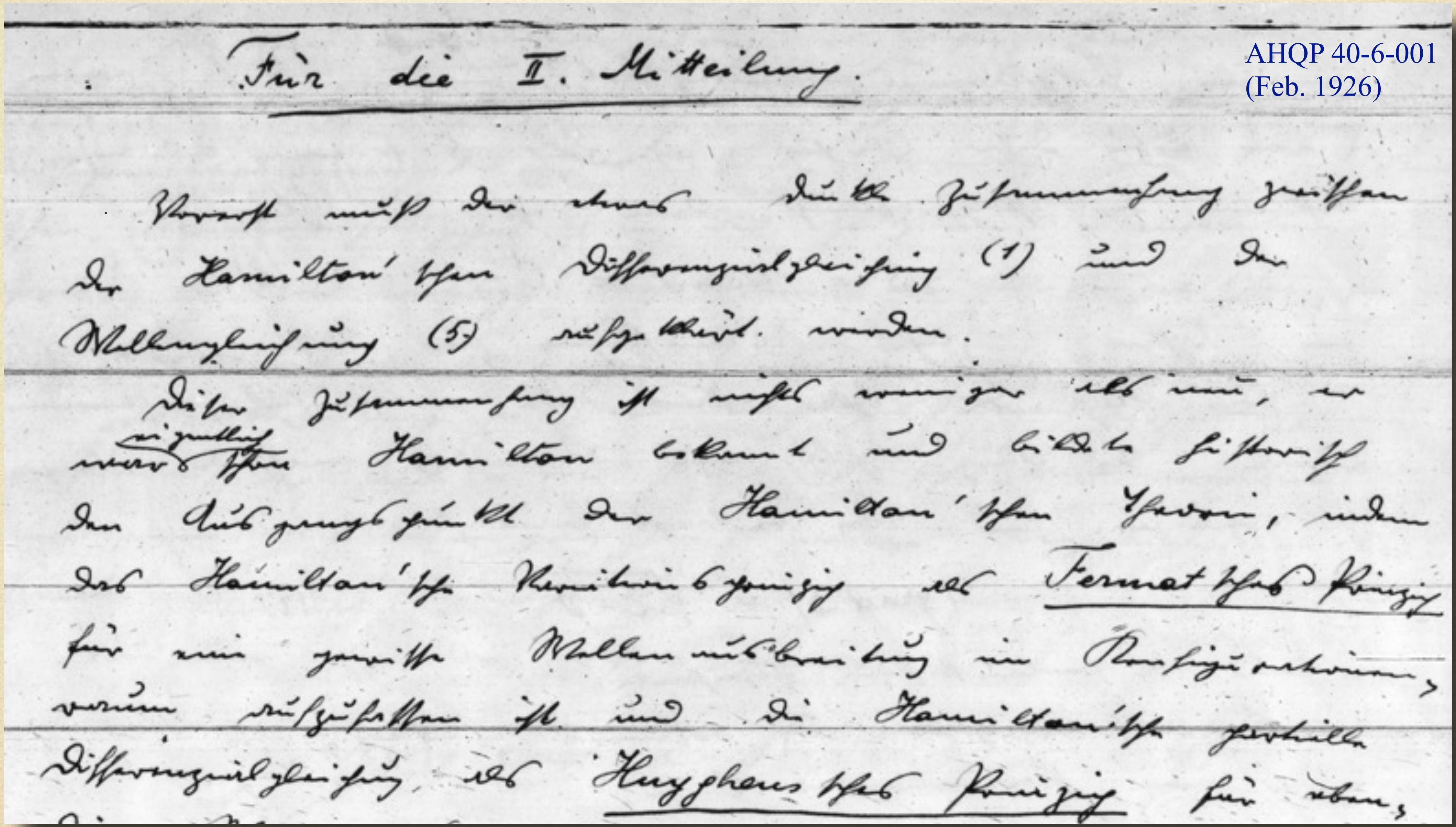
$$H = m_0 c^2 \sqrt{1 + \frac{1}{m_0^2 c^2}\left[\left(p_x - \frac{e}{c}\mathfrak{A}_x\right)^2 + \left(p_y - \frac{e}{c}\mathfrak{A}_y\right)^2 + \left(p_z - \frac{e}{c}\mathfrak{A}_z\right)^2\right]} + eV \qquad (6)$$

AHQP 40-6-001
(Jan. 1926)

"The equation, however, turns out to be unsymmetrical and incomplete, it has rudimentary character due to nonapplication of the theory of relativity. Therefore, we immediately turn to [the relativistic case] ..."

The genesis of wave mechanics

The second communication



AHQP 40-6-001
(Feb. 1926)

"For the second communication.
First, the somewhat obscure connection between the Hamiltonian differential equation and the wave equation needs to be explained. This connection is nothing less than new, it was actually already known to Hamilton and historically constituted the starting point for the Hamiltonian theory..."

The genesis of wave mechanics

An "undulatory mechanics"

February 1926: When preparing his second communication, Schrödinger **suddenly** realizes that Hamilton's action function only describes the phase (and not the amplitude) of the wave fronts. If the amplitude varies, e.g., in the case of interference, ψ is not a simple function of the action.

