Christian Huygens:

The Measurement of Time and of Longitude at Sea

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> First at the request of the Directors of the Indies Company I undertook for finding longitudes to construct clocks of which the sure and constant motion would be equal to a three-foot pendulum, and they would not be disturbed by the agitation of the sea. Which task I found to more difficult that I had initially thought, and it is not finished yet; but there is no small hope that it will succeed.[1]

1. Huygens to Fullenius, 12.XII.1683, in Christiaan Huygens, *Oeuvres complètes* (The Hague, 1888-1950); hereafter HOC), Vol. VIII, 475.

1. Archimedes' Clock

From the very beginning of Christiaan Huygens' career as a mathematician and natural philosopher his father referred to him as "my Archimedes", and friends and admirers soon followed suit. But not blindly: the Republic of Letters in the late seventeenth century chose its classical images carefully. The name of Archimedes conjured up not only the genius that established the mathematical principles of stability in the physical world but also the ingenuity that transformed those principles into machines that put nature to work. Archimedes was an engineer as well as a mathematician. It did not matter that, as Plutarch reported, the real Archimedes had disdained to commit his mechanical inventions to writing.[2] To have recognized the equal importance of works and words was the advantage the moderns enjoyed over the ancients. Yet, elevating mechanics to the rank of learning did not make any less rare those with the special talent (as Pierre Petit put it), "pas permis � tout le monde, de bien penser et de bien faire."[3] Christiaan Huygens obviously had that talent, and contemporaries looked to it to yield fruits comparable to those found in Archimedes' writings and described in reports about him.

It would surely deepen our understanding of Huygens to know precisely what the cognomen meant to him. The repeated use of ευρηκα in his notes to signal the moment or occasion of discovery suggests that he took the comparison as more than a doting father's hyperbole. But to know how it influenced his choice of problems, his methods of investigation, and his style of presentation would require a study as yet undone. To do justice to the question, the study would have to explore also the differences that separate Huygens from Archimedes. These, I suspect, would turn out to have greater historical significance than have the similarities. In retrospect from today Huygens looks less like the reincarnation of Archimedes than like the prototype of Coulomb, Carnot, and the practitioners of the Ecole polytechnique's characteristic *m�lange de math�matique et physique*. No facet of his career shows this more clearly than his research into the measurement of time and of longitude at sea. It established a model for a new sort of mechanics, a truly mathematical physics rooted firmly in the physical world.

From his first pendulum clock in 1657 to his last sketches for a "perfect marine balance", Huygens' efforts to capture the uniform flow of time in a reliable mechanism, and especially in a device immune to disturbance from its (possibly moving) surroundings, ranged from the most general theory to the most intricate practice. His explorations took him into uncharted regions of abstract mechanics, where he applied the newest mathematics of his day to the analysis of the constrained motion of points along curves and of points linked in rigid configurations; at one point, he had to create a new branch of mathematics, the theory of evolutes. But those explorations began with the actual physical mechanisms that were his successive clocks, and his discoveries took concrete form in practical improvements of the clocks. The theoretical research and the practical application of its results had one purpose: to increase the clocks's accuracy. Each step forward defined in precise quantitative terms the limits within which the next step would have to fall. Taking turns, theory and practice narrowed the tolerance between the abstract and the physical to fractions of seconds and inches. If predecessors such as Galileo, first emulating Archimedes and then going beyond him, had already shown how to reduce bodies in motion to geometrical configurations, none had yet insisted that the reduction satisfy such exacting standards of measured fit between the real and the ideal. In demanding that precision, Huygens did not follow the past; he led the future.

Huygens' work on clocks and on the method of longitudes has another facet that looks toward the future rather than reflecting the past. Although as mathematician Huygens aspired in the best classical tradition to fame, as mechanic he also jealously guarded the fortune that might result from his inventions.[4] In the spirit of Archimedes he insisted that his *magnum opus*, the *Horologium oscillatorium*, wait for publication until he could present its theoretical content in the finished form of a mathematical treatise. In quite another spirit, he postponed publication when it seemed that his first marine clock would sustain sea trials and form the instrumental core of a practical (and profitable) method of longitudes. A written description of the clock might interfere with the exclusive license Huygens had awarded to a Hague clockmaker, and the successful outcome of even more rigorous trials might heighten the clock's value. For two years in the mid-1660s, while colleagues eagerly awaited the long-promised treatise, Huygens worked instead to secure patents and *privil�ges* from the French, English, and Dutch governments, not to mention the reward offered by the Spanish throne for a reliable method of longitudes.[5] Although by no means unprovoked, Huygens' disputes with the clockmakers Douw and Thuret over patent rights display an acrimony sharper than any of his contests over issues of mere theoretical consequence.[6] The

2. For a scientific biography of Archimedes that emphasizes his mechanical investigations, see Ivo Schneider, *Archimedes* (Darmstadt, 1979).

3. Petit to Huygens, 18.X.1658, HOC.II.257. As in all quotations in French below, I have modernized the orthography.

4. Huygens began seeking patents for his method of determining longitude by means of clocks even before the first sea trials had taken place. The matte is a recurrent theme in his correspondence with his brother Constantijn, with Robert Moray, and with Jean Chapelain during 1664-5; see. HOC.V *passim*. The whole question of the patents and *privil�ges* sought by Huygens and awarded to him for his clocks would reward careful study.

5. According to J.H. Leopold, Huygens' designs did not suffice to determine the actual construction of his clocks. With regard to escapements in particular, he needed the expertise of the various skilled clockmakers with whom he was associated. He did not entirely appreciate the extent of their contributions and hence was quick to interpret their claims of invention as infringements on his own. Douw in 1658 and Thuret in 1675 were the only two to stand up to Huygens in public. Douw lost his case before the States o Holland, but Thuret had enough powerful friends in Paris to force a compromise, even though he had to withdraw any claim to the invention of the spring-regulated balance. See the pertinent exchanges in HOC.II and HOC.VII. On the issue in general, see J.H. Leopold, "Christiaan Huygens and his instrument makers," in *Studies on Christiaan Huygens*, ed. H.J.M. Bos et al. (Lisse, 1980), [(Added 2007) On the dispute with Thuret, see my "Drawing Mechanics" in *Picturing Machines*, 1400-1700, ed. Wolfgang Lefebvre (Cambridge, MA: MIT Press, 2004), pp. 297-304.]

6. Harrison's first design, a marine pendulum, dates from 1730; the successfi clock, regulated by a spring balance, was built in 1759 and tested on a voyage to Jamaica in 1762. On Harrison's clocks and the determination of longitude, see William J.H. Andrewes (ed.), *The Quest for Longitude* (Cambridge, MA, 1996); for a popular account based on the contributions tc that volume, see Dava Sobel, *Longitude* (New York, 1995) and Sobel and Andrewes, *The Illustrated Longitude* (New York, 1998).

new *m�lange de math�matique et physique* that brought the mechanic into the academy also lured the mathematician into the marketplace.

