Module 4 Wave Theory of Light



UNIVERSITY OF COPENHAGEN

Experiment 1

Take two pieces of thin glass and **gently** press one against the other with the tip of a pen/pencil. Change some parameters and describe what you see.

Color patterns in thin films



Explanation 1 (Hooke, 1665)

Micrographia: or Some Physiological Descriptions of Minute Bodies Made by Magnifying Glasses. With Observations and Inquiries Thereupon.



MICROGRAPHIA: OR SOME Phylohogical Defiritions OF MINUTE BODIES MADR BY MAGNIFYING GLASSES WITH OBJERVATION, and IN QUINTES theretoon. By R. HOOKE, Fellow of the ROYAL SOCIETY Design of the most of the ROYAL SOCIETY Design of the R



LONDON, Frinted by Jo. Martyn, and Ja. Alighy, Printers to the RoxAL Society, and set to be fold at their Shopar the Bell in Starls Clurch-yard. M DC LX V.





- Pulse **ab** (white light) falls on the surface **AB**, which reflects a portion of it as the pulse **cd**.
- The rest is refracted and then reflected by the back surface EF as the pulse ef
- Pulse ef is weaker than cd
- When the combination of the two pulses enters the eye, it perceives different colors (*blue, red, purple, yellow, green*) depending on the relative position between **cd** and **ef**
- The latter depends on the thickness of the plate





Different colors Thickness is the key Qualitative Why only thin plates?

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Source: Kipnis (1992)

Explanation 2 (Newton, 1704)

Newton's rings



- **Regularities** in the experimental data: Ratio of the areas (bright 1, 3, 5... dark 2, 4, 6...), thus same ratio of thicknesses

- The unit in these progressions is 1"/178.000, $\ .$ which is the thickness of the film at 1st order bright for yellow (calculated for other colors)



Explanation: Light can **periodically change** its property of being reflected or refracted

Fits of easy reflection and refraction



- After entering a refracting medium at every distance from the surface multiple of 2"/178.000 light acquires a property to be transmitted further
- In the middle between these distances it can be reflected back
- Distance between two consecutive points of easy transmission/reflection is *interval of fits* and it depends on the color and index of refraction (λ ?)
- Light either passes or returns, if the thickness is an odd or even number of 1"/178.000



Fits??? Why only thin plates? Source: Kipnis (1992)

Thomas Young

- Eldest of 10 children in a family of Quakers
- Grandfather stimulated in classic literature, neighbor in math/physics
- [At 14] versed in Greek, Latin, French, Italian, Hebrew, Persian and Arabic. (Later involved in deciphering the Rosetta Stone)
- 1792 begins medical studies in London
- Interested in physics "important for a good physician"
- 1796: studied the formation of human voice (acoustics)
- 1799: Studying beats of sound, discovered principle of interference (two sounds can also destroy one another)
- 1801: Generalized the idea to light



Explanation 3 (Young, 1801)



Ray 1 travels *SCO*, Ray 2 travels *SAEFO* Path difference is $\Delta = SA + AE + EF + FO - (SC + SO)$ $SA \approx SB$ $t_{1BC} = t_{2AD}$, thus $\Delta = 2nDE$ (n ref index) Since DE = CE cosr (r angle of refraction) If *e* is the plate's thickness $\Delta = 2.e.n.cosr$

Problem: Doesn't match Newton's results

Solution: π phase shift in reflection from rarer to denser medium

$$\Delta = 2.e.n.\cos r + \lambda/2$$

Bright fringes when $\Delta = m\lambda$ (m = 0,1,2...)

Dark fringes when $\Delta = (2m+1/2)\lambda$

If the incident light is white, the rays producing maxima are reflected under different angles for different wave-lengths, resulting fringes of different colors. The spectra of higher order overlap and colors mix producing white. Thus, normally only spectra of the first few orders are seen. The number of spectra increases with path difference, which depends on thickness. This explains why fringes appear only in thin plates

Quantitative predictions Precise measurements Why only thin plates?



Source: Kipnis (1992)

Experiment 2 (Young, 1807)

When a beam of homogeneous light falls on a screen in which there are two very small holes or slits, from whence the light is diffracted in every direction. In this case, when the two newly formed beams are received on a surface placed so as to intercept them, their light is divided by dark stripes into portions nearly equal, but becoming wider as the surface is more remote from the apertures, so as to subtend very nearly equal angles from the apertures at all distances, and wider also in the same proportion as the apertures are closer to each other. The middle of the two portions is always light, and the bright stripes on each side are at such distances, that the light, coming to them from one of the apertures, must have passed through a longer space than that which comes from the other, by an interval which is equal to the breadth of one, two, three, or more of the supposed undulations, while the intervening dark spaces correspond to a difference of half a supposed undulation, of one and a half, of two and a half, or more.