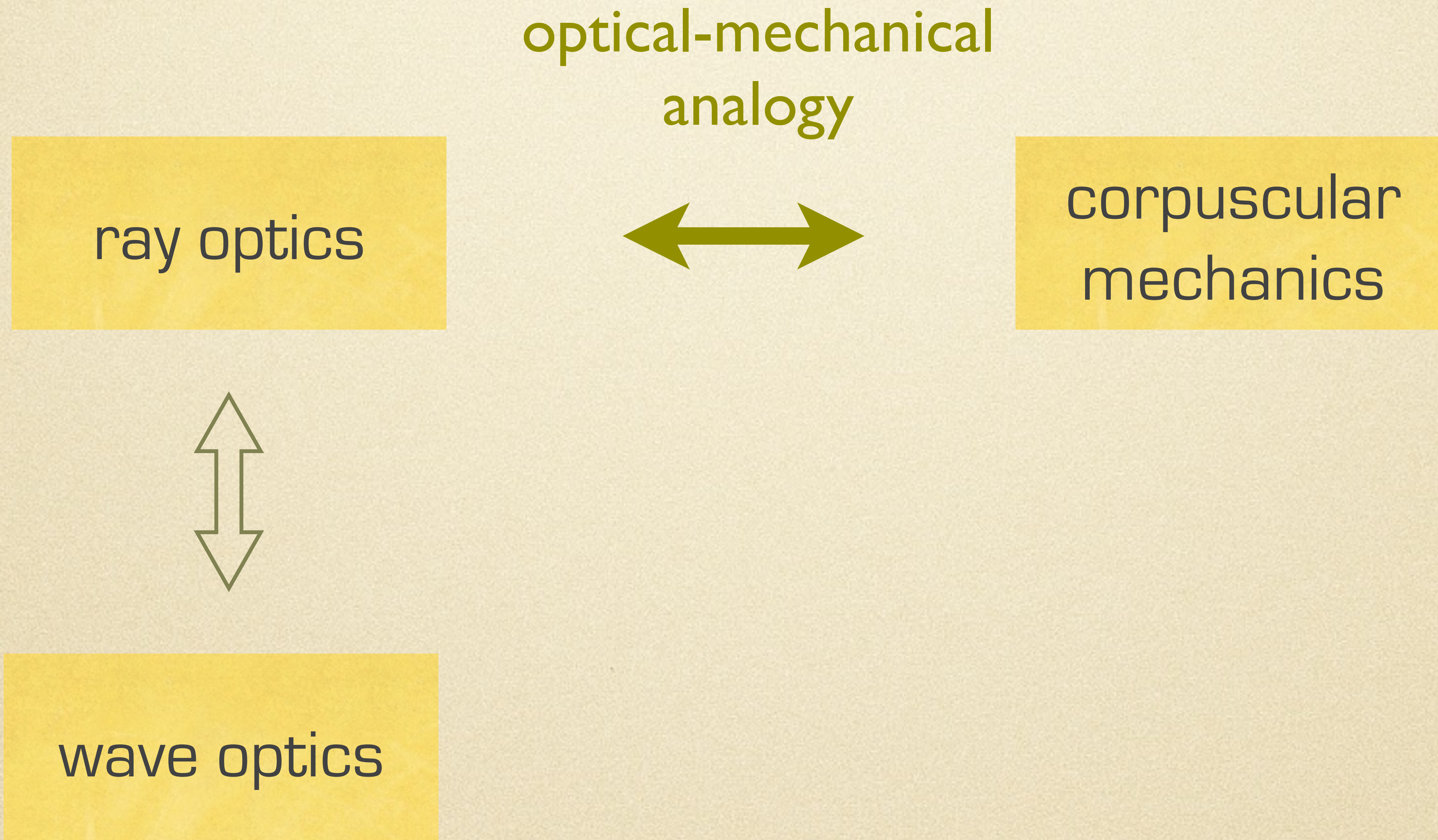
> "Maybe our classical mechanics is the **full** analog of geometrical optics, and, as such, wrong, not in agreement with reality. It fails as soon as the radii of curvature and the dimensions of the trajectory are not large anymore compared to a certain wavelength, to which one can attribute a certain reality in q-space. In that case, one has to search for '**undulatory mechanics**'—and the obvious way to this end is the wave-theoretical **extension** of Hamilton's picture."

Schrödinger (1926, second communication)