Huygens' life-long engagement with clocks passed through several distinguishable stages. The first begins in 1657 with his invention of a means of combining the pendulum and the mechanical clock into a single long-running and reliable device and extends through 1661, by which time he had discovered the tautochrony of the cycloidal pendulum, the cycloidal cheeks that make a pendulum cycloidal, the conical pendulum, the center of oscillation of a compound pendulum, and the sliding weight to vary the pendulum's period. The second stage covers the decade from 1662 to 1672 during which Huygens worked out, alone and in collaboration with others, means of suspending the clock from a shipboard mounting and the details of the measurement of longitude by means of clocks. The period ends with the final preparation of the *Horologium oscillatorium*, which Huygens felt forced to bring out after sea trials had proved only partly successful. The third stage stretches from his invention of a spring balance in 1674 to his final sketches of a marine clock in the years just preceding his death. Huygens' research during this period consisted largely of studies of physical mechanisms embodying his principle of *incitation parfaite d�croissante*, laid down in 1675 to explain the tautochrony of the spring. But Huygens' investigations also reflect a troubling awareness that his earlier successes were illusory and that subsequent modifications were proving ineffectual. Sea trials on Dutch vessels in 1686-87 and 1690-92 did not live up to expectations. The fame Huygens so avidly sought as the inventor of the first reliable clock for determining longitude at sea eluded him.[7] It would continue to elude horologers for almost fifty years more, to be captured ultimately by John Harrison in 1762. But if Harrison found the answer in metallurgy, he had Huygens to guide him through the mechanics.

7. Huygens to Carcavi, 26.II.1660, HOC.III.28

2. The *Horologium* of 1658: Harnessing Driving Force and Regulating Motion

Commenting on the many rival claims to have invented the pendulum clock, Huygens wrote to Pierre Carcavi in 1660:

> "C'est une chose �trange, que personne devant moi n'ait parl� de ces horloges, et qu'� cette heure il s'en d�couvre tant d'autres auteurs."[8]

8. Cf. OC.II.109.

He was willing to grant that others might also have hit upon the idea of using a pendulum to regulate the motion of a mechanical clock, for example Gilles Personne de Roberval. However, as he pointed out to Chapelain, Roberval's mechanism failed to solve the basic technical question and hence to carry out "la principale industrie", namely, how to maintain the pendulum's swing with the help of the clock's driving weight. That was, Huygens had to admit, not such a great trick; indeed, "il me semble toujours que j'ai � tr�s bon march� la r�putation que cette fabrique m'a donn�e."[9]

9. Cf. similar remarks to Petit, OC.II.271.

Huygens may have belittled his achievement because neither of its main components was itself new. Mechanical clocks, driven by the force of a falling weight or of an uncoiling spring, had existed for centuries; in both the large format of elaborate orreries and the small format of intricate watches, they have reached a high level of sophistication.[10] The pendulum was of more recent origin, having been brought to wide attention by Galileo. Astronomers had quickly adopted it for their observations; as Huygens described their procedure, "they impelled by hand a weight hung from a light chain and, counting its single vibrations, they measured as many equal moments of time."[11] The device had already produced improved measurements of eclipses and of celestial dimensions.

10. An excellent history is Giuseppe Brusa, *Orologi europei* (Milan, 1978).

11. Huygens, *Horologium* (Den Haag, 1658), in HOC.XVII.55. Huygens credited Galileo with the technique. For accounts by contemporary astronomers, see the sources cited on pp. 4-5 of the *Avertissement* to the 165 clock in HOC.XVII.

But each instrument had inherent flaws that counteracted its basic function. The clock lacked a regulator that by its intrinsic period of reciprocating motion could both advance the escapement in uniform steps and overcome the perturbing effect of variations in the force transmitted from the drive owing to imprecisely cut gears and uneven friction. The pendulum ran down unless repeatedly sustained by impulses from the operator's hand. Moreover, its use by astornomers meant tedious hours of counting its swings, with the attendant risk of error.[12] The pendulum required a motor and a counter, the clock a regulator. Huygens devised a means of combining them so that each supplied the other's need without otherwise interfering with it.

12. *Horologium*, HOC.XVII.55.

From Huygens' first pendulum clock, through all its subsequent versions, "the principle of its motion, and indeed of the whole invention," remained the same.[13] The pendulum hung from a flexible cord clamped in a vise mounted on the clock's frame. The pendulum's motion was transmitted to the escapement via a small crank, or crutch, through one end of which the pendulum's rod passed freely yet snugly; the other end was joined to the verge either directly or through gearing. The crutch in turn transmitted from the escapement to the pendulum the small force necessary to sustain the latter's swing.

13. *Horologium*, HOC.XVII.59-61. In the original version of 1657 the crutc led directly to a horizontal verge moving a horizontal escapement wheel. Unable to discover a means of rendering the pendulum tautochronic, Huygens chose to restrict its swing and to gear up the rotation of the crutch, which he therefore separated from the verge and escapement wheel now standing vertically in the clock. The cycloidal pendulum removed the need for such gearing, and Huygens returned to the original horizontal escapemer after 1660.

The independent suspension and the crutch linked the pendulum and the mechanical clock while still keeping them essentially separate and hence separately adjustable. For example, increasing the weight of the bob to resist small variations in the force delivered by the crutch, or to decrease the retarding effect of the air's resistance, did not add to the friction on the verge and hence to the resistance offered by the palettes to the escapement wheel.[14] Similarly, changes in the driving force could be accommodated in the escapement and the crutch before reaching the pendulum. On the one hand, then, the small perturbations to which the pendulum in itself was subject were not overridden by grosser variations transmitted to it from the clock's works. On the other hand, the crutch transmitted those perturbations to the clock, which was thereby made sensitive to them.[15] Without that arrangment of one-way sensitivity, precise analysis of small changes in the pendulum's motion served little practical purpose. With it, such theoretical analysis promised practical means of

14. The designs of both Galileo and Roberval joined the pendulum's rod and the palette to a common hollow verge turning on an axle. Increasing the weight of the bob meant increasing the friction on that axle and hence the force necessary to move the palettes. The rigid connection also made the pendulum quite sensitive to changes in the impulse delivered by the clock's works to the palettes. Huygens learned of these other designs in 1659; Boulliau transmitted Leopoldo de' Medici's account of Galileo's pendulum escapement, and Chapelain described Roberval's. For descriptions and contemporary drawings, see HOC.II *passim*.

15. In particular, the long pendulum and the cycloidal pendulum eliminated the need for a stackfreed or fus�e in a spring-driven clock. From the outset of correspondence with Huygens in 1658, Petit pressed him for a precise analysis of the relation between the weight driving the clock and the weight of the bob. Huygens waved off the problem with references to empirical dat

calibration. Thus Huygens' innovation made the clock an interface for theory and practice in mathematical physics.