Did Young perform Young's experiment?



John Worrall: NO

Young's double-slit experiment was an intuition of the truth, not a real experiment. Young does not explicitly state that he did the experiment; Young provides no numerical data; Young says nothing about the light source he used and the other experimental conditions; and Young never again refers to the experiment.



Nahum Kipnis: YES, but

Young did not interpret his experimental observations correctly. Young did experiment with two slits, and used both white and monochromatic light. However he did not discover the interference fringes: he confused them with diffraction fringes. Because the interval between the slits was too large, he could see only diffraction fringes produced by each slit separately. His observation was qualitative: if he had measured the distance between the observed fringes, he would have realized that they were of the wrong kind.

Then Fresnel enters the scene...

Excerpts from Fresnel's originals

35. Given the intensities and relative positions of any number of trains of light-waves of the same length and travelling in the same direction, to determine the intensity of the vibrations produced by the meeting of these different trains of waves, that is, the oscillatory velocity of the ether particles.

36. It is natural to suppose that the particles whose vibrations produce light perform their oscillations like those of sounding bodies that is, to suppose that the acceleration tending to make a particle return to its position of equilibrium is directly proportional to the displacement.

 $v = \sqrt{U} \sin\left(2\pi t\right)$

37. The energy of motion in the ether at any point on the wave depends upon the velocity of the point-source at the instant when it started a disturbance which has just reached this point. The velocity of the ether particles at any point in space after an interval of time *t* is proportional to that of the point-source at the instant $t - x/\lambda$.

$$u = a \sin \left[2\pi \left(t - \frac{x}{\lambda} \right) \right]$$
 Makes sense?

38. Let us first determine the velocity of a luminous particle in a vibration which results from the interference of two trains of waves displaced, one with respect to the other, by a quarter of a wave-length [i.e., differing in phase by 90°], and having intensities which we shall denote by *a* and *a*'.

$$u = a \sin \left[2\pi \left(l - \frac{x}{\lambda} \right) \right]$$
 and $u' = a' \sin \left[2\pi \left(\frac{t - x + \frac{\lambda}{4}}{\lambda} \right) \right]$

WHY by a quarter of a wave-length?

$$u' = -a' \cos\left[2\pi \left(t - \frac{x}{\lambda}\right)
ight] \qquad a \sin\left[2\pi \left(t - \frac{x}{\lambda}\right)
ight] - a' \cos\left[2\pi \left(t - \frac{x}{\lambda}\right)
ight]$$

$$A \cos i \sin \left[2\pi \left(t - \frac{x}{\lambda}\right)\right] - A \sin i \cos \left[2\pi \left(t - \frac{x}{\lambda}\right)\right] \qquad A \sin \left[2\pi \left(t - \frac{x}{\lambda}\right) - i\right]$$

38. Let us first determine the velocity of a luminous particle in a vibration which results from the interference of two trains of waves displaced, one with respect to the other, by a quarter of a wave-length [i.e., differing in phase by 90°], and having intensities which we shall denote by a and a'.

$$A \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)-i\right]$$





39. The solution of this particular case for waves differing by a quarter of a wave-length suffices to solve all other cases. In fact, whatever be the number of the trains of waves, and whatever be the intervals which separate them, we can always substitute for each of them by its components referred to two reference points which are common to each train of waves and which are distant from each other by a quarter of a wave-length; [...] but this is exactly the method employed in statics to fid the resultant of any number of forces; here the wave-length corresponds to one circumference in the static problem, and the interval of a quarter of a wave-length between the trains of waves to an angular displacement of 90° between the components.

Are you convinced? Need to see mathematically?

$$\frac{a \sin \left[2\pi \left(t-\frac{x}{\lambda}\right)\right]}{\operatorname{cr} \operatorname{cr} \left(t-\frac{x}{\lambda}\right)} \frac{a' \sin \left[2\pi \left(t-\frac{x+e'}{\lambda}\right)\right]}{\operatorname{cr} \operatorname{cr} \left(t-\frac{x}{\lambda}\right)} \frac{a \sin \left[2\pi \left(t-\frac{x}{\lambda}\right)\right]}{\operatorname{cr} \left(t-\frac{x}{\lambda}\right)} \frac{1+a' \sin \left[2\pi \left(t-\frac{x+e'}{\lambda}\right)\right]}{\operatorname{cr} \left(t-\frac{x}{\lambda}\right)} \frac{1}{\operatorname{cr} \left(t-\frac{x}{\lambda}\right)} \frac{1}{\operatorname$$

But this is precisely the value of the resultant of two forces, *a* and *a'*, inclined to each other at an angle $2\pi(c/\lambda)$.

