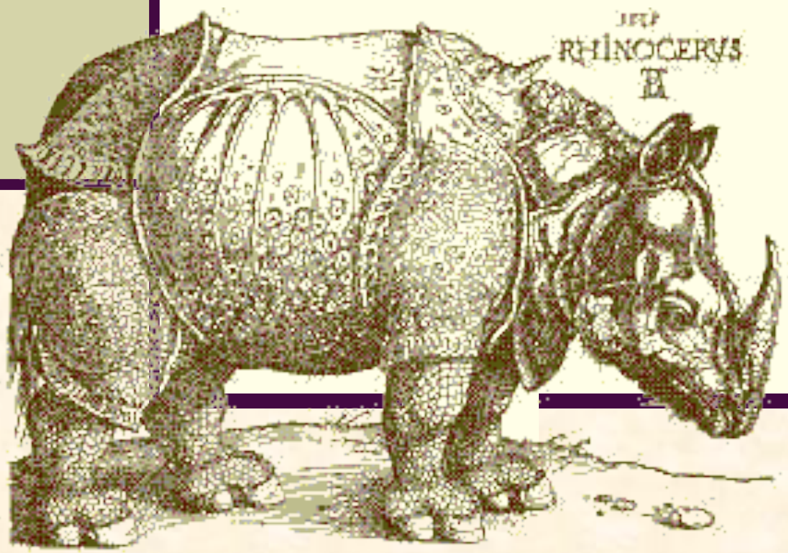
Louis de Broglie’s struggle with the wave-particle dualism, 1923-1925

Roberto de Andrade Martins

Group of History and Theory of Science
Unicamp, Brazil



Early influences

Louis de Broglie's motivation to deal with the wave-particle dilemma came from two sources.

- His study of Einstein's 1905 and 1909 papers on the nature of light
- His involvement with X rays and their properties

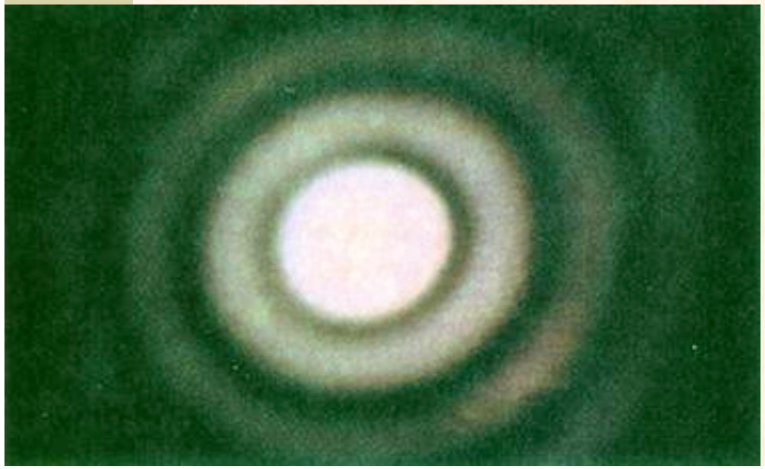
Einstein

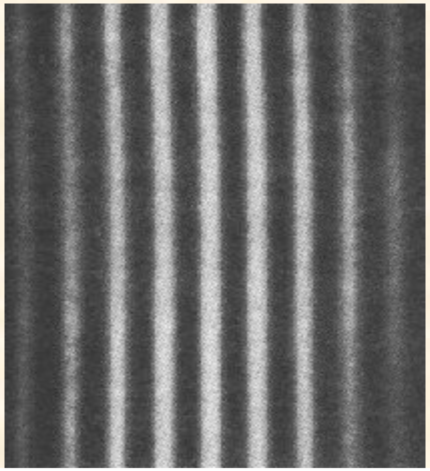
Contrary to widespread lore, Einstein did not propose a wave-particle theory of light in the first decade of the 20th century.

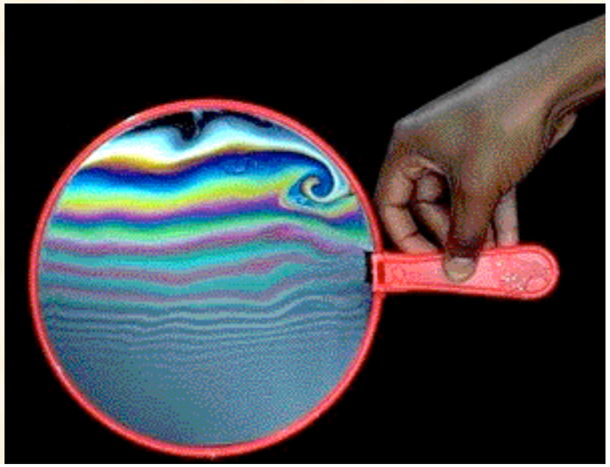
In his 1905 paper he used a corpuscular hypothesis, grounded upon Wien’s law of blackbody radiation, to explain some phenomena of interaction between matter and radiation.

Einstein (1905)

Einstein did not (and could not) attempt to account for the typical wave properties (interference, diffraction, polarization, etc.) using his hypothesis of independent point-like energy quanta.



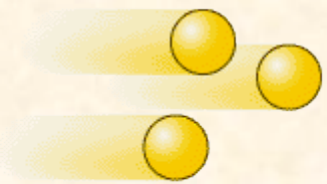




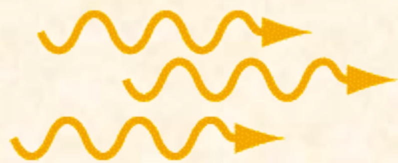
Einstein (1909)

In his 1909 papers, Einstein used Planck's law of blackbody radiation (instead of Wien's), and analysed the energy and momentum fluctuations.

He obtained equations with two distinct terms, which he interpreted as due to corpuscular and wave properties of light.