3. Toward the *Horologium oscillatorium*

3.1 The Tautochronic Pendulum

In the late fall of 1659 two problems, one practical and one theoretical, met on that interface. The first stemmed from the clock: how can one control the wide excursions of the pendulum so as to make all its swings take the same time? Huygens knew from Mersenne and others that, despite Galileo's claims, the period of a pendulum varies with its amplitude. Only for swings in the immediate neighborhood of the centerline is the variation negligible and the pendulum effectively tautochronic.[16] The second problem arose out of current research into the nature of accelerated free fall: what is the distance through which a body falls from rest in one second?[17] The new clock seemed to offer a contribution to the latter questions by providing an independent means of calibrating a one-second pendulum, but the practical need for wide swings during experiments with falling objects overrode that help. What was needed was a solution to the problem Huygens set himself sometime in November: he sought "the ratio of the time of a very small oscillation ... to the time of perpendicular fall from the height of the pendulum."[18] That resort to theoretical analysis yielded unexpected practical results.

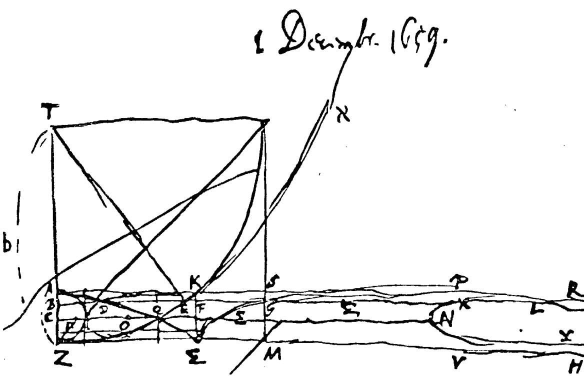

Figure 1: Huygens' original diagram; HOC.XVI.392

Because of the dependence of period on amplitude, Huygens expected to have to make an approximation in his analysis of pendular motion. It turned out to lie deep in the mathematics of the problem. He began (Figure 1) with a bob swinging from rest at K over a small arc KZ and sought an expression for the time of motion over any infinitesimal segment E of that arc. Assuming that the bob moves uniformly over the segment at the speed acquired by fall along the curve from K to E --which is the same as the speed acquired in free fall through the vertical distance AB-- and taking as a base of comparison the bob's uniform motion over an infinitesimal segment B of the vertical AZ at the speed acquired in free fall through the whole interval AZ, he used Galileo's law of uniform motion to determine that $\frac{time(E)}{time(B)} = \frac{length(E)}{length(B)} x \frac{BF}{BD}$. By the tangent properties of the circle, $\frac{length(E)}{length(B)} = \frac{TE}{BE}$, which by constructing BG = TE one may express as $\frac{BG}{BE}$. Then, on the figure of the pendulum Huygens superimposed a parabola representing the relation of the successive distances AB to the speeds acquired by a body falling freely over those distances from rest at A; he chose the units of measure so as to set ZΣ --i.e. the speed of the body at Z-- equal to AK. That

his papers give no evidence of his having attempted a theoretical analysis of the transmission of forces within the clock itself.

16. Marin Mersenne, *Reflectiones physico-mathematicae* (Paris, 1647), Chap XIX. It was to restrict the swing that Huygens introduced the geared escapement into the 1658 clock (see above, n. 13), but at the time he had nc theoretical measure of the variation.

17. Joella Yoder pointed out to me the role of this second problem in Huygens' discovery of the tautochronic pendulum. She pursues the subject in depth in her *Unrolling Time: Christiaan Huygens and the Mathematization of Nature* (Cambridge, 1988).

18. The following discussion of Huygens' discovery of the tautochronic pendulum is based primarily on the worknotes published in HOC.XVI.392-413. It recapitulates the structure of the derivation and employs Huygens' notation and mathematical language, but it does not follow his precise line of reasoning. What is said here benefitted greatly from critical remarks by JoAnne Morse and Geoffrey Sutton. [Added 2004: For more detail, see my later essays, "Huygens and the Pendulum: From Device to Mathematical Relation", in E. Grosholz and H. Breger (eds.), *The Growth of Mathematical Knowledge* (Dordrecht, 2000), pp. 17-39 and "Drawing Mechanics" , in *Picturing Machines, 1400-1700*, ed. Wolfgang Lefebvre (Cambridge, MA, 2004).]
Note that the manner in which Huygens sets up the problem keeps the solution, so to speak, internal to the pendulum. That is, the precise parameters of its period are in part canceled out by referring its motion to free fall through a vertical height equal to its length. In modern terms, the result will appear as the ratio of $\pi\sqrt{\frac{l}{g}}$ to $\sqrt{2\frac{l}{g}}$, or $\frac{\pi}{\sqrt{2}}$. Although the dependence of the period on $\sqrt{l}$, announced by Galileo in *Discorsi ... intorno* � *due nuove scienze* (Leiden, 1638, p. 96; *Opere*, ed. Favaro, VIII, 139), ha been confirmed by experiment and by theoretical derivation from the law of fall, the dependence on the constant of free acceleration had not yet been specified, and Huygens' derivation continued to mask it.

19. On the conceptual structure of this approach to quadrature, see Michael Sean Mahoney, *The Mathematical Career of Pierre de Fermat* (Princeton, 1973; 2nd ed. 1994), 253ff. The first of the lemmas "sine quibus motus aequabilis in cava cycloide inveniri not potest" (HOC.XVI.398) figures prominently in Fermat's work on quadrature and may bear witness to the course of Huygens' mathematics here.

20. In the course of the derivation Huygens subtly altered the precise nature of the substitution. At first he took AK as an ordinate common to both circle and parabola. From having already set AK = Z he then had $\frac{AK^2}{AZ} = \frac{Z\Sigma^2}{AZ} = p$ the latus rectum of parabola AD and hence also of parabola ZEK. Hence, BE taken as ordinate of the parabola ZEK would be expressed by

$$BE^2 = \frac{AK^2}{AZ} \cdot BZ = \frac{(2TZ \cdot AZ - AZ^2) \cdot BZ}{AZ} = 2TZ \cdot BZ - AZ \cdot B$$

whereas BE taken as ordinate of the circle would be given by $BE^2 = 2TZ{\cdot}BZ - BZ^2$. Later in the derivation, however, Huygens assigned 2TZ as the common latus rectum of the two parabolas, i.e. $\frac{AK^2}{AZ} = 2TZ$ or $AK^2 = 2TZ{\cdot}AZ$. In that form, the circula and parabolic arcs are no longer coterminous, and the value of BE^2 changes to 2TZ·BZ for E on the parabola Huygens did not discuss the shift directly, but he did defend the second form of the substitution by pointing to the assumption of a "minimal" arc and hence of a very small value for AZ, on which the difference between the curves depends.