Huygens principle described mathematically Fresnel integrals



Huygens principle described mathematically Fresnel integrals

$$m's' \approx \frac{z^2(a+b)}{2ab}$$
 ???





x/Am´=(a+b)/a Am' ≈ z

Huygens principle described mathematically Fresnel integrals (Buchwald, 1989)

 $m's' \approx \frac{z^2(a+b)}{2ab}$



$$\sin\left[2\pi\left[t-\frac{CM+m's'}{\lambda}\right]\right]$$



$$\left[\int dz \, \sin\left[\pi z^2 \, \frac{a+b}{ab\lambda}\right]\right]^2 + \left[\int dz \, \cos\left[\pi z^2 \, \frac{a+b}{ab\lambda}\right]\right]^2$$

Experiment 3

Look at some objects (e.g. text, point, window) through the crystal. Play a bit, move it around, rotate, etc. What do you observe? Take another crystal and put on top of the first one. What do you expect to see? What do you see? Observe light through the linear polarizer. Make your own experiments with this stuff. Be prepared to share your findings.



Bartholini (1669): Experiments with Iceland spar

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ERASMI BARTHOLINE

oculis nudis confpiciuntur, vel per aliud corpus pellucidum videntur. Hine fpecierum, EF & CD, aliqyando apparebit una pars altera dilutior. Ut, fi in lig. præcedente, fuerit objecti lo-



co linea aliqua dentior A; dum circumvolvitor Prisma ei incumbens, fuperficie eadem deorfum vergente, animadvertemus in certo aliquo fitu, apparentiam objecti A, repræfentari in fuperficie R SPQ per DE, ita ut pars F C fit obfeuriore colore, quan extremitates DF & CE.

EXPERIMENTUM IX.

A Nimum & aciem oculorum probe intendentibus apparet una ex duabus hifee imagini-

Huygens (1678/90): Traité de la lumière

CHAPTER V: ON THE STRANGE REFRACTION OF ICELAND CRYSTAL

- Explains birefringence with wave theory of light
- Principal section (only one image)
- Different refraction indexes in different directions (anisotropic)
- Ordinary ray (circle) and Extraordinary ray (ellipse)
- Describes experiments with two calcite crystals



Before finishing the treatise on this Crystal, I will add one more marvelous phenomenon which I discovered after having written all the foregoing. For though I have not been able till now to find its cause, I do not for that reason wish to desist from describing it, in order to give opportunity to others to investigate it. It seems that it will be necessary to make still further suppositions besides those which I have made;

Explanation ? (Newton, 1704)

Book 3 Opticks: Queries

When I made the foregoing Observations, I designed to repeat most of them with more care and exactness, and to make some new ones for determining the manner how the Rays of Light are bent in their passage by Bodies, for making the Fringes of Colors with the dark lines between them. But I was then interrupted, and cannot now think of taking these things into farther Consideration. And since I have not finished this part of my Design, I shall conclude with proposing only some Queries, in order to a farther search to be made by others.

Qu. 26. Have not the Rays of Light several sides, endued with several original Properties?





Important discovery (Malus and Brewster)



Modern explanation

- Light is a transverse wave
- Wire-grid polarizer absorbs E field in all directions except the \perp
- In the iceland spar, the ordinary and extraordinary rays are polarized in perpendicular planes



Not so fast! Fresnel reappears triumphantly...

Fresnel equations

MÉMOIRE

SUR LA LOI DES MODIFICATIONS

QUE LA RÉFLEXION IMPRIME A LA LUMIÈRE POLARISÉE (*),

LU À L'ACADÉMIE DES SCIENCES, LE 7 JANVIER 1823.

Goal: Equations describing what fraction of light is reflected (Fresnel equations).

$$r = r(\theta_1, n)$$







Assumptions

1 60



Modern version in Optics lecture notes (Jan Thomsen NBI)

$$r_{\perp} = \frac{n\cos(\theta_1) - \sqrt{1 - n^2\sin^2(\theta_1)}}{n\cos(\theta_1) + \sqrt{1 - n^2\sin^2(\theta_1)}}$$

When
$$n^2 \sin^2(\theta_1) = \begin{cases} < 1 & |r_{\perp}| < 1 \\ = 1 & r_{\perp} = 1 \\ > 1 & r_{\perp} \text{ is not real!} \end{cases}$$

Critical angle (c)

Total internal Reflection

Refraction

Cependant, en vertu de la loi générale de continuité, si elles étaient une expression exacte des lois de la réflexion jusqu'à la limite dont nous venons de parler, elles doivent encore l'être après; mais l'embarras est de les interpréter et de deviner ce que l'analyse annonce dans ces expressions imaginaires.