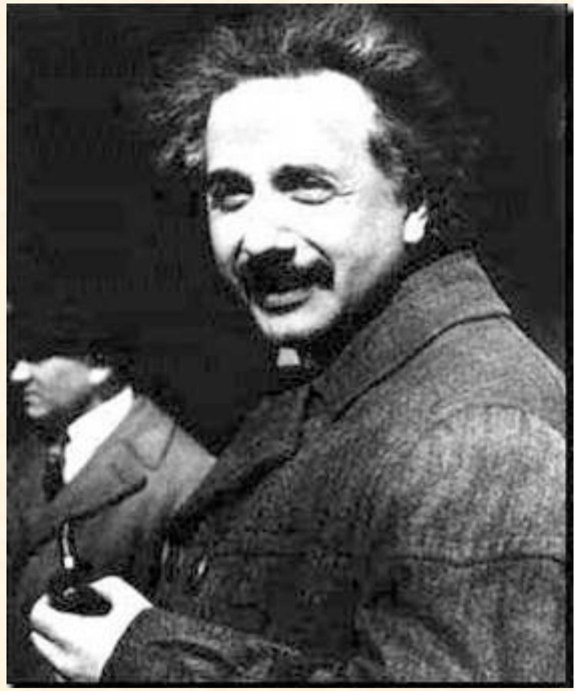


$$\mathring{a}^2 = \frac{R}{Nk}\left\{ \text{í} h \text{ç}_0 + \frac{1}{8\partial}\frac{c^3}{\text{í}^2 d\text{í}}\frac{\text{ç}_0^2}{V}\right\}$$



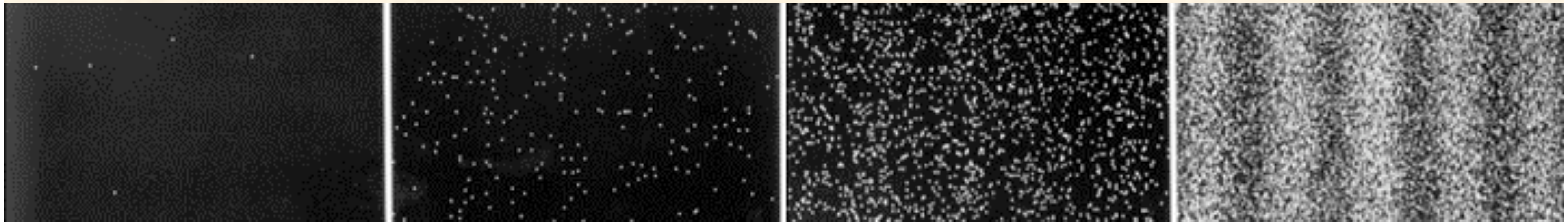
Einstein (1909)

- He suggested the necessity of developing a theory that included both features of radiation, and sketched a proposal where the wave properties would be a collective feature of a large number of interacting quanta.
- A single quantum, however, would have no wave properties.



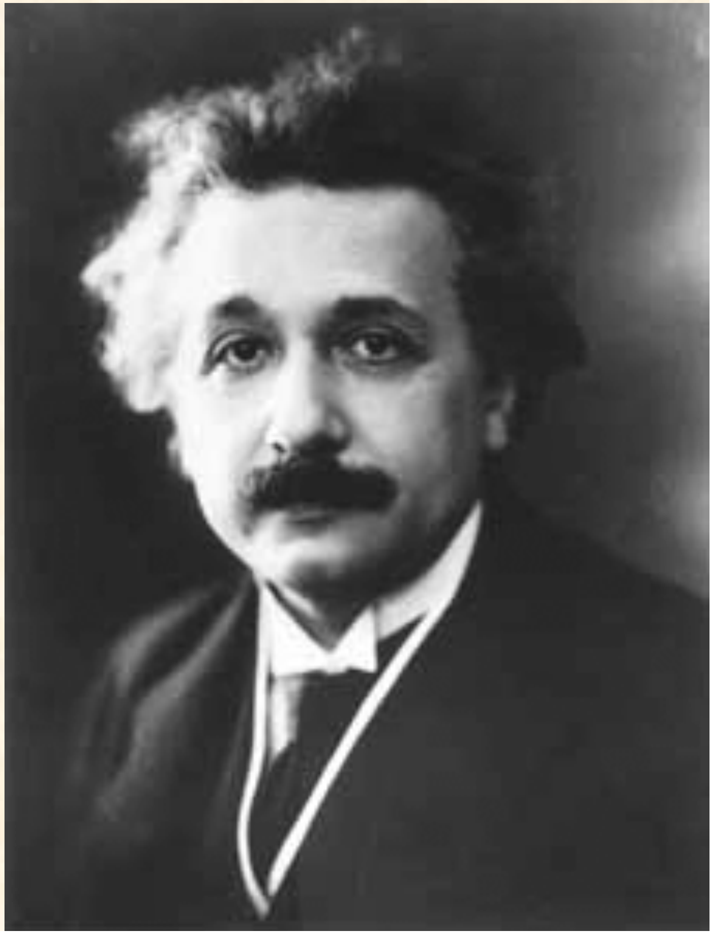
Taylor (1909)

- According to Einstein's proposal, light of very low intensity would not exhibit the usual interference phenomena.
- Taylor's experiments, published in that same year, showed that such a view was untenable.



Einstein’s quanta

- It is well known that Einstein's quantum theory of light, regarding quanta as energy atoms, was rejected by most physicists during the two first decades of the 20th century and was not very influential.