21. The reduction and the ensuing quadrature rest on the sequence of lemma referred to in n. 19. Huygens offered no proofs; in some cases they are straightforward, in others they require the techniques of quadrature and rectification recently developed by Fermat and others. Here, to begin with, since the parabolas have a common latus rectum, $p = \frac{BD^2}{AB} = \frac{BE^2}{BZ}$,
$BD \cdot DE = \sqrt{p \cdot AB} \cdot \sqrt{p \cdot BZ} = p\sqrt{AB \cdot BZ} = p \cdot BI$. Then, for an curve $\frac{BX}{BY} = \frac{AC}{BI}$, where BI is the ordinate of a circle on diameter 2AC = AZ, $\frac{\text{area under } BX}{AZ \cdot BY} = \frac{\text{semiperimeter } AIZ}{\text{diameter } AZ} [= \frac{\pi}{2}]$. For the specific curve , the two lemmas combine to give
$\frac{BX}{BF} = \frac{BG \cdot BF}{BE \cdot BD} = \frac{BG \cdot BF}{p \cdot BI} = \frac{BG \cdot BF}{\frac{BF^2}{AZ} \cdot BI} = \frac{BG \cdot AZ}{BF \cdot BI}$; whence
$\frac{BX}{BG} = \frac{AZ}{BI}$, or $\frac{BX}{2BG} = \frac{AC}{BI}$, and
$\frac{\text{area under } BX}{AZ \cdot 2BG} = \frac{\pi}{2} = \frac{\text{all } BX \text{ over } AZ}{2AZ \cdot BF} \times \frac{BF}{BG}$.

permitted him to express the ratio of the speeds over B and E as $\frac{BF}{BD}$, where $\frac{BF^2}{BD^2} = \frac{AZ}{AB}$. The resulting expression, $\frac{BG \bullet BF}{BE \bullet BD}$, for the ratio of the times reduced the physical situation to a mathematical configuration to which Huygens then applied the analytical tools at his disposal, among them recent results by Fermat and others on the transformation and quadrature of curves. For, by constructing a curve of ordinates BX such that $\frac{BX}{BF} = \frac{BG \bullet BF}{BE \bullet BD}$, he had

$$\frac{\textit{time over arc KZ}}{\textit{time of a'cceld motion over AZ}} = \frac{\textit{time over arc KZ}}{\textit{2 x time of uniform motion over AZ}} = \frac{\textit{all BX over AZ}}{2 \bullet BF \bullet AZ},$$ where "all BX over AZ" is the sum of all ordinates BX standing on the base AZ, i.e. the area under the curve of ordinates BX.[19] Determining the period of the pendulum meant finding that area.

It was that mathematical problem that required an approximation, thus reflecting the physical situation. Huygens could not find the area under the curve as given. Its ordinate is inversely proportional to the product of the ordinate of two base curves, a circle and a parabola, and it appeared to have no properties that would permit its reduction to simpler terms. But he could handle a curve which in the immediate neighborhood of Z --i.e. for small AK-- lay quite close to the original one. That new curve replaced the circular arc ZEK by a parabolic arc congruent to arc ADΣ.[20] For E lying on the parabola rather than the circle, the product BD.BE reduced to a constant multiple of the ordinate BI of a semicircle AIZ on AZ as diameter, and the quadrature of the adjusted curve of ordinates BX reduced to the rectification of the semi-perimeter AIZ; that is, $\frac{\textit{time over arc KZ}}{\textit{time of a'cceln over AZ}} = \frac{BG}{BF} x \frac{\pi}{2}$.[21] To relate the time over KZ to that over TZ required a turn back to the physical situation: by Galileo's laws of fall, $\frac{\textit{time of accel. over AZ}}{\textit{time of accel. over TZ}} = \frac{\sqrt{AZ}}{\sqrt{TZ}} = \frac{\sqrt{2TZ \bullet AZ}}{TZ\sqrt{2}} = \frac{BF}{BG\sqrt{2}}$, and the final ratio reduces to $\frac{\pi}{2\sqrt{2}}$.

The approximation of the period of minimal oscillation assumed, then, for the purposes of mathematical analysis that the relation between the normal and the ordinate of the bob's circular path could be represented by the relation between that normal and the ordinate to a parabola. As Huygens then turned to finding the physical situation for which the result just derived would be an exact solution, he looked for a path of descent for which the parabola would exactly represent the relation between the normal and the ordinate. He had to retain the parabola for both mathematical and physical reasons: by mirroring the parabola ADΣ it made the quadrature possible, and the parabola it mirrored was fixed by the kinematics of free fall. "From this," he recorded in the same worknote of 1 December 1659,

22. On Pascal's challenge, see R. Taton, "Pascal, Blaise", *Dictionary of Scientific Biography*, Vol. X (New York, 1974), 335ff.

23. See his "Demonstratio melior huc tandem redacta", HOC.XVI.405ff., dated 15 December 1659.

> I saw that, if we want a curve on which the times of descent are equal over any arcs terminated at Z, it must be of such a nature that, if one sets the ratio of the curve's normal ET to the ordinate EB equal to that of a given line GB to another EB, the [second] point E falls on a parabola with vertex Z. This I found to agree with the cycloid, by a know method of drawing the tangent.

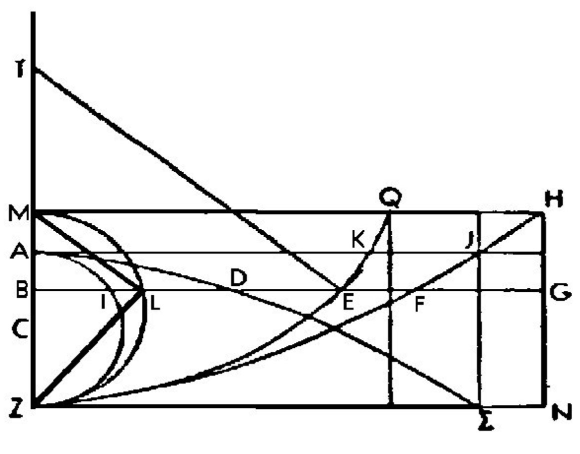

Figure 2

Suppose, that is, that ZKQ (see Figure 2) is the cycloid generated by circle MLZ, E any point on the cycloid, BE the ordinate to that point. The "known method of drawing the tangent" consists of drawing a line through E parallel to chord ZL of the circle. Hence, the normal ET is parallel to chord ML and $\frac{TE}{BE} = \frac{ML}{BL}$. By the properties of chords in circles,

Figure 2

$$\frac{ML}{BL}=\frac{MZ}{LZ}=\frac{LZ}{BZ},$$ whence $\frac{TE^2}{BE^2}=\frac{MZ}{BZ}$. Now, through any point H on MQ draw HN parallel to MZ and intersecting BE at G, and construct point F on BG such that $\frac{BF}{BG}=\frac{BE}{TE}$. Then $\frac{BF^2}{BG^2}=\frac{BF^2}{MH^2}=\frac{BZ}{MZ}$, which holds if and only if F and H lie on a parabola with vertex at Z.

If, then, in the derivation of the pendulum period one replaces circle ZEK by cycloid ZEKQ and constructs parabola ZFJH congruent to the parabola of free fall ADΣ, it follows *mutatis mutandis* that the time over arc KZ of the cycloid is the same for any starting point K; it stands to the time of free fall over diameter MZ as the semiperimeter of a circle to its diameter, i.e. as $\frac{\pi}{2}$. Hence the time over any arc is to the time of fall through 2MZ as $\frac{\pi}{2\sqrt{2}}$, and the approximately constant period of a narrowly oscillating simple pendulum of a given length becomes exactly constant when the bob is constrained to follow an inverted cycloid, the diameter of whose generating circle is half the given length.