Schrödinger ca. mid-1926

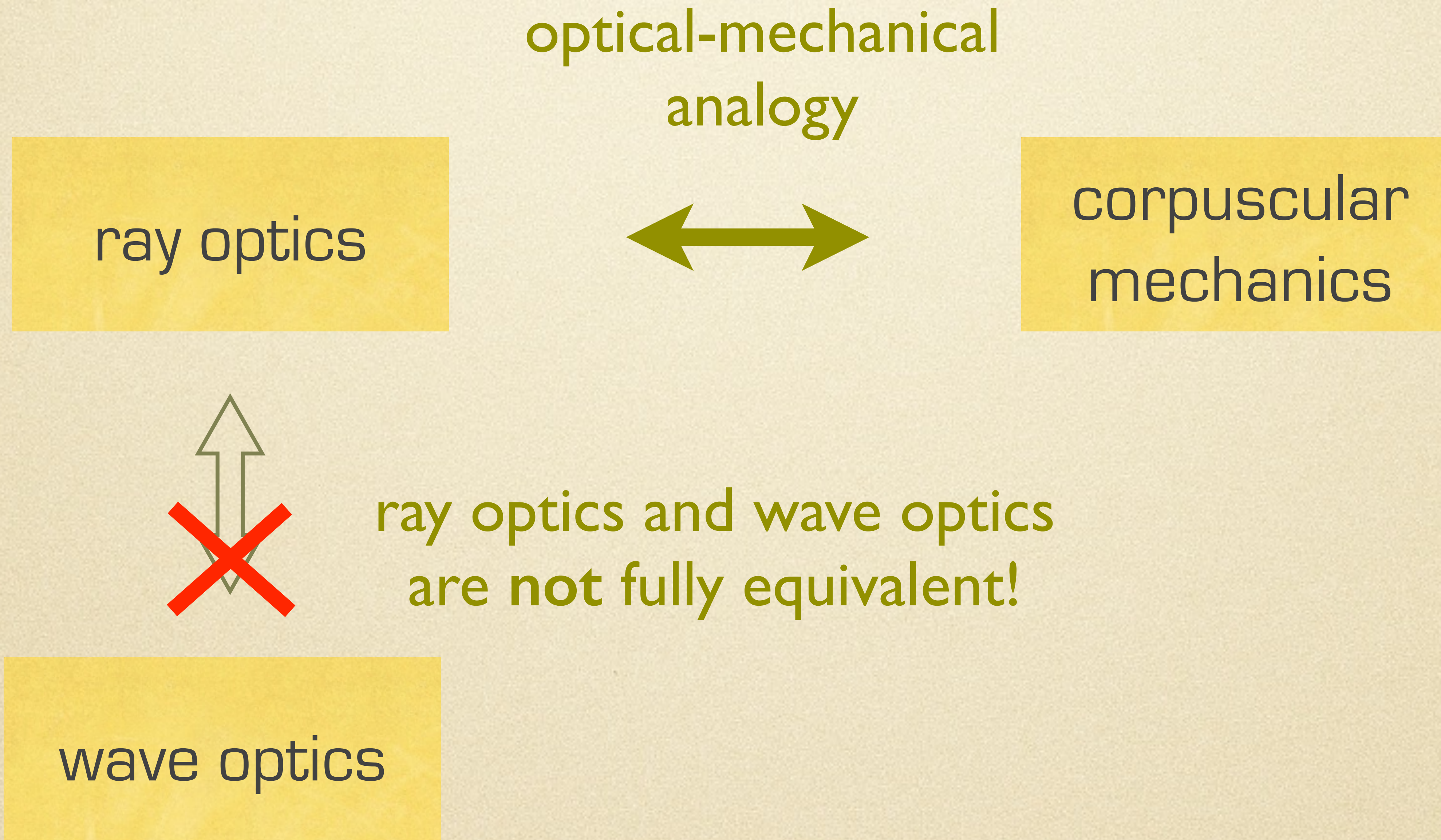
The Genesis of Wave Mechanics

Schrödinger's Completion of Hamilton's Analogy



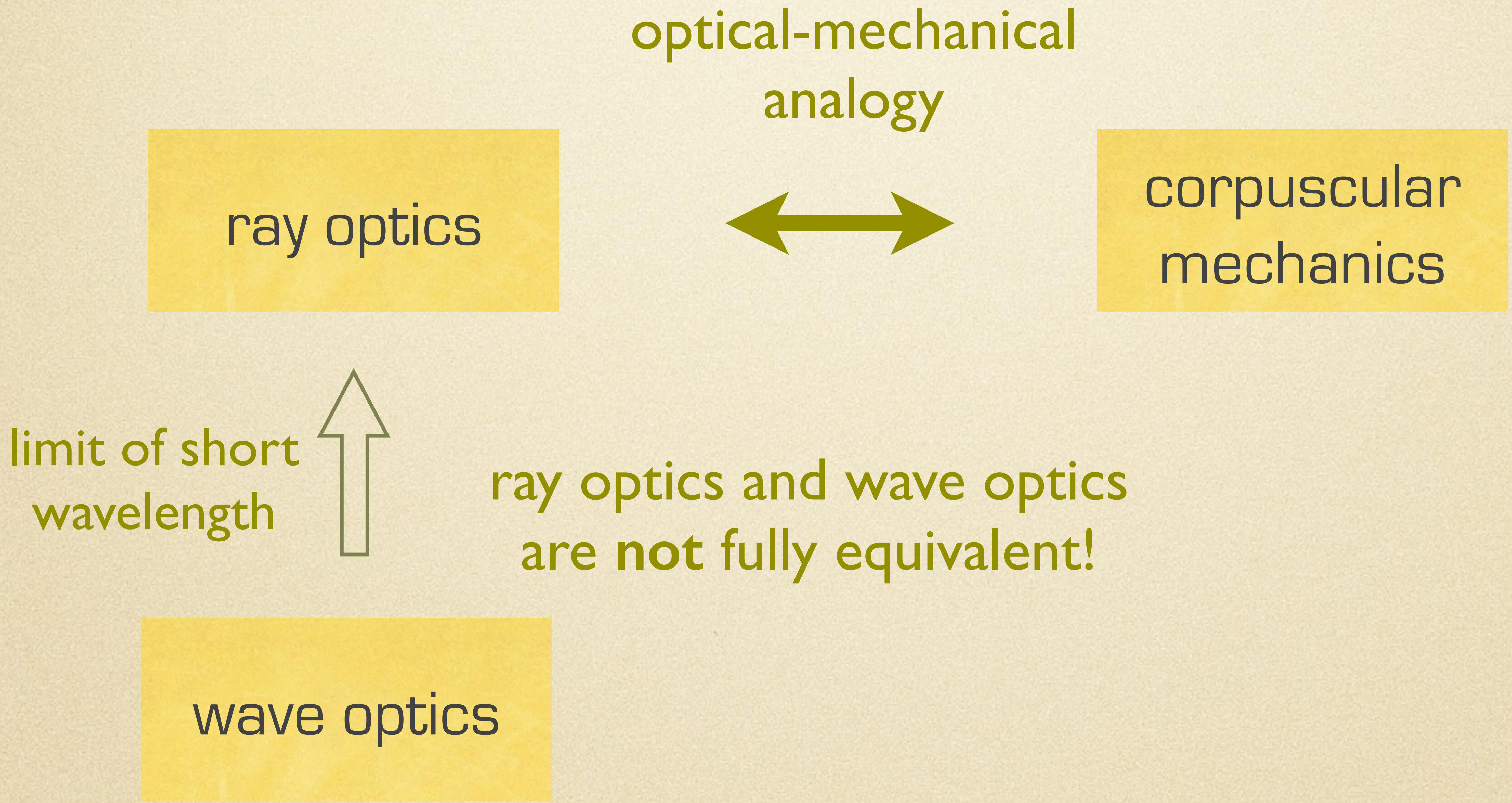
The genesis of wave mechanics