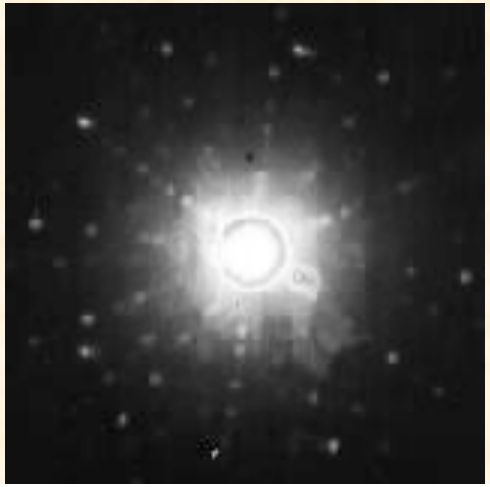


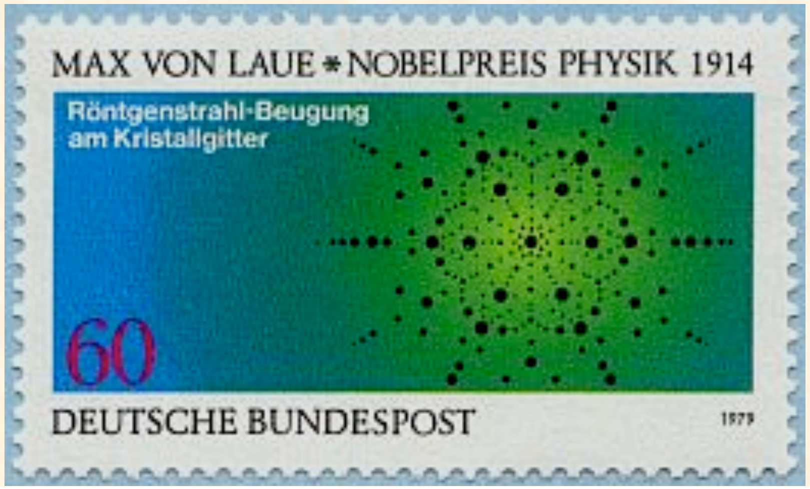
X-rays and duality

- Independently of Einstein's ideas, the physicists who dealt with X rays – such as Maurice and Louis de Broglie – had a clear perception that radiation had both wave and particle properties.

X-rays and duality

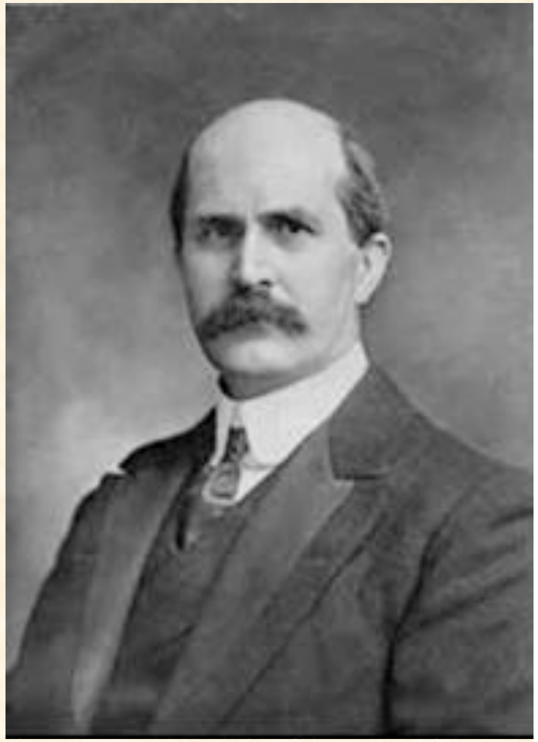
- Diffraction of X rays in crystals had shown their wave behaviour.
- Absorption and emission, on the other hand, exhibited a corpuscular behaviour (point-like energy concentration).





X-rays and duality

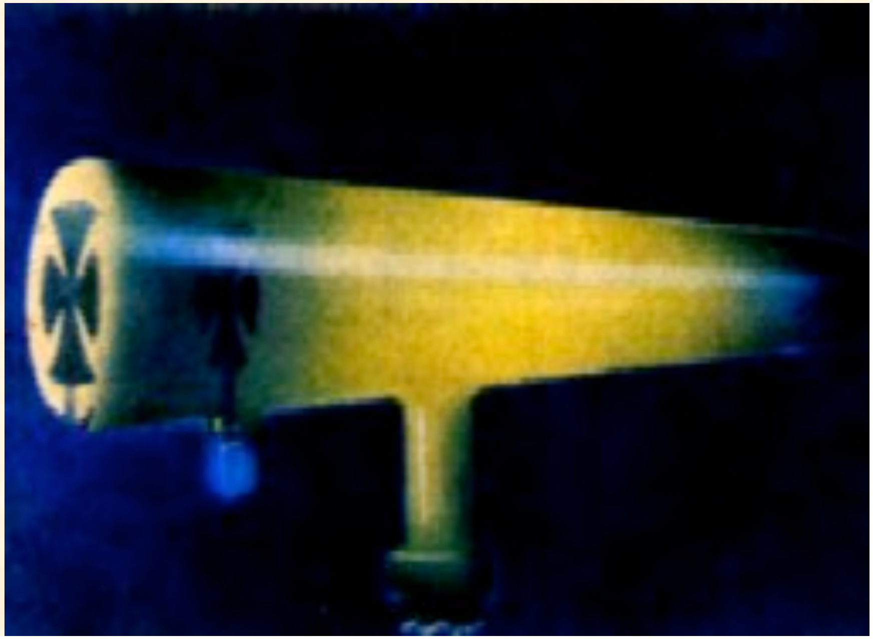
- There was no *theory*, however, that could account for both corpuscular and wave properties.
- William Bragg commented that on Monday, Wednesday and Friday one had to use one of the hypotheses and on Tuesday, Thursday and Saturday the other one.



X-rays and duality

- That was the situation around 1920:

Physicists involved in the study of X rays knew that a synthesis was necessary, but such a theory was not available.



De Broglie (1922)

- Louis de Broglie’s attempted to deal with the wave-particle dilemma in 1922, when he published his first theoretical analysis of light.
- He used a corpuscular hypothesis, together with the conjecture that the “atoms” of light, with energy $E=h\nu$, could join to produce “light molecules” with energy $E=nh\nu$.

De Broglie (1922)

- If the light molecules had *n* atoms, the energy of each molecule would be **nhν**, and it should obey the following energy density equation:

$$du/d\nu = 8\pi\frac{h}{c^3}\nu^3 e^{-\frac{nh\nu}{kT}}$$

De Broglie (1922)

- For a mixture of light molecules with all possible numbers of atoms, the equation of the total energy density would be:

$$du/d\nu = 8\pi \frac{h}{c^3} \nu^3 \left[e^{-\frac{h\nu}{kT}} + e^{-\frac{2h\nu}{kT}} + e^{-\frac{3h\nu}{kT}} + \ldots\ldots \right]$$

De Broglie (1922)

$$du/d\nu = 8\pi \frac{h}{c^3} \nu^3 \sum_n e^{-\frac{n.h\nu}{kT}}$$

- This sum can also be written in a different way:

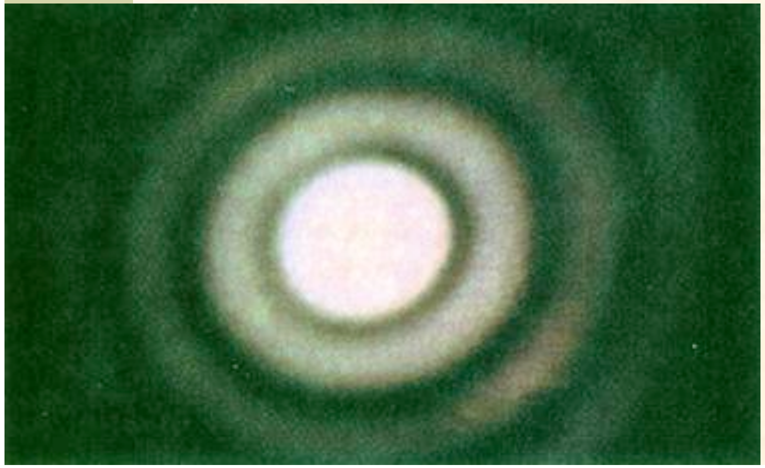
$$du/d\nu = 8\pi \frac{h}{c^3} \nu^3 \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

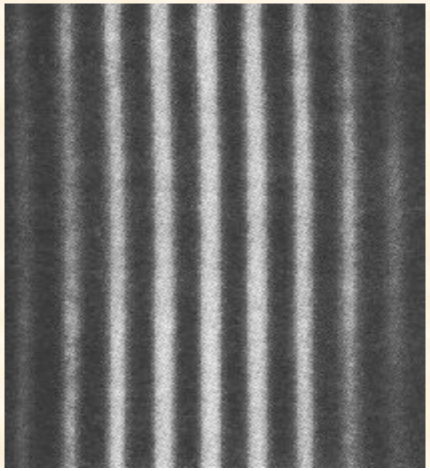
De Broglie (1922)

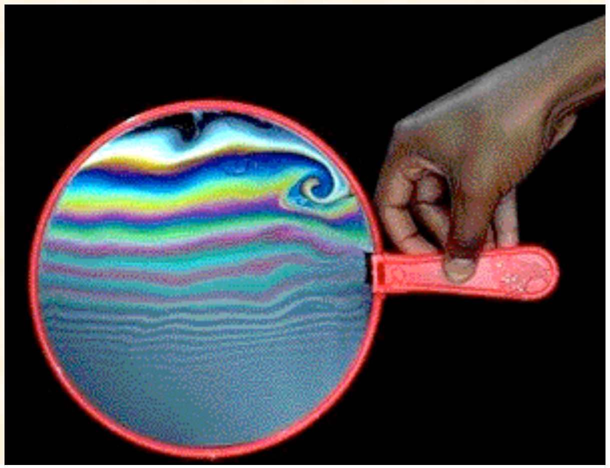
- Using those assumptions he was able to derive Planck's law of blackbody radiation.
- According to his interpretation, this law did not contain both corpuscular and wave features, as interpreted by Einstein.
- It was possible to explain it fully without assuming any wavelike property of light.

De Broglie (1922)

- However, it was not obvious that those ideas could be applied to explain the classical wave phenomena of radiation.
- De Broglie’s analysis of light did not awake strong interest.







De Broglie (1922)

- The proposal had, however, one feature that was crucial in the development of his wave-particle theory, next year.
- De Broglie’s atoms of light were relativist particles, with a non-null rest mass and speed smaller than *c*.

De Broglie (1922)

- De Broglie called *c* the "limiting speed" of the theory of relativity, since it was not equal to the speed of light in the vacuum.
- The rest mass of the light atoms was chosen as a very small one (about 10^{-50} g) so that it would not introduce any conflict with the observed properties of light.

De Broglie (1922)

- In his 1922 papers, de Broglie applied relativistic mechanics to compute the dynamical properties of the light quanta (mass, momentum, energy).
- In particular, he applied the relation $E=mc^2$ to the light atoms.

De Broglie (1922)

- He also associated a *frequency* to the light atoms and molecules, otherwise it would be impossible to apply $E=h\nu$.
- However, he did not attempt to explain the meaning of this magnitude, as applied to his relativistic particles.

Stark (1907)

- It was not usual to associate a frequency to electrons.
- In 1907 Johannes Stark had applied both $E=mc^2$ and $E=h\nu$ to electrons, and was led to ascribe a frequency to them (he conjectured it might be a rotation frequency).
- However, he made no further use of this idea, and it was soon forgotten.

From 1922 to 1923

- Between the 1922 papers on light molecules and his first 1923 paper on the wave-particle proposal, de Broglie did not publish any other theoretical paper.
- The next slides show a reconstruction of the path that led him to his theory.

Quantum and relativity

- Starting from those early papers, de Broglie began to think about light quanta and other particles (such as electrons) in a unified way.
- Both obeyed the theory of relativity and – so he thought – both should also obey Planck's relation $E=h\nu$.

Quantum and relativity

- In the rest frame of the particle, one should have $E_0 = m_0 c^2 = h\nu_0$, of course.
- In other reference systems, the correct equation should be $E = mc^2 = h\nu$.
- Was that a valid relativistic relation?

Quantum *versus* relativity

- No, it did not seem correct.
- Indeed, according to special relativity the mass and energy of a particle should increase with its speed, but the frequency associated to the particle should decrease.

$$m = \frac{m_0}{\sqrt{1-\hat{a}^2}} \qquad \acute{\imath}' = 1/T = \acute{\imath}_0\sqrt{1-\hat{a}^2}$$

Quantum *versus* relativity

$$m = \frac{m_0}{\sqrt{1-\hat{a}^2}} \qquad \acute{\imath}' = 1/T = \acute{\imath}_0\sqrt{1-\hat{a}^2}$$

- From de Broglie's later accounts we know that he struggled for some time with his difficulty.
- After a few months he found a solution to this paradox by introducing a wave associated to the pulsating particle.

Quantum and relativity

$$í = \frac{m_0 c^2}{h\sqrt{1-\hat{a}^2}} \qquad í' = 1/T = í_0\sqrt{1-\hat{a}^2}$$

- The frequency ν of the wave would vary as the energy of the particle.
- The frequency ν’ of the particle would obey the other equation.