Huygens did not include among the lemmas in his worknote the above derivation linking the parabola and cycloid. The nonchalance with which he adduced the relation was sincere. His recognition of the cycloid behind the conditions of the problem sprang from familiarity, no genius. Quite independently of his work on the clock, he had been following closely the controversy over the cycloid unleashed by Pascal in 1658.[22] The results exchanged had honed his knowledge of the curve's properties to the extent that he could work backward from them. Indeed, his intimate knowledge of the curve enabled him soon to abandon the hybrid configuration of spatial path and kinematic diagram and to construct a "demonstration rendered better than heretofore" that used Galileo's results to embed the kinematics in ratios among chords of the cycloid's generating circle.[23] That better demonstration became the skeleton of the formal exposition published in the *Horologium oscillatorium*.

Huygens' knowledge of the cycloid perhaps also explains his quick recognition of another property of the curve pertinent to his needs. Beginning with the first clock, he had tried to control unequal excursions of the pendulum by enclosing the suspension cord between two strips of metal, or cheeks, thus pulling the bob back from the circle of its unrestricted swing. The shape of the cheeks had been a matter of cut-and-try, and Huygens reported having achieved some success.[24] But he could not precisely define the experimentally determined shape and in the *Horologium* of 1658 replaced the cheeks by gearing to keep the oscillations narrow, even at the expense of increasing the

24. See, for example, his letter of 1 November 1658 to Pierre Petit, HOC 2, 271.

25. Cf. *Horologium* (1658), HOC.XVII.69.

replaced the cheeks by gearing to keep the oscillations narrow, even at the expense of increasing the clock's sensitivity to disturbance.[25] Nonetheless, the cheeks remained on his mind, especially as he pondered the implications of motion along a cycloid.

The transition from circle to cycloid in the derivation of the period of a pendulum had replaced the fixed length and center of the former by a normal that varied in both length and intersection with the cycloid's vertical axis (which was also the centerline of the pendulum). A cycloidal pendulum seemed at first to have no fixed length to serve as its parameter and to satisfy Galileo's law relating the period of a pendulum to the square root of its length.[26] Yet, precisely because Huygens constructed his cycloid to fit the parabola of free-fall velocities to which the circular arc had been an approximation, the new pendulum and the old shared within the close neighborhood of the centerline a common period measured by the fall of a body through a height equal to twice the diameter of the cycloid's generating circle. Hence, about the centerline, the cycloidal pendulum acted as if it were a simple pendulum of given length. For wider amplitudes, its arc lay within that of the simple pendulum, and it swung over a shortened radius about a center other than the point of suspension.

The cheeks themselves suggested to Huygens how to analyze that shortening and displacement. By winding along the surface of a cheek, the flexible cord followed the arc of the cheek's curve and was thus shortened by the difference between that arc and its subtending chord. As the cord wound around the cheek, the centrifugal force of the bob kept the free portion of the pendulum taut and thus tangent to the leaf at the cord's last point of contact, which momentarily acted as the center about which that free portion was swinging. For that moment, then, the bob followed a small circular arc coincident with the pendulum's trajectory and hence was normal to the latter. If the cheek extended all the way to intersect with the trajectory, and one began with the pendulum wound about it, then one could view the trajectory as the curve traced by the endpoint of the cord as it is unwound from the cheek while being kept taut and tangent to it. This physically inspired construction permitted exact mathematical definition in terms of tangents, normals, and arc lengths, and the cheeks of the clock thereby became a particular physical instance of a quite general mathematical concept. But the mathematical theory as first set out by Huygens retained a vestige of its inspiration. He called the curve corresponding to the leaf the *evoluta* or *d�velopp�e*, the "unwound [curve]."[27]

The pendulum clock gave rise, then, to the theory of evolutes which, especially in its application to the rectification of curves, yielded results far in excess of what Huygens had needed for the clock itself. Indeed, he admitted to the reader of the *Horologium oscillatorium* that most of Part III, "On the Evolution of Curves", was included for its "elegance and novelty" rather than for its application to present needs. Certainly, the determination of the shape of the leaves governing a cycloidal pendulum required very few of the mathematical techniques he eventually devised and published.[28] The initial conditions imposed ont he leaves all but dictated their form. First, each had to be tangent to the centerline at the point of suspension. Second, when extended to meet the cycloid at its widest point, each leaf had to be tangent to the baseline of the cycloid and perpendicular to the curve itself. Third, since at that point the leaf would coincide with the full length of the pendulum, it had to have an arc-length equal to twice the diameter of the cycloid's generating circle. But, as Christopher Wren had just shown, that was precisely the length of one half of the cycloid measured from base to vertex. In sum, then, the leaf had to follow a curve having the same base, height, and length as the cycloid to be traced. That it might be the cycloid itself was quickly confirmed in a rough sketch of a demonstration that (see Figure 3):

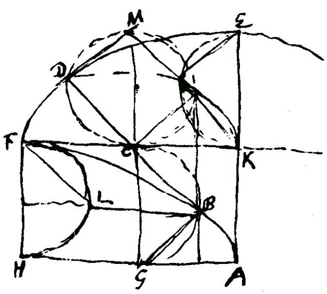

Figure 3 (HOC.XIV.404)

> If *ABF* is half a cycloid, about which a cord or flexible line *ABF* is laid and secured at *A*, and the other end moving away from *F* while the cord remains taut describes some curve, that curve, namely *FDE*, will also be half a cycloid equal and similar to *FBA*.[30]

The cycloidal pendulum provided Huygens with the practical means of constructing a portable clock capable of keeping the same beat despite sudden wide excursions of the pendulum. But it also laid the theoretical bases of two new areas of mathematical and mechanical investigation. Characteristically, at the same time that Huygens had some clocks built to the new design and began experiments with them, he also drafted two short treatises, "On the Fall of Heavy Bodies and Their Motion on a Cycloid" and "On the Evolution of Curves," which he intended for a new and expanded version of his *Horologium*. He began revisions in 1660, but his plans for quick publication were overtaken by even more rapid new developments in the clock's design and use.[31]

3.2 The Problem of Longitude

In the original *Horologium* Huygens had pointed to the development of a portable version of his clock as the key to transforming it from an astronomical instrument to a navigational tool.[32] As an astronomical instrument, the stationary, long-pendulum clock making very narrow oscillations provided a uniform measure of time independent of the motions of the stars and planets and hence suitable for determining irregularities in those motions. In particular, since it was accurate to within seconds over a day, the pendulum clock could measure small variations in the length of the solar

26. Cf. above, n. 18.

27. The initial inspiration is also captured by an omission of nomenclature that persisted for several decades and became more noticeable as the theory grew more sophisticated, especially in the new infinitesimal calculus. The curve generated by evolution had no name other than the description just used. It was not the "evolvent", nor was it, as it is now called, the "involute" Although modern theory treats the two curves as a pair, Huygens was originally interested in only one of them, the evolute. Its place in clock design explains why. On the theory's mathematical development, see H.J.M. Bos, "Huygens and Mathematics", in *Studies on Christiaan Huygens* (Lisse, 1980), 126-146.

28. As Huygens told Moray (OC.IV.51), the shape of the leaves was easy. It was the "methodical" exploration of the evolute that caused trouble.