Due to the general law of continuity, if there is an accurate expression for the laws of reflection just before the limit, it should remain valid afterwards; the challenge is to interpret/guess these imaginary expressions.

$$r_{\perp} = \frac{n\cos(\theta_1) - \sqrt{1 - n^2\sin^2(\theta_1)}}{n\cos(\theta_1) + \sqrt{1 - n^2\sin^2(\theta_1)}}$$

$$\sqrt{1 - n^2 \sin^2(\theta_1)} = \sqrt{n^2 \sin^2(\theta_1) - 1} \cdot \sqrt{-1}$$

$$a = n\cos(\theta_1)$$
 $b = \sqrt{n^2 \sin^2(\theta_1) - 1}$

$$r_{\perp} = \frac{a - bi}{a + bi} = \frac{a^2 - b^2}{a^2 + b^2} + \frac{-2abi}{a^2 + b^2}$$

$$|r_{\perp}| = \sqrt{\left(\frac{a^2 - b^2}{a^2 + b^2}\right)^2 + \left(\frac{-2ab}{a^2 + b^2}\right)^2} = 1$$

$$r_{\perp} = |r_{\perp}| [\cos \alpha + i \sin \alpha]$$

$$r_{\perp} = |r_{\perp}| [\cos \alpha + i \sin \alpha]$$

$$r_{\perp} = \frac{a^2 - b^2}{a^2 + b^2}$$

$$r_{\perp} = \frac{-2ab}{a^2 + b^2}$$

$$r_{\perp} = \frac{-2ab}{a^2 + b^2}$$

Pour les simplifier, remplaçons la constante n^2 par c et la quantité variable sin² i par x, alors ils deviennent :

$$\frac{(c^2+1)x-c-1}{(c-1)[(c+1)x-1]}, \quad \text{et} \quad \frac{2\sqrt{c(1-x)(cx-1)}}{(c-1)[(c+1)x-1]}.$$

Par le même changement de lettres dans la formule (A), on a :

$$\frac{c+1-2cx}{c-1}$$
, et $\frac{-2\sqrt{c(1-x)(cx-1)}}{c-1}$.

pour les coefficients correspondants, dans le cas où la lumière incidente est polarisée suivant le plan d'incidence.

17. On sait que, pour déterminer la position de chacun des deux lle; systèmes d'ondes résultants, quand ces deux systèmes composants sont comme ici séparés par un quart d'ondulation, le calcul d'interférence est absolument semblable au calcul qu'on fait en statique pour trouver la direction de la résultante de deux forces rectangulaires. Ainsi. la longueur d'ondulation étant représentée par une circonférence entière, pais Ć si nous représentons par l'angle α la distance qui sépare les points ho- qui 10 mologues du système résultant et du système composant réfléchi à la na-S'(surface, nous aurons, pour le cas où la lumière incidente est polarisée 'me ra suivant le plan de réflexion. V

$$\cos \alpha = \frac{c+1-2cx}{c-1}, \quad \text{et} \quad \sin \alpha = \frac{-2\sqrt{c(1-x)(cx-1)}}{c-1}$$

et représentant par l'angle β la distance du système résultant au système composant réfléchi à la surface, dans le cas où les rayons ont été polarisés perpendiculairement au plan d'incidence, nous aurons :

$$\cos\beta = \frac{(c^2+1)x-c-1}{(c-1)[(c+1)x-1]}, \quad \text{et} \quad \sin\beta = \frac{2\sqrt{c(1-x)(cx-1)}}{(c-1)[(c+1)x-1]}.$$