Quantum and relativity

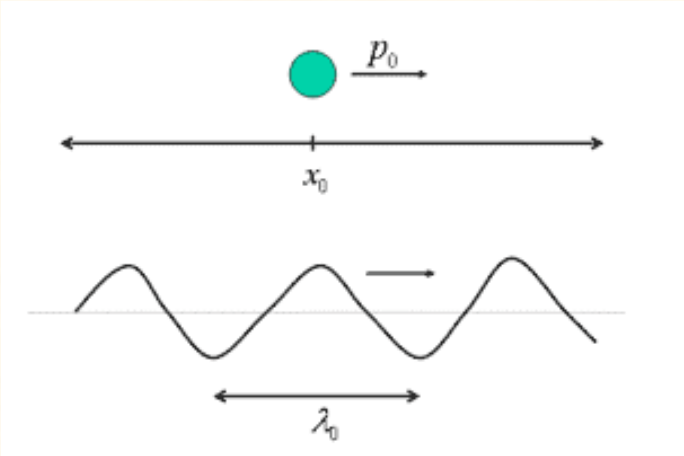
$$\text{í} = \frac{m_0 c^2}{h\sqrt{1-\hat{a}^2}} \qquad \text{í}' = 1/T = \text{í}_0\sqrt{1-\hat{a}^2}$$

- Using this strategic move, de Broglie was able to satisfy equation $E=mc^2=h\nu$ for all reference systems, interpreting ν as the frequency of the wave, not of the particle.
- De Broglie kept the idea of a pulsating particle, and assumed that the wave should have the same phase as the particle.

Quantum and relativity

- Using those assumptions he was able to prove that the wave could not be moving together with the particle.
- The speed of the wave *U* should be related to the speed *v* of the particle according to $U=c^2/v$.
- Since *v* is smaller than *c*, the speed *U* of the wave is always greater than *c*.

The speed of the wave



$$U = \frac{c^2}{v} > c$$

- This introduced a new problem of interpretation, as velocities greater than *c* were incompatible with the theory of relativity.
- However, de Broglie soon perceived how this problem could be solved.

The speed of the wave

- Marcel Brillouin (the father of de Broglie's friend Léon Brillouin) had studied electromagnetic waves that could travel in matter with speeds greater than *c*.

The speed of the wave

- Brillouin proved that the speed of energy transportation of those waves was equal to the group velocity of the waves, and that the group velocity was smaller than *c*.

Wave groups

- Applying the same idea, de Broglie introduced the idea of wave groups, and proved that the group velocity *V* (and energy velocity) was equal to the velocity of the particle.
- Besides that, the wave group also allowed de Broglie to ascribe a localisation to the electron.

$$\frac{1}{V} = \frac{d\left(\frac{\nu}{U}\right)}{d\nu} = \frac{1}{v}$$

Wave groups?

$$\text{í} = \frac{m_0 c^2}{h\sqrt{1-\hat{a}^2}}$$

- This introduced a new conceptual problem.
- In a classical framework, to each electron should be assigned a well-defined speed, and therefore a well-defined energy and frequency.
- It was difficult to understand what meaning could be ascribed to a wave group, with a set of similar but different frequencies

Wave groups?

$$\Delta p \, \Delta x \geq \frac{1}{2} \hbar$$

$$\Delta E \, \Delta t \geq \frac{1}{2} \hbar$$

- Notice that this problem occurred four years before the proposal of Heisenberg's uncertainty principle, and de Broglie did not arrive to similar ideas, at that time.

The extended "particle"

- A more satisfactory view was presented one year later (1924), in de Broglie's thesis.
- In that work, he proposed a new approach to the problem, describing the electron itself as an extended system, since its electromagnetic energy is not concentrated in a point, but is spread over the whole space around the charge (with a stronger concentration around a centre).

The extended "particle"

- Notice that this concept is not equivalent to Schrödinger's later proposal of an electron with extended *charge*
- It was not altogether clear in de Broglie's thesis whether the charge itself was localised or spread around a centre.

The extended “particle”

- In the rest frame of the electron, its whole (infinite) structure was supposed to be pulsating in synchrony, with a frequency given by $h\nu_0=m_0c^2$.

$$\Psi_0 = A\,\mathrm{sen}\,2\pi(\nu_0 t_0)$$

- Relative to other reference systems, the synchrony would be lost, of course.

The extended “particle”

- Applying the Lorentz transformation to this pulsation, de Broglie easily showed that the oscillation would transform to a wave, relative to other reference frames, and obtained the speed, frequency and other properties of the wave.

$$Ø_0 = A\,\mathrm{sen}\,2∂\left[í_0 \frac{1}{\sqrt{1-â^2}}\left(t-\frac{âx}{c}\right)\right]$$

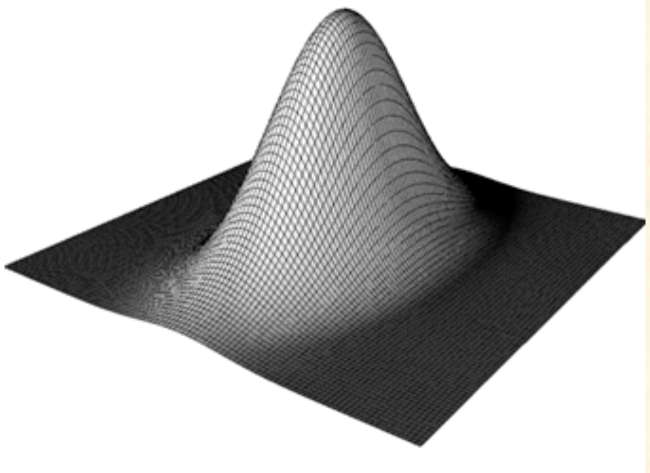
The extended “particle”