29. Specifically, Wren showed that any arc FB of a cycloid FBA (see diagram immediately following in the text) is twice the chord FL subtending the arc FL through which the generating circle has rolled. Hence, the total length of semicycloid FBA is twice the diameter FH. Pascal's *Histoire de la roulette* (1658) made particular note of Wren's rectification (see n. 39 of Taton's article on Pascal, cited above, n. 22), and Wallis published it in his *Tractatus duo, prior de cycloide* ... (Oxford, 1659; cf. HOC.XIV.367, n.5). For Huygens' acknowledgement of Wren's contribution, see the letter to Robert Moray, 10 February, 1662, HOC.IV.51.

30. *OC. 17*, 404; the drawing inverts the physical situation. The proof rests on the relation DB = 2BC = 2FL = arc FB.

31. Huygens first announced a new treatise in a letter to Ismael Boulliau, 22 January 1660, HOC.III.13. A draft of the revision (HOC.XVII.120-123) shows that it was to include the cycloid as tautochrone, the cycloid as volute of itself, and the general theory of volutes, with a final section on the equation of time (see next section). Although Huygens told Chapelain on 2 September 1660 (HOC.III.120) that the treatise had been finished "il y a longtemps, " he told Moray a year later that the work was still in progress (HOC.III.297). In October 1662, he again spoke of delay (HOC.IV.51). Thereafter, he said nothing about the treatise until 1665, by which time the work had been expanded to include the center of oscillation and the compound pendulum. But the results of the first sea trials of Huygens' marin clock prompted him to withhold the text for commercial reasons (see below

32. *Horologium* (1658), HOC.XVII.57.

33. The phenomenon of the inequality of the solar day had been known sinc antiquity. Ptolemy rendered an account of it in Chapter 9 of Book III of the *Almagest*, and sophisticated sun dials had incorporated that account in the form of a figure-8 mean time meridian. But Ptolemy's explanation could not be tested closely; as he noted, the difference over one solar day or even ove several was too small to be measured. Hence, although he set out the mathematics for determining cumulative inequality over long periods, he di not compose a table of daily values. Copernicus followed the same form of presentation. Huygens' clock changed that. Although Parisian clockmakers

day as marked by the sun's successive passages through the meridian. When compared with theoretical calculations, the measurements offered astronomers empirical means of resolving longstanding questions about the parameters governing the inequality of the solar day and hence of settling on values of the equation of time to be used with the clock in recording solar events.[33] Huygens emphasized this new contribution to astronomical theory at the same time that he exploited its practical benefits. An accurate table of the equation of time made it possible to use the sun and its shadow, instead of the stars, to regulate the clock. That made regulation easier and more accessible to those using the clock.[34] More importantly, however, it completed the theoretical basis of a method of determining longitude by clocks and left open only the practical problem of portability.[35]

had already begun to claim for their product that "solis menaces arguit horas" (*The Flammarion Book of Astronomy*, New York, 1964, p. 23), it was the accuracy of the pendulum clock that first made measurement of the daily inequality possible. The resulting corrections in solar time made measurements of the moon's motion and of eclipses correspondingly more accurate.

34. Huygen's various instructions for the clocks' use, especially his *Kort Onderwijs* (see below, n. 56) set out several of his techniques of regulation in some detail (itself a mark of their novelty).

35. The principle of determining longitude is straightforward. The earth turn uniformly from west to east through 360^o in one mean solar day of 24 hours For every hour's difference in local time between two points there corresponds a difference of 15^o in longitude. If, then, a traveler carries with him a clock regulated to mean solar time and set to the sun at his starting point, he can determine his longitudinal distance from that point by comparing the time of the sun where he is to the time given by the clock. After only a few days' travel, however, he will have to correct local time for the accumulated inequality of the solar day.

Huygens first began investigating the equation of time as soon as he had built his first clock. He found some modern astronomers like Ismael Boulliau at odds with their predecessors over the structure of the phenomenon, and he used his clock first to decide between them.[36] He concluded that Ptolemy and Copernicus had been right: the inequality of the solar day stemmed solely from the eccentricity and obliquity of the ecliptic.[37] If the sun's circle coincided with the equator, then its annual motion, combined with the daily motion of the heavens, would produce a constant daily advance eastward of 59'8"20"' in right ascension.[38] But the angle of the ecliptic and its eccentricity join to produce a daily advance in longitude that does not correspond to that difference in right ascension. As the dialy differences accumulate, a clock regulated to the mean solar day (I.e. to the constant sidereal day plus $3m\ 56.5^{+}s$) and set to the sun at noon on 10 February will fall behind it almost $20m$ by 14 May. The clock will again catch up to within $9m$ by 25 July, only to fall behind again by $31^{+}m$ on 1 November; by 10 February it will have caught up completely.[39] The choice of starting point is arbitrary unless one wants to compare astronomical observations with tables and parameters set to a standard epoch. For 10 February as base the cumulative inequality remains positive throughout the year, and Huygens chose it for that reason.

36. For Huygens' first studies, see HOC.XV.527ff. On Boulliau's differing view of the phenomenon - "in�galit� des conversions journelles de la terr que vous voulez introduire avec Kepler" (Huygens to Boulliau, 29 January 1660, HOC 3I, 17) - see Boulliau to Huygens, 27 February 1660, HOC.III.29, and Boulliau's note of 26 March 1660, *ibid*., 51-55. Huygens proposed on 22 April that the two postpone further discussion until they could converse directly in Paris, and the subject disappears from his correspondence until the spring of 1662, when he sent out his table and explained its construction to Petit (25 April 1662, HOC.IV.138-142).

37. Or, in Copernican terms, from the eccentricity of the earth's orbit and the inclination of its axis to the plane of the orbit. Huygens, like navigators dow to the present, found it convenient to analyze the phenomenon in Ptolemaic terms. Ignoring for the moment the factor of eccentricity, imagine the sun to move uniformly along the ecliptic at a daily rate of $\Delta\lambda = 360/365.242$. For any $\lambda_n = n\Delta\lambda$, the corresponding right ascension α_n measured along the equator is given by the relation $\tan \alpha_n = \tan\lambda_n \cos\delta$, where δ is the angle of inclination of the axis (the relation ignores small variations in the sun's latitude). The factor of eccentricity means in addition that $\Delta\lambda$ will vary from day to day.

38. The value corresponds to a tropical year of 365.242 days.

39. The dates of the turning points are Huygens', which differ from the modern values reported in Flammarion's *Astronomy* (New York, 1964, p. 22 as 11 February, 15 April,27 July, and 4 November.

Huygens spent two years calculating and confirming his table of cumulative inequality, which by subtraction provided the inequality over any number of days, and sen out the first public copies in February 1662.[40] To use it in calibrating the clock, one set the clock to the sun on any given day. After the clock had run continuously for several days, one then noted the time of the sun's passage through the meridian. To that time one added the difference of the cumulative equation for the day of the second reading less that for the day of the setting; if the former was less than the latter, one subtracted the difference. The result was supposed to be 12:00; any discrepancy, divided by the number of days the clock had been running, gave the daily advance or retardation. When applied to finding longitude, the table first aided in calibrating the clock and then in correcting its readings for the elapsed inequality before they were compared with local sun time at sea. Hence the table formed as much a part of the navigator's kit as did the clock and the quadrant.