Pour avoir l'intervalle qui sépare les points correspondants des deux systèmes d'ondes résultants, c'est-à-dire leur différence de marche, il suffit de calculer $\alpha - \beta$, ce qu'on peut faire aisément au moyen de la formule

$$\cos{(\alpha-\beta)} = \cos{\alpha}\cos{\beta} + \sin{\alpha}\sin{\beta};$$



Circular polarization \perp and || phase difference = 90°



Fresnel Rhomb



792 THÉORIE DE LA LUMIÈRE. — DEUXIÈME SECTION.

D'après la valeur maximum que nous venons de trouver pour $\alpha - \beta$, et qui excède à peine d'un degré le huitième de la circonférence, il est clair que, pour avoir entre les deux faisceaux une différence de marche égale à un quart d'ondulation, il faut au moins deux réflexions totales dans l'intérieur du verre. J'ai voulu déduire de la formule (C) l'incidence exacte qui satisfaisait à cette condition, c'est-à-dire donnait rigoureusement un huitième d'ondulation de différence à chaque réflexion, et, pour que la formule pût servir à d'autres expériences où le nombre des réflexions serait plus considérable, j'ai résolu le problème d'une manière générale en représentant par *a* le cosinus de la partie quelconque de circonférence à laquelle on voulait que l'arc $\alpha - \beta$ fût égal, et, égalant la valeur de cos $(\alpha - \beta)$ au cosinus donné *a*, j'ai eu l'équation :

$$\frac{2cx^{2}-(c+1)x+1}{(c+1)x-1}=a, \text{ ou } 2cx^{2}-(c+1)x+1=a(c+1)x-a,$$

ou enfin,

$$x^{2} - \frac{(c+1)(a+1)x}{2c} + \frac{a+1}{2c} = 0;$$

d'ou l'on tire

$$x = \frac{(c+1)(1+a) = \sqrt{(1+a)[(c+1)^3(1+a) - 8c]}}{4c} = \sin^2 i \cdots \cdots (D)$$

On voit que x, ou sin² i, a en général deux valeurs différentes, qui ne deviennent égales que dans le cas du maximum de la différence de marche $\alpha - \beta$, parce qu'alors a étant égal à

$$\frac{8c}{(c+1)^3} - 1, \text{ ou } a+1 \text{ à } \frac{8c}{(c+1)^3}, (c+1)^2(1+a) - 8c = 0,$$

et le radical s'évanouit.

Quand on fait *a* égal à cos 45° ou $\sqrt{\frac{1}{2}}$, on trouve pour les deux valeurs correspondantes de l'angle d'incidence, $i = 48^\circ 37' \frac{1}{2}$ et $i = 54^\circ 37' \frac{1}{3}$.

La première des valeurs étant plus voisine que l'autre de la première limite de la réflexion complète, qui est différente pour les diverses espèces de rayons, on sent aisément que, calculée d'après le rapport de réfraction des rayons jaunes, elle devra donner des résultats moins semblables pour les rayons de différente réfrangibilité; c'est donc la

Fresnel Rhomb



I found the two images uncolored and with the same intensity regardless of the azimuth



II MÉMOIRE SUR LA RÉFLEXION DE LA LUMIÈRE POLARISÉE. 793 seconde valeur qu'il faut adopter de préférence, si l'on veut avoir plus d'uniformité dans les modifications imprimées aux diverses espèces de rayons colorés qui composent la lumière blanche. J'ai fait tailler un



parallélipipède de verre de Saint-Gobain, dont les faces d'entrée et de sortie étaient inclinées de 54° 37' sur les deux autres, de manière qu'elles

fussent perpendiculaires au faisceau polarisé dans l'azimut de 45°, qui éprouvait successivement deux réflexions intérieures sur celles-ci, sous l'incidence calculée de 54° 37'. Alors, analysant les rayons émergents avec un rhomboïde de spath calcaire, j'ai trouvé les deux images sensiblement incolores et d'égale intensité, dans quelque azimut que je tournasse sa section principale.

22. Cette expérience, n'étant guère qu'une répétition de celles que j'avais faites anciennement, mais seulement plus exacte et éclairée par la théorie, ne pouvait en être considérée comme une vérification nouvelle; c'est pourquoi j'ai essayé de produire la même modification, ou d'obtenir une différence de marche d'un quart d'ondulation, d'abord par trois et ensuite par quatre réflexions totales.

Dans le premier cas, il faut que $\alpha - \beta$ soit égal à un tiers de quadrant, ou que *a* soit égal à cos 30° : cette valeur, substituée dans la formule (D), donne pour l'angle d'incidence *i*, qui satisfait à cette condition, 43° 10' $\frac{1}{3}$ et 69° 12' $\frac{1}{3}$. J'ai voulu vérifier par l'expérience ces deux valeurs de *i*, et pour cela j'ai fait tailler deux verres trapézoïdaux,



dont les faces d'entrée et de sortie étaient inclinées en sens contraires sur les deux faces réfléchissantes, dans l'un de 43° 11' et dans l'autre de 69° 12', de sorte qu'elles fussent perpendiculaires aux rayons incidents et émergents réfléchis dans le premier verre sous l'incidence de 43° 11', et dans le second sous celle de 69° 12'.

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