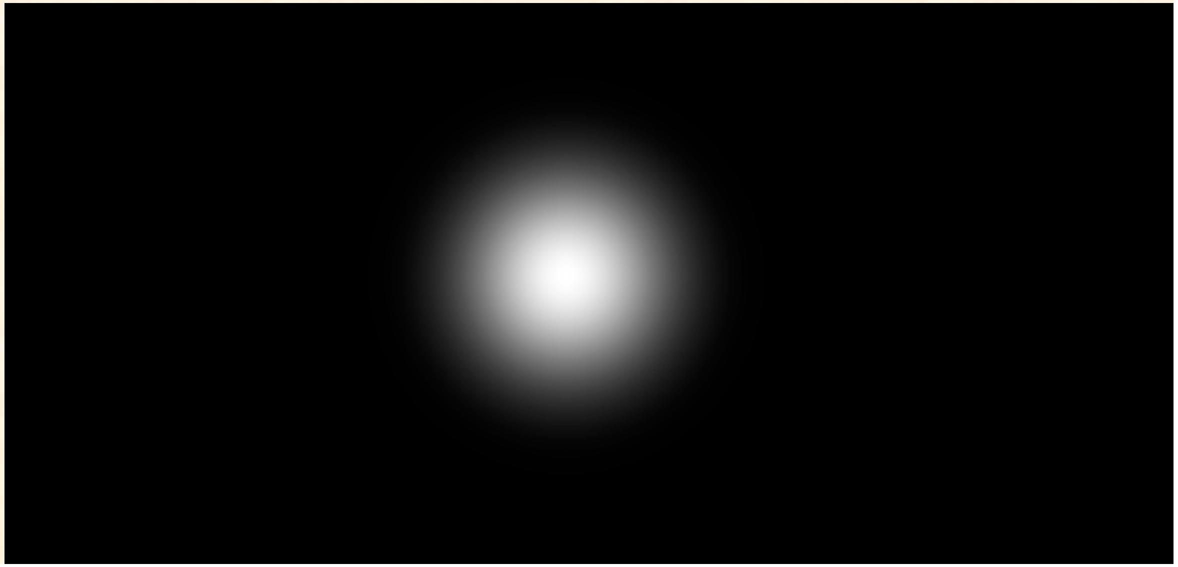
$$\text{Ø}_0 = A\,\text{sen}\,2\partial\left[\text{í}_0 \frac{1}{\sqrt{1-\hat{a}^2}}\left(t-\frac{\hat{a}x}{c}\right)\right]$$

$$\Psi_0 = A\,\text{sen}\,2\pi\left[\nu\left(t-\frac{x}{V}\right)\right]$$

The extended "particle"

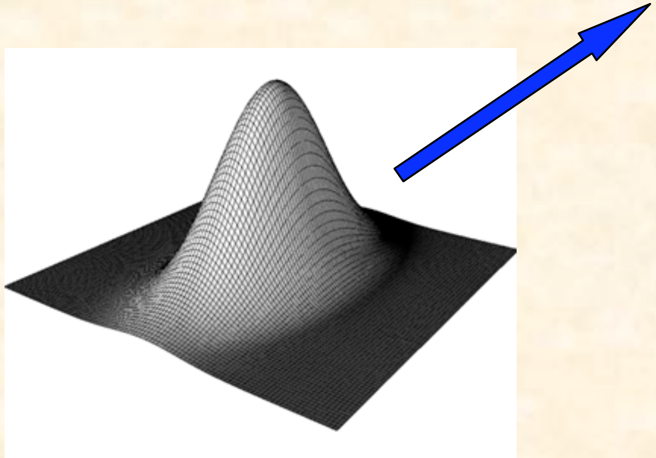
In this new approach, the electron has a strong energy concentration around a centre, and the wave associated to its pulsation will also have a strong amplitude concentration around a centre travelling with the speed of the electron.





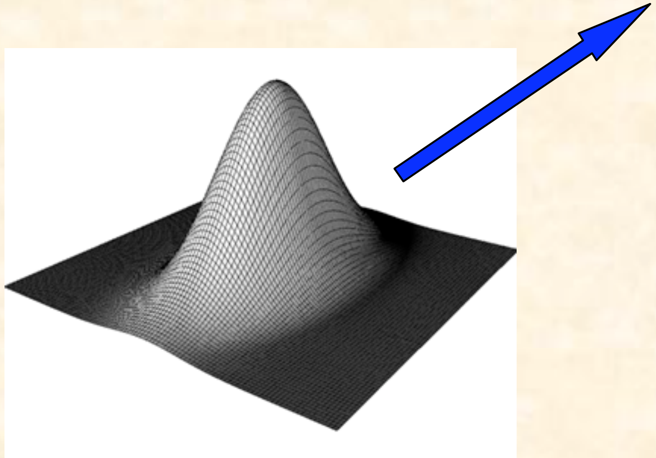
The extended "particle"

This is mathematically equivalent to a wave group, but conceptually it is quite different, because in the rest frame it does have a single, well-defined frequency, maintaining the classical (non-probabilistic) outlook that guided de Broglie's work.



The extended “particle”

$$\text{Ø} = A_{(x,y,z)}\,\text{sen}\,2\partial\left[\text{í}_{0}\frac{1}{\sqrt{1-\text{â}^{2}}}\left(t-\frac{\text{â}x}{c}\right)\right]$$



Electromagnetism?

- De Broglie was attempting to develop a theory that could describe both electromagnetic radiation (light quanta) and other particles such as electrons.
- He envisaged light quanta as energy chunks with speed smaller than *c* (but very close to *c*) associated to electromagnetic waves with speed slightly greater than *c* (of course, this required a correction of Maxwell's equations).

Electromagnetism?

- However, changing Maxwell's equations was a terrible task.
- De Broglie did not suggest how that change should be made.

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

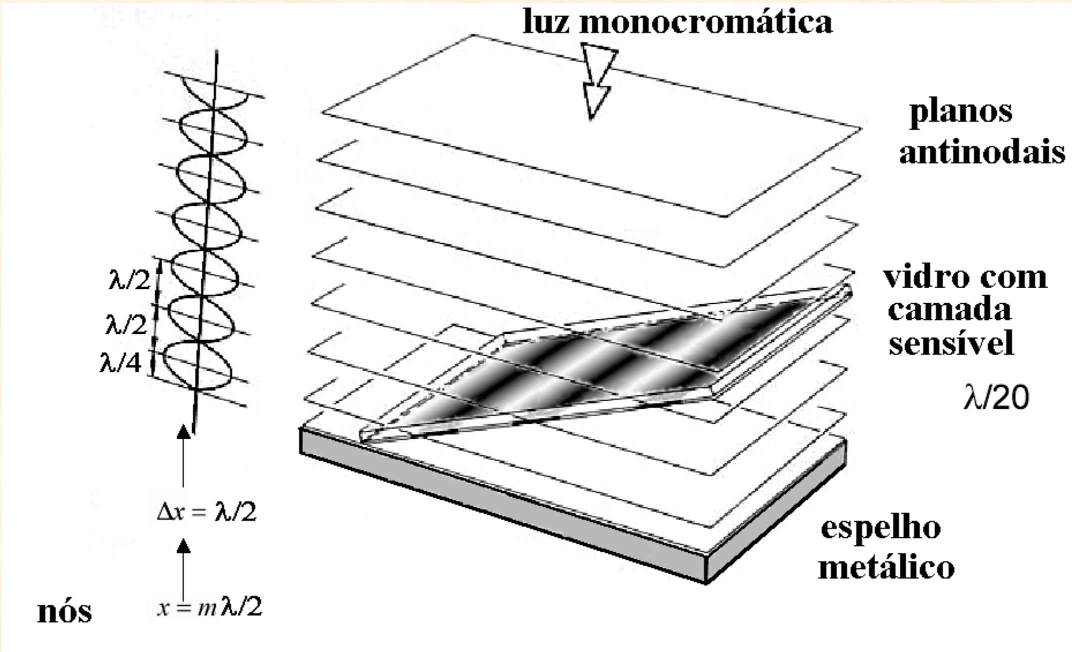
$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