40. Cf. Huygens to Lodewijk Huygens, 15 February 1662, HOC.IV.54-57; Huygens to Moray, 17 February 1662, *ibid*., 60. The table remained unchanged through its publication in the *Kort Onderwijs* (1665; see below, n.56), its translations, and the *Horologium oscillatorium* (1673, p.15; HOC.XVIII.112-113).

3.3 Adjusting the Pendulum

At about the same time that Huygens made public his table of equation of time, he also announced an addition to his clock that made its regulation by the table even easier and more accurate: the sliding weight. It was the practical payoff of another theoretical breakthrough, the determination of the center of oscillation of a compound pendulum. Neither the simple nor the cycloidal pendulum clock in itself demanded that breakthrough. Each already had provision for moving the bob up and down and thus for adjusting the clock to agree with the stars, with the corrected sun, or with another clock. But in musing on the wider implications of his invention Huygens had imagined that the length of a simple pendulum that beat isochronically with the 1-second pendulum of a precisely calibrated clock could serve as a universal standard of measure: one third of that length would be a *pes horarius*, a 'clock-foot'. Huygens' colleagues in the Royal Society proved especially receptive to the notion and in 1661 began experiments on both simple and cycloidal pendulums.[41] They soon reported significant divergences between Huygens' theory and their observations. In particular, pendulums of different materials seemed to behave differently.

41. Cf. Moray to Huygens, 23 December 1661, HOC.III.427; Huygens' repl of 30 December, HOC.III.438; and his own description of the project in the *Horologium oscillatorium* (1673, pp. 151-154; HOC.XVIII.349-355). The topic recurred in the two men's correspondence until about 1665, when the English began to lose confidence in the idea. Ultimately, measurements by French scientists of the length of a one-second pendulum at various location revealed significant geographical variations; see below, section VI.

It is not clear from the evidence when Huygens hit upon the notion of the compound pendulum and the associated concept of the center of oscillation.[42] By 1661, however, he had gone far enough to explain the English experiments, at least in part. His working notes show that he began with the case of two dimensionless weights swinging together on an inflexible weightless rods, and his solution established the paradigm for analyzing all more complex systems.[43] It relied on a principle Huygens had already used with success in deriving the laws of impact: when the center of gravity of a system falls from rest and then rises again to rest, it begins and ends at the same height, irrespective of the changes of relative position within the system itself.[44]

42. The dating of Huygens' first efforts is problematic. None of the ms. page containing the evidently early calculations bears a date, but they fall togethe between pages dated 1659. The draft preface of 1660 (HOC.XVII.121) includes a *regula inveniendi longitudinem penduli*, but that probably refers t a formula, based on the relation between period and free fall through the length, that applies to a simple pendulum. The relation of his work to earlier attempts to determine the center of percussion, attempts to which he refers i the preface to Part IV of the *Horologium oscillatorium*, and the subsequent controversies with Roberval and the Abb� de Catelan fit more properly in a study of Huygens' mechanics than in the present account.

43. "De centro oscillationis sive ad invenienda perpendicula [= pendula] simplicia isochrona propositis perpendiculis compositis [1661], HOC.XVI.415-427; cf. a sketchier version in HOC.XVII.149-152, as well a Prop. XXIII or Part IV of the *Horologium oscillatorium*. What follows here preserves the content, but not the precise order of Huygens' derivation.

44. On this fruitful principle, named variously after Torricelli and Mersenne and on its central role in Huygens' mechanics, see Alan Gabbey, "Huygens and Mechanics", *Studies on Christiaan Huygens* (Lisse, 1980), 166-199.

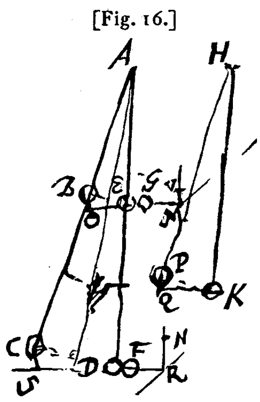

Figure 4: The two-bob pendulum; HOC.XVI.415.

Huygens imagined (see Figure 4) weights *B* and *C* lying on rod *AC*, which swings about point *A* through angle *CAD* to centerline *AD* isochronically with the simple pendulum *HK*. That is, suppose that "pendulum *AED* has vibrations equally as fast as [those of] pendulum *HK*," or that "pendulum *AED* is moved at the same speed as the simple pendulum *HK* is moved," or (to state explicity Huygens' implicit criterion of isochrony) that the two pendulums swing through equal angles CAD and PHK in equal times. If $AC = AD = a$, $AB = AE = b$, and $HP = HK = x$, then at any time, $vel(B):vel(C):vel(P) = b:a:x$. Moreover, since *HK* is a simple pendulum, the velocity acquired by *P* at *K* depends entirely on the vertical height *QP* through which it has fallen, and $QP:CS = AD:HK$, or (if $CS = d$) $QP = \frac{xd}{a}$. [45]

To consider the dynamical effects of these kinematical results, suppose that, on reaching centerline *AD*, the weights *B* and *C* strike directly weights *G* and *F* respectively equal to them and standing at rest. By the laws of impact, *B* and *C* will stop, and *G* and *F* will move off at speeds of *vel*.(B) and *vel*.(C), respectively. Suppose further that *G* and *F* are then reflected upward at those speeds.[46] Let *MV* and *RN* be the vertical heights to which they respectively rise. By Galileo's law those heights are as the squares of the speeds, and *OP* provides a basis of comparison. Therefore, $\frac{RN}{QP} = \frac{vel.(C)^2}{vel.(P)^2}$, or $RN = \frac{a^2}{x^2} \bullet \frac{dx}{a} = \frac{ad}{x}$; similarly, $\frac{MV}{QP} = \frac{b^2}{x^2} \bullet \frac{dx}{a} = \frac{b^2 d}{ax}$.

From the situation of bodies falling through certain heights, communicating their motion to other bodies, and thereby driving the latter to certain heights, it follows, Huygens asserts,

> that the composite center of gravity of the spheres G and *F*, after they have taken motion from *E* and *D* and have converted it upward as far as they can, that this center (I say) rises to the same height as that of the center of gravity of the spheres *B* and *C*.

That is, $RN \bullet wgt\ F + MV \bullet wgt\ G = CS \bullet wgt\ C + BO \bullet wgt\ B$, or (for e = *wgt* F = *wgt* C and c = *wgt* G = *wgt* B) $\frac{ead}{x} + \frac{b^2dc}{ax} = ed + \frac{bdc}{a}$, whence $x = \frac{ea^2+b^2c}{ea+bx}$. [47] As an immediate corollary,

> if given length *AD* and weight *D*, and it is required to attach the given weight *E* to a certain point of the pendulum such that the compound pendulum is isochronous with a pendulum of given length *HK*, it follows from the preceding equation

that $b = \frac{1}{2}x + \sqrt{\frac{1}{4}x^2 + \frac{ea}{c}x \ \frac{ea^2}{c}}$.