Schrödinger’s completion of Hamilton’s analogy



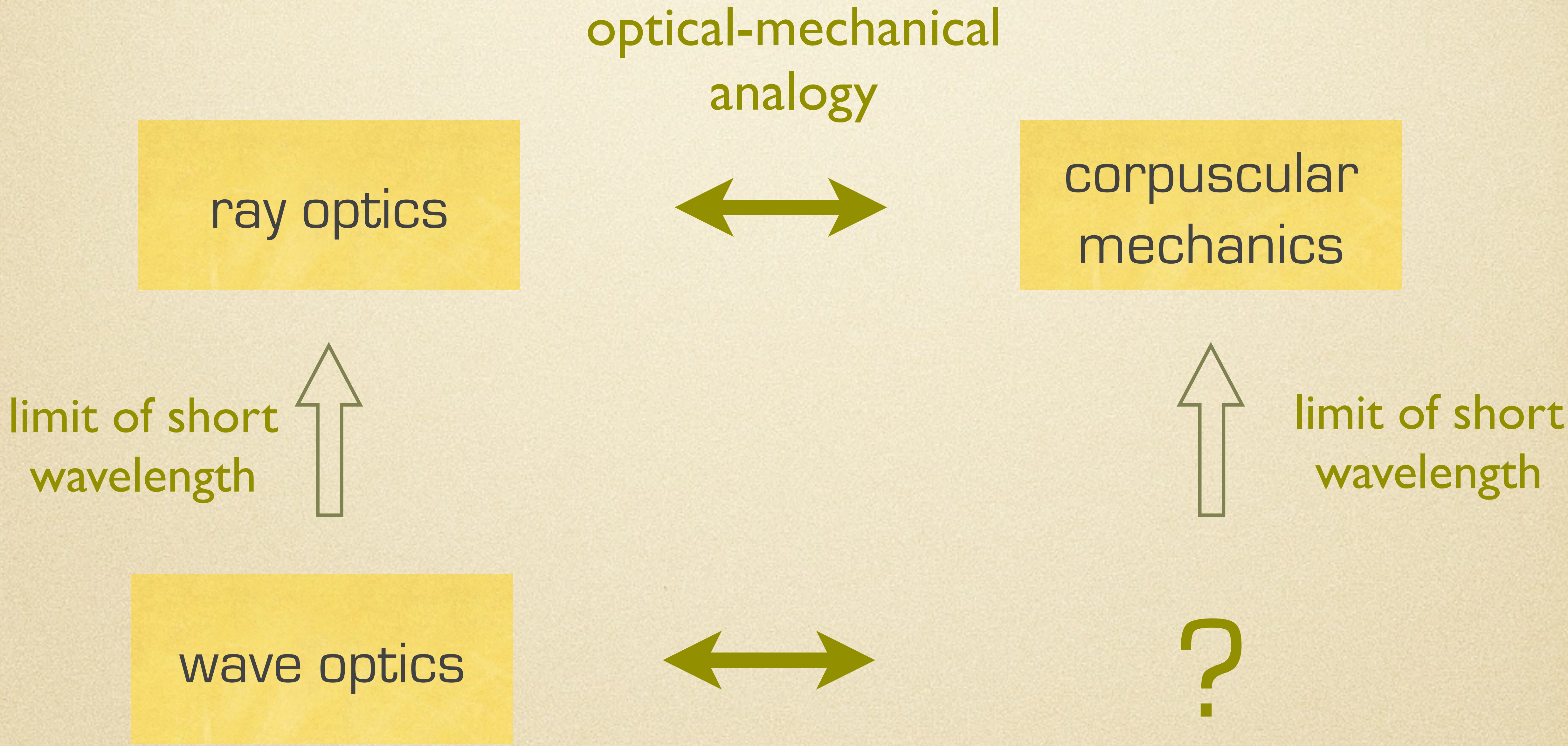
The genesis of wave mechanics

Schrödinger's completion of Hamilton's analogy



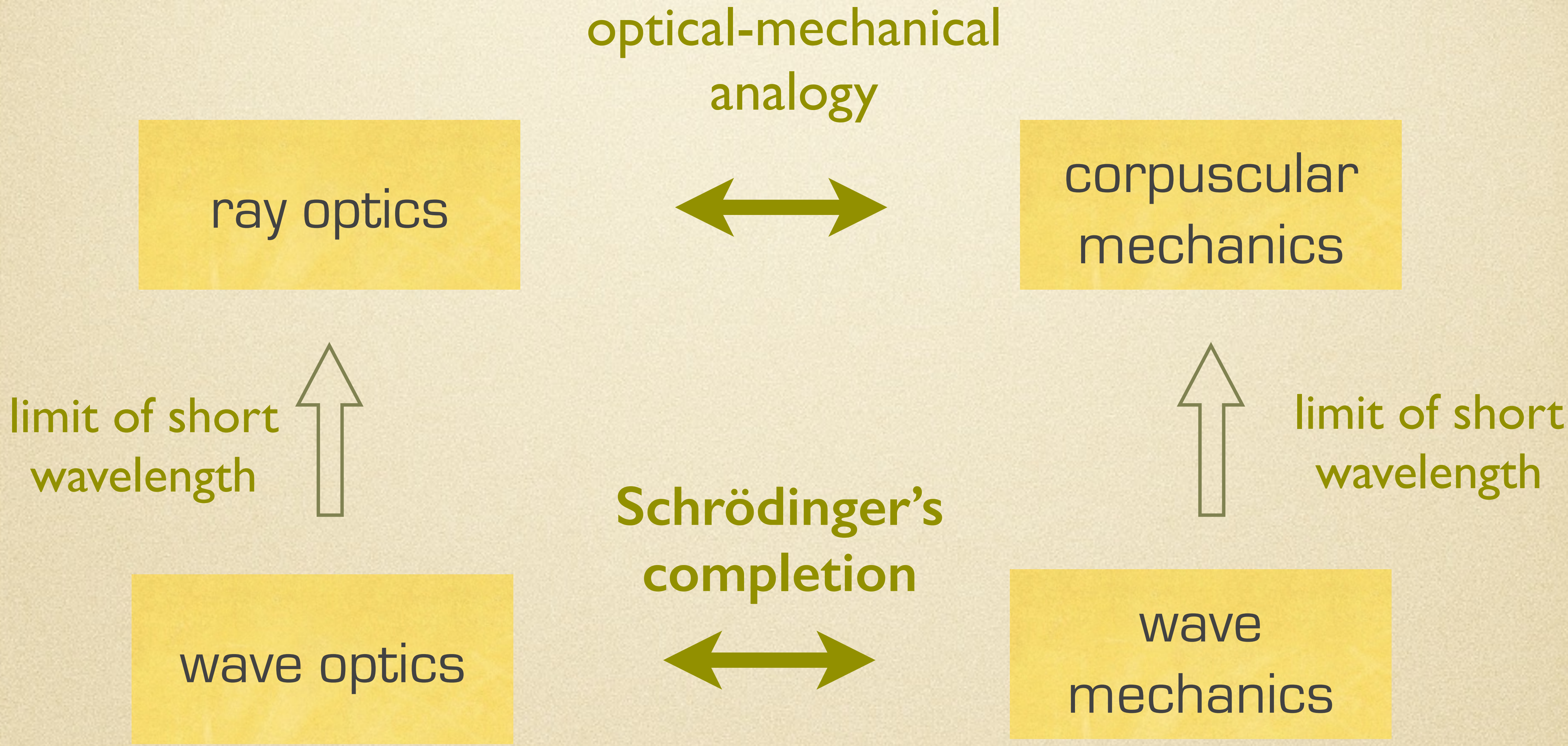
The genesis of wave mechanics