Interference

- To explain interference phenomena – both in light experiments and with electrons – he introduced a probabilistic relation between waves and particles.
- The probability of absorption or emission of a light quantum (or electron) should be proportional to the intensity (square of the amplitude) of the wave at each region.

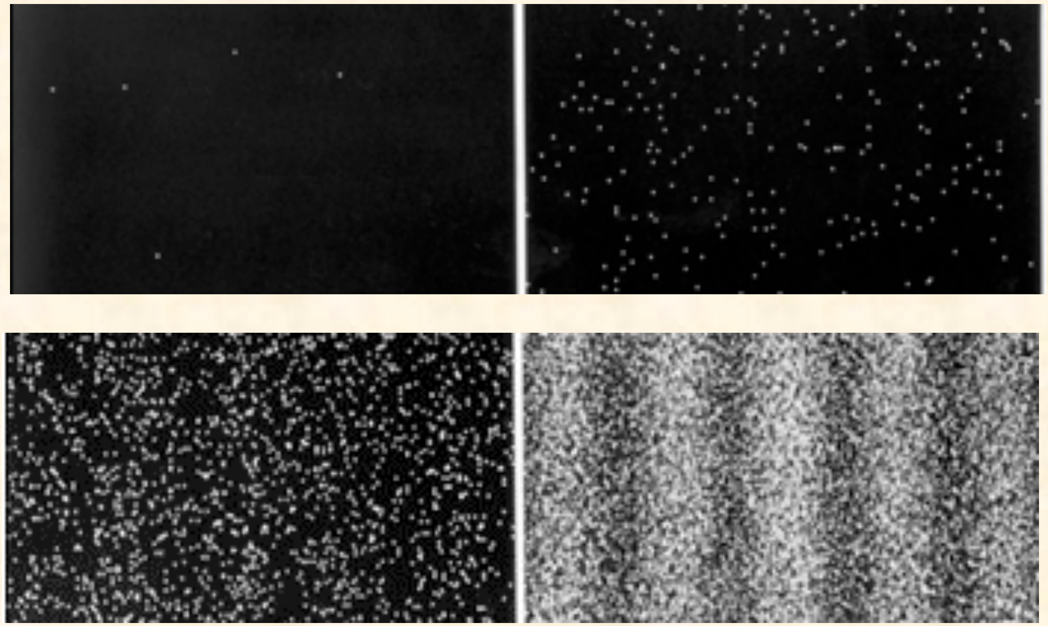
Probability

- This hypothesis was necessary to explain how the quanta could pass through photographic plates without interaction, in Otto Wiener's experiments.

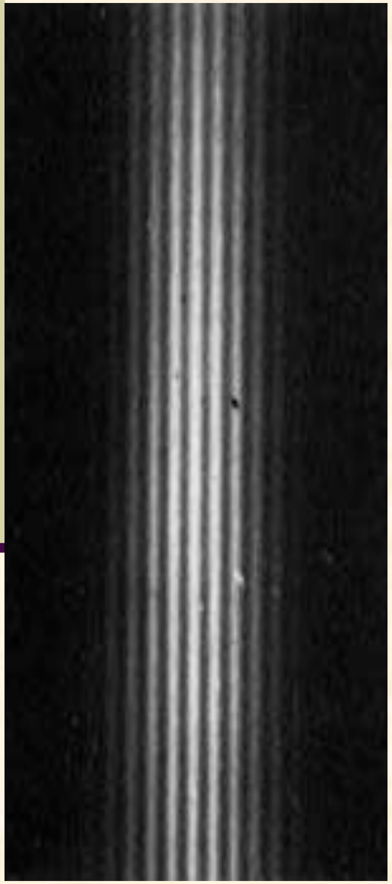


Probability

- Since each particle is associated with a wave, the theory predicted low-intensity interference effects, such as Taylor’s experiments



Probability



- The probability hypothesis was necessary to obtain an agreement between his theory and the classical optical results
- However, it could not be derived from (and was hardly compatible with) his fundamental concept of wave-particle duality.

A work in progress

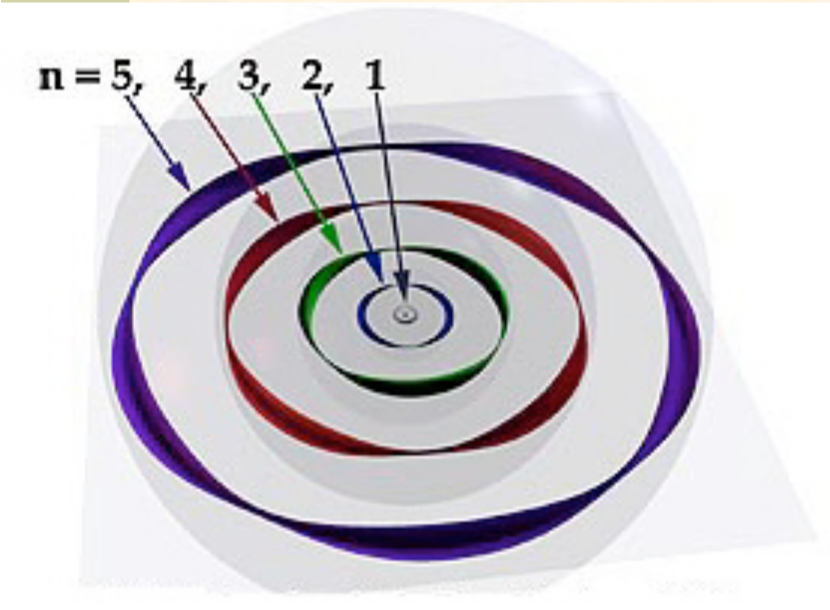
- In the papers he published before the thesis, in his thesis, and in papers published shortly after it, de Broglie was fighting against severe conceptual difficulties such as those shown here.

A work in progress

- He kept changing some of his fundamental hypotheses, maintaining only a few basic assumptions, such as relativistic dynamics and the relation $E=h\nu$.
- Instead of a coherent and final theory, his papers exhibited a changeable work in progress, with deep and unsolved conceptual problems.

A work in progress

- He obtained important results – such as the wave explanation of Bohr's atomic orbits – but the continuous changes of his theory show that he was not satisfied with its basic assumptions.



A work in progress

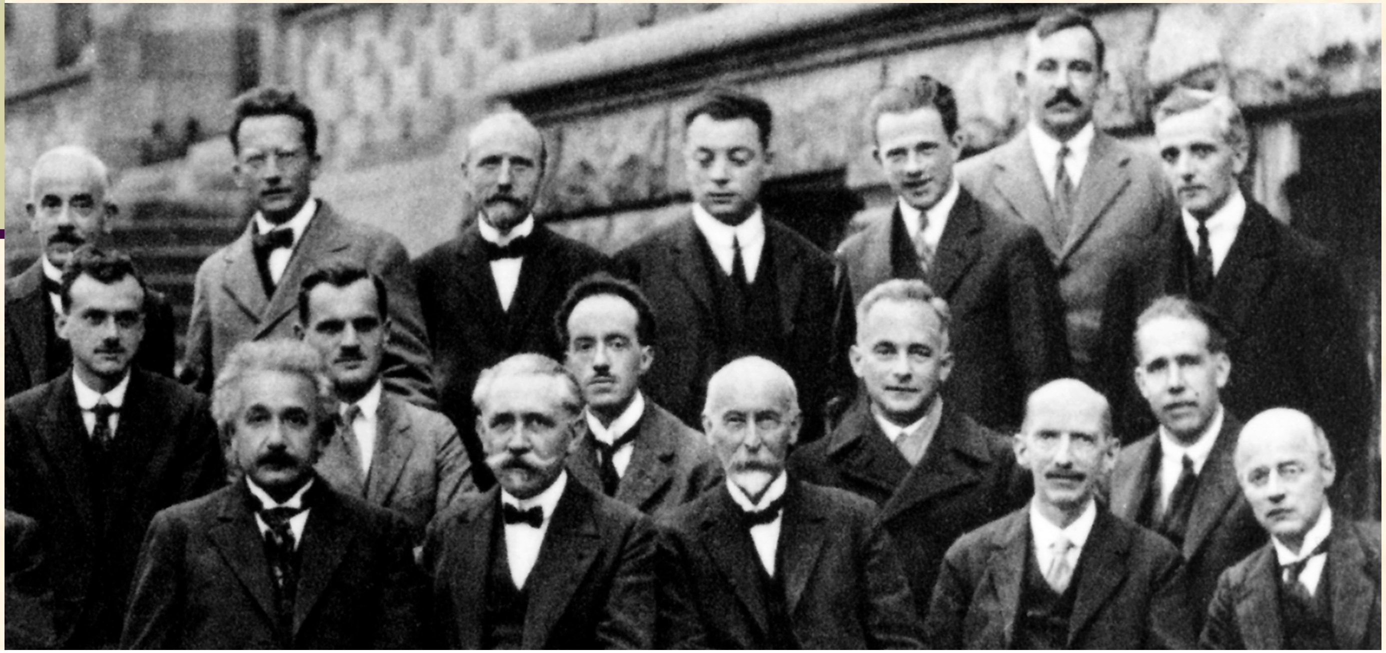
- Although Schrödinger's work was based on his ideas, and led to very important results, de Broglie did not accept it, because he thought it was not conceptually satisfactory ... and did not comply with the theory of relativity.

A work in progress

In the 1927 Solvay conference, de Broglie did not fight for his new theory (or theories), perhaps because he did not believe it was sound.

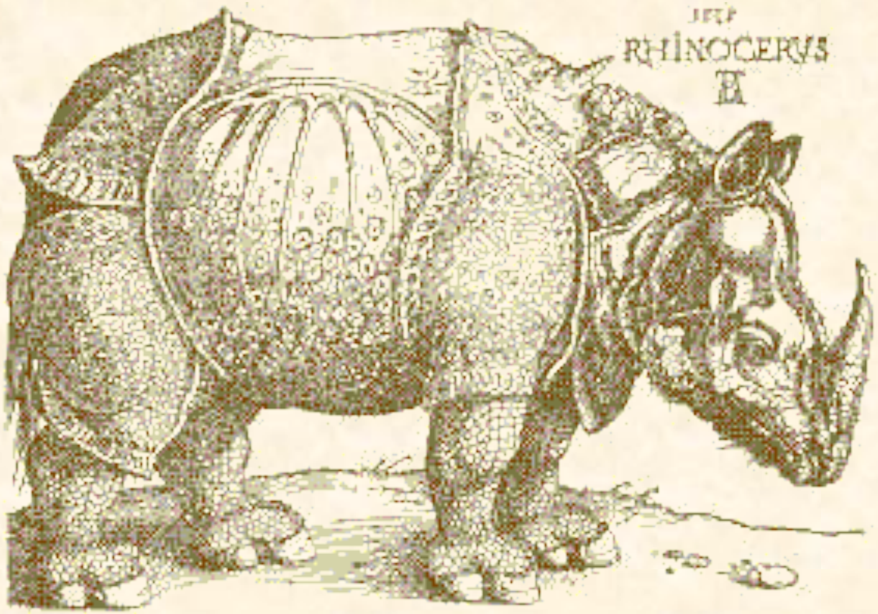
A work in progress

Afterwards, he accepted and taught the Copenhagen interpretation of quantum mechanics, never presenting his own ideas to his students, until the decade of 1950.



But that is another story!

The end



Group of History and Theory of Science, Unicamp

http://www.ifi.unicamp.br/~ghtc/