Schrödinger’s completion of Hamilton’s analogy



The genesis of wave mechanics

Schrödinger's completion of Hamilton's analogy

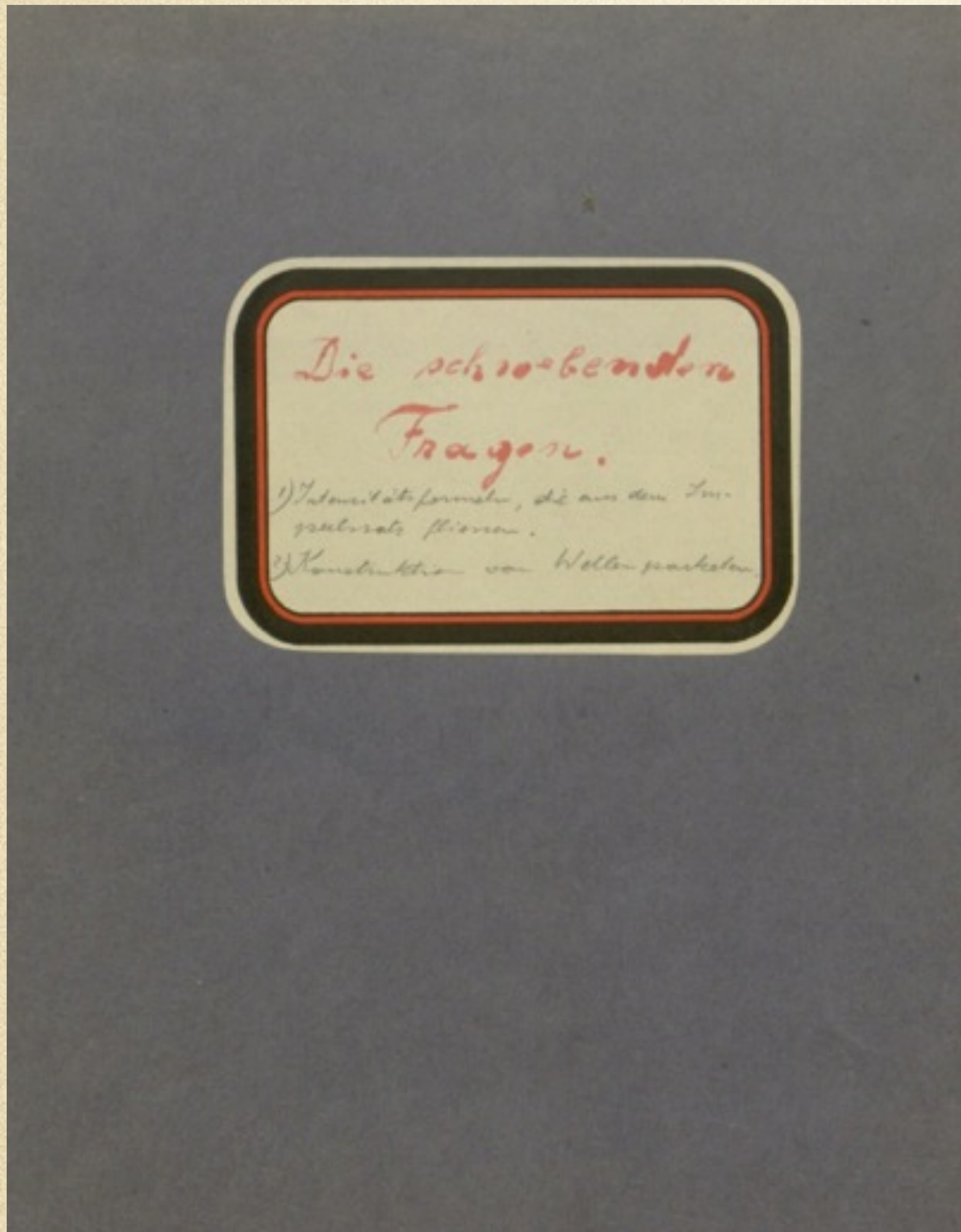


Corpuscular mechanics is merely a limiting case of a more general wave mechanics!

Part III:

The ”pending questions“

Extension and Interpretation
The Pending Questions



AHQP 41-2-002 [begun in early 1926]

In the spring of 1926, Schrödinger identifies the following "**pending questions**" in his research program:

- **relation** between wave mechanics and matrix mechanics
- **relativistic extension** of wave mechanics
- **coupling** of matter field to electromagnetic field, unified field theory
- **interpretation** of the wavefunction
- **many-particle problem** within wave mechanics

Extension and Interpretation

- During the year 1926, Schrödinger's private and public research programs begin to **diverge**.
- While focusing on successful **applications** of the nonrelativistic wave equation in his published work, Schrödinger's notebooks show him working much more intensively on **fundamental** questions of his more general program.
- He relies on the physical reading of the completed optical-mechanical analogy and searches for a "realist" **wave interpretation** of quantum mechanics.

"Baptism of the *wave packet*" (undated)

Extension and Interpretation

Relation to Matrix Mechanics

Schrödinger initially hopes that the optical-mechanical analogy ultimately will lead to a wave mechanics that is both **more general** (relativistic) and **heuristically more productive** than matrix mechanics.

In March 1926, he publishes "On the Relation of the Heisenberg-Born-Jordan Quantum Mechanics to Mine," (first published account of **equivalence** between matrix and wave mechanics).

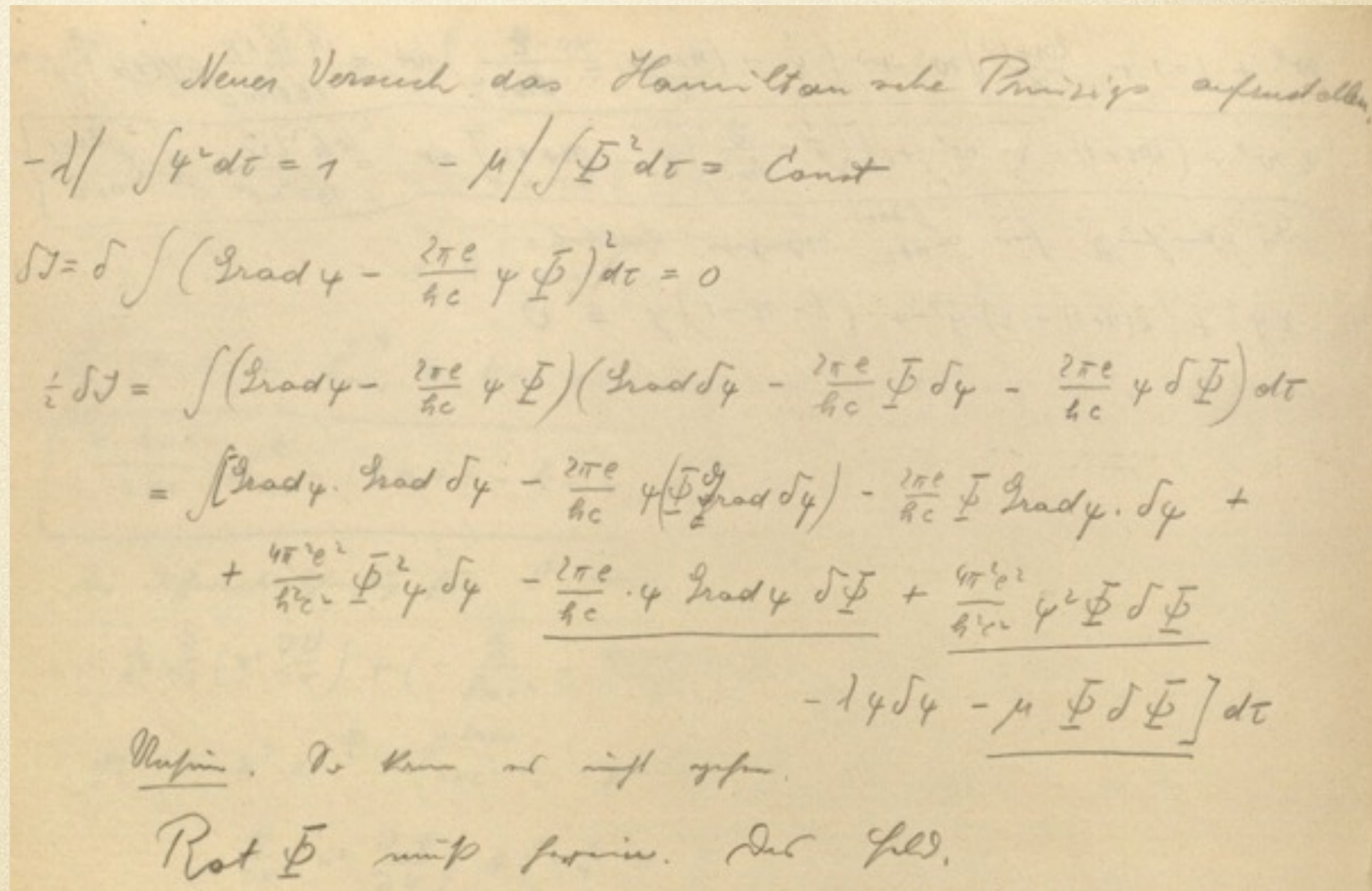
unknown date and occasion

Extension and Interpretation

Interaction of Matter Field and Electromagnetic Field

Schrödinger keeps searching for the **relativistic generalization** of the wave equation. His attempts get ever more desperate.

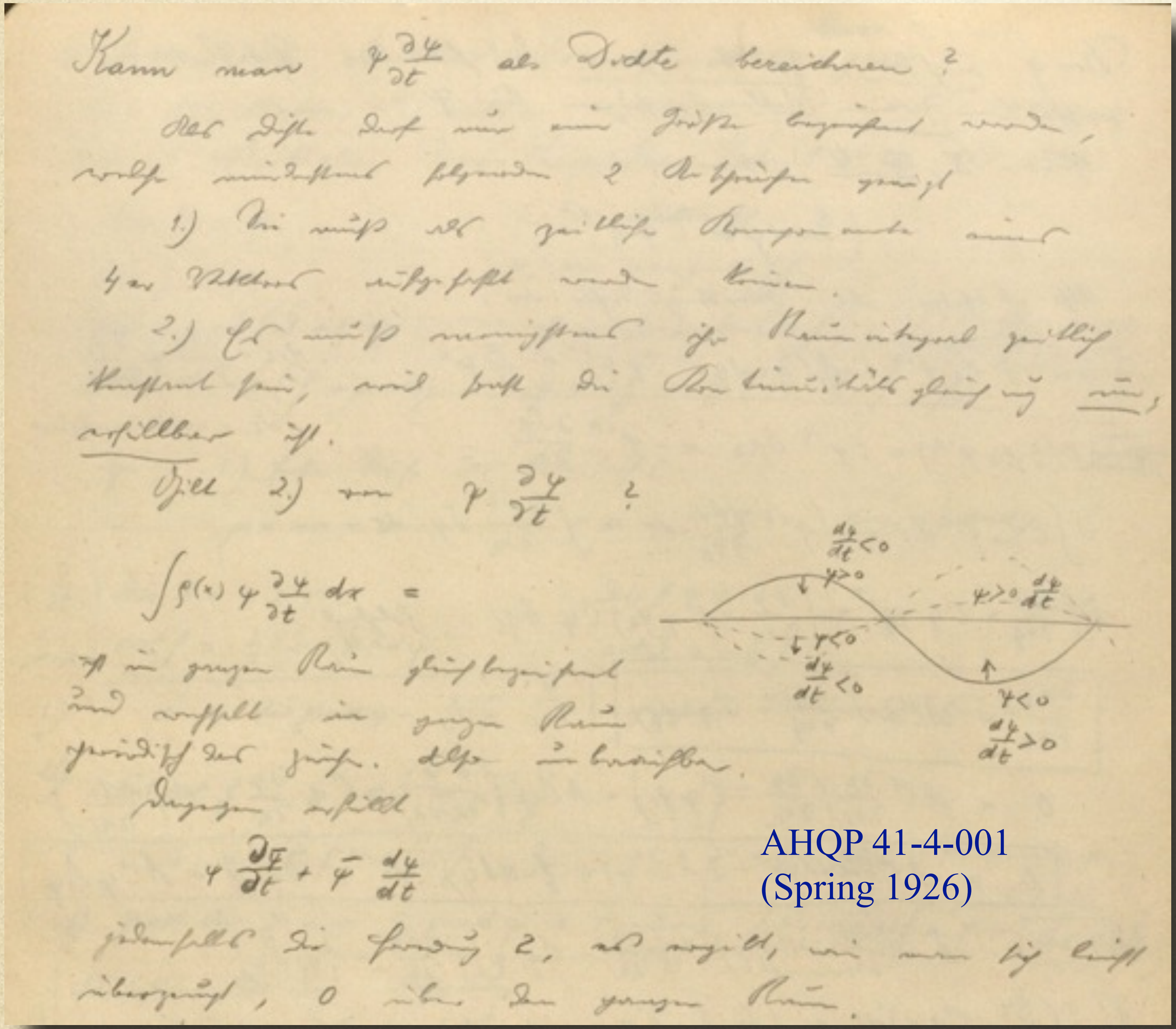
A central question becomes the problem of formulating the **coupling** of the matter field to the electromagnetic field, e.g., to explain the Zeeman effect.



"New attempt to formulate Hamilton's principle" [including the interaction between the matter field and the electromagnetic field]

Extension and Interpretation

Interpretation of the Wave Function

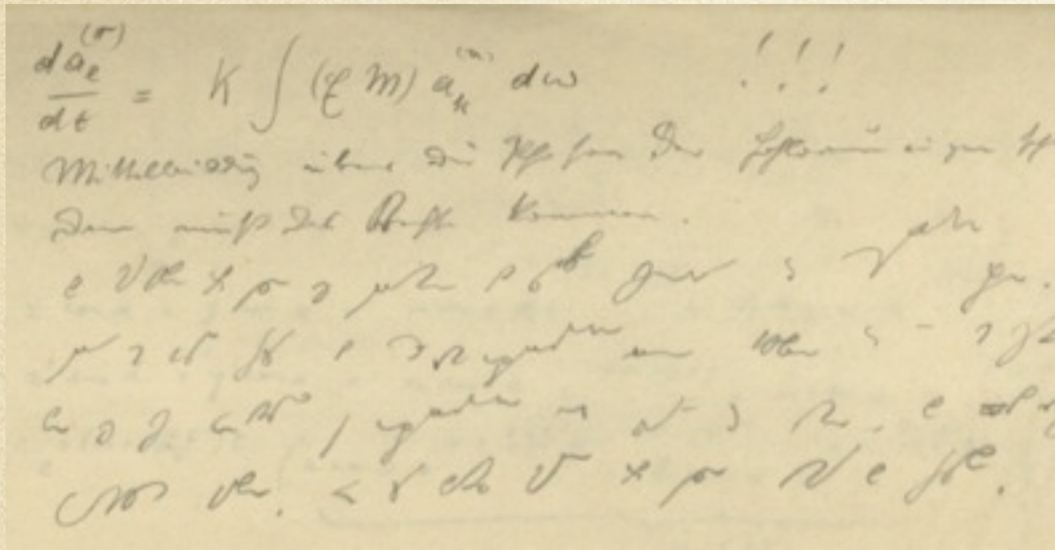


The problem of the coupling leads Schrödinger to ponder the question of the **physical meaning** of the wave function.

The notebooks show numerous attempts at deriving a **charge-current density** from the wave function, interpreting it as a physical wave.

In the meantime, Born proposes his **probabilistic interpretation** of the wave function. Schrödinger objects vehemently—the **interpretation debate** begins.

Interpretation of the Wave Function



AHQP 41-1-002
(after Nov. 1926)

pack.18, n.4 (Einlageblatt)
Stenogrammübertragung

Das Mittelbilden über die Phasen muß vollkommen die statistische Schweinerei aus Göttingen ersetzen.

Vielleicht am besten zuerst die Hohlraumoszillatoren allein behandeln und ganz schwache Wechselwirkung zu ihnen annehmen,

dann muß schon wenigstens für die Oszillatoren einer Sorte heraus= kommen, daß die Amplitudenquadrate eine geometrische

Progression bilden. Aber erst durch das Mitteln über die Phasen kommt das zustande.

A·D· Jan. 1981

Shorthand transcription:
"Averaging over the phases has to replace completely the Göttingen obscenity."

Extension and Interpretation

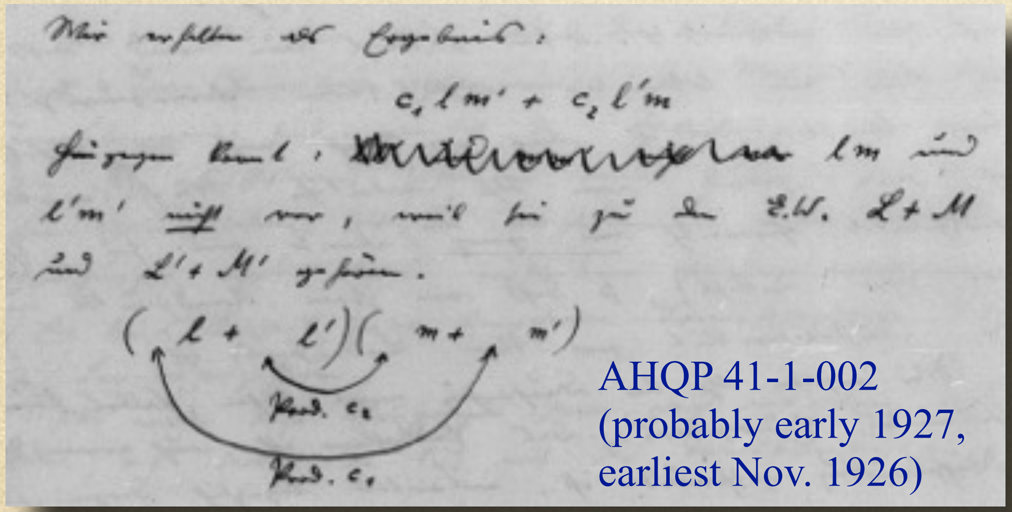
Many-Particle Problem

Schrödinger has one key problem with his interpretation of the wave function as a physical wave: How to make sense of its definition in **configuration space**?

In the case of N particles, what is the relation between his 3N-dimensional wavefunction and a physical wave in 3 dimensions?

By mid-1926, Schrödinger attacks the **many-particle problem**, studying coupled oscillators within wave mechanics, in a hope to find a **real-space interpretation** of the many-particle wavefunction. He discusses coupled systems with Heitler and London, directly before their paper on the covalent bond (beginning of **quantum chemistry**).

In notebooks on coupled systems, Schrödinger by early 1927 has first premonition of quantum-mechanical **entanglement**.



AHQP 41-1-002 (probably early 1927, earliest Nov. 1926)

"The combined oscillatory state, which is immediately realized upon interaction, [...] is such that it cannot be resolved anymore into the states of the component systems."

Conclusion

- Success of explaining “unnatural” Bose-Einstein statistics of particles as Boltzmann statistics of waves convinced Schrödinger of **wave nature of matter**.
- **First communication.** Using Hamilton‘s analogy as a **heuristic** device, he derives nonrelativistic wave equation **explaining** quantum conditions of old quantum theory.
- **Second communication.** Only now, Schrödinger understood full impact of his new wave mechanics: Classical mechanics merely **limiting case** to new wave mechanics.
- Schrödinger found his own **“sharpened”** correspondence principle: the optical-mechanical analogy relates classical physics to quantum theory.
- **Classical knowledge** (Hamiltonian mechanics, Hamilton’s analogy) **crucial** for the discovery of wave mechanics.
- For Schrödinger, completed analogy crucial also for its **interpretation**: His **physical reading** of the analogy explains why attempts to reduce the new mechanics to a particle ontology must have appeared **absurd** to him.
- **Bottom line:** Even in a fundamental change of the scientific world picture, it is insufficient to describe the theoretical development as a **wholesale replacement** of one conceptual system by another. Rather, one needs to pay close attention to the intricate process of **transformation and reinterpretation** of previously accepted knowledge.