In later versions Huygens eliminated the medium of impact and simply imagined the bodies constituting the compound pendulum to be simultaneously released from constraint and to be individually directed upward at the speeds acquired at the point of release. But the basic principle remained the same. At the start of the swing the pendulum's center of gravity, which is fixed by the weights of its constituent parts and by their distribution over it, lies at a certain height. Whether or not the parts remain connected over the length of the swing, they come to rest only when their common center of gravity has again reached that initial height. The difficulties of applying the principle lie in determining the initial and final positions of the center of gravity under the assumption of dissolution during the swing. In the case of a uniform rod of length *a* swinging about one end, Huygens expressed these two positions by a triangular and a parabolic area, respectively, equating an initial condition of $\frac{1}{2}ad$ and a final condition of $\frac{1}{3}\frac{a^2d}{x}$ to determine that $x = \frac{2}{3}a$. [48]

As a corollary to this last derivation, Huygens noted that "whatever weight is added to the rod at that place, it will make the swings no faster or slower." The center of oscillation, he clearly implied, is the point at which the whole weight of the body concentrates to form a simple pendulum, the period of which is independent of its weight. From here he could move toward his goal of regulating pendular motion. Clearly, weight added to the bar below the center of oscillation would slow its swings, and added above the center would speed them up. Consider first a weight *Q* added to the very bottom (see Figure 5). If *a* is again taken as both the length and the weight of the rod, and *n* is to *AD* as the weight *Q* is to the weight of the rod, then *n* simply represents the weight of *Q*. To the determination of the initial center of gravity, *Q* adds the term *nd*; to that of the final center of gravity, the term $\frac{nad}{x}$. Hence, $\frac{1}{2}ad + nd = \frac{1}{3}\frac{a^2d}{x} + \frac{nad}{x}$, whence a value for *x* directly follows.

Adding another weight $H = h$ at distance $AC = c$ from *A* introduces two more terms to the equation: $\frac{hcd}{a}$ to the initial conditions on the left, $\frac{hc^2d}{ax}$ to the final conditions on the right. They now yield the equation: $\frac{1}{2}ad + nd + \frac{hcd}{a} = \frac{1}{3}\frac{a^2d}{x} + \frac{nad}{x} + \frac{hc^2d}{ax}$. Of greater interest here than the solution of *x* is that for *c*:

45. At the risk of misleading the modern eye, I retain Huygens' use of the letter *d* here to denote a finite line segment.

46. As the diagonal lines in the diagram suggest, Huygens imagined the bodies to bounce off immobile inclined planes. He had used the same analytic device in his treatise on impact.

47. For *n* bodies m_i placed at distances r_i from the fixed end of the rod AD (considered as a weightless line), the proof immediately generalizes to the form $x = \frac{\sum_{i=1}^{n} m_i r_i^2}{\sum_{i=1}^{n} m_i r_i}$.

48. Here Huygens imagined the uniform rod to consist of "an infinite numbe of little spheres conjoined in a rigid straight line." Each of these little spheres, which are thought of as equal, has an equal counterpart lying on *CI* equal to *AB* and lying immediately beside it. Concentrating first on the bottommost sphere, setting $AB = a$, $HK = x$, and $OV = d$, and following the previous pattern of analysis, one has $\frac{ad}{x}$ = height to which sphere *D* is impelled by the motion acquired by *B* in falling through *OV*. If $BS = OV$, then for any sphere *N*, *MN* will be the height through which it falls. To determine the height *RP* to which *N* impels *R*, set $\frac{HK}{AN} = \frac{NM}{RP}$ (following shortcut established earlier). Then, $HK{\cdot}RP = AN{\cdot}NM$. But $HK{\cdot}DE = AB{\cdot}BS$, whence $\frac{RP}{DE} = \frac{AN \cdot NM}{AB \cdot BS}$. But by similar triangles $\frac{AN}{AB} = \frac{NM}{BS}$ whenc $\frac{RP}{DE} = \frac{AN^2}{AB^2} = \frac{CR^2}{CD^2}$. Therefore *P* lies on a parabola with vertex at *C*, ax *CF* parallel to *DE*, and *latus rectum* = $\frac{CD^2}{DE}$. The ordinates of that parabola represent, then, the heights to which the corresponding bodies are raised by the velocities acquired by constrained fall through the heights represented b the ordinates of the straight line *AS*. To set the compound centers of gravity equal: "because the weight of all the spheres [on the one side and other] is equal, let us think of the weights of the individual [spheres] as expressed by the equal lines that lie between two contiguous centers." That is, let the unit of weight be so chosen that $AB = a$ represents the total weight of the bar, as well as its length. Then the sum of the products of the spheres times their initial heights is the triangle *ABS*, and the sum of the products of the spheres and their final heights is the parabolic area *CPED*, and by the basic principle those areas must be equal. But the area of the triangle is $\frac{ad}{2}$ and that of the parabolic figure $\frac{1}{3}\frac{a^2d}{x}$ (i.e. $\frac{1}{3}CD \cdot DE$). Therefore $x = 2a/3$.

49. Huygens took explicit note of the explicit corollary to this last result: for a given *x* less than the length of the simple pendulum isochronic with the compound pendulum consisting of weight Q and rod AD alone, *c* has two positive values and hence determines two different positions for weight H.

$c = \frac{1}{2}x + \sqrt{\frac{1}{4}x^2 + \frac{naxa^2n + \frac{1}{2}aex\frac{1}{3}a^2e}{h}}$, where *e* expresses the weight of the rod directly, rather than by means of *a*. *c* is the distance at which a given weight *h* must be placed in order to render the compound pendulum of weight *H*, weight *Q*, and suspension rod *AB* isochronous with a simple pendulum of known length *x*.[49]

But if one knows the length of a simple one-second pendulum, one also knows the length of a simple pendulum that deviates from it by any fixed interval, say, 1' over 24 hours. Moreover, from the results obtained above one can determine the distance along a rod at which to place a fixed weight so that the compound pendulum has a period of 1 second (i.e. so that the distance from the point of suspension to the center of oscillation is the length of a simple 1-second pendulum). A small weight added to that pendulum at the center of oscillation will not affect its period. If placed elsewhere, the weight will change the period, and one can calculate where to place it to cause a precise change. One can, that is, mark off on either side of the center of oscillation the lengths *c* at which the small weight effects any desired deviation.

That is the principle behind the *poids curseur*, or sliding weight, that Huygens first announced to Moray on 30 December 1661:

> J'ai trouv� depuis quelque temps le moyen d'ajuster fort pr�cis�ment � son heure mon horloge par un petit plomb mobile que j'applique � la verge de cuivre du pendule, le plomb d'en bas demeurant toujours ferme. C'est ainsi que j'ai dress� son cours aux jours m�diocres ...[50]

The phrase "depuis quelque temps" cannot point very far back, since by all evidence the revision of the *Horologium* carried out in 1660 did not include any consideration of centers of oscillation. Hence, although the notes and calculations that appear to date from the fall of 1659 indicate that Huygens had made a start on the problem, he did not gain full control of it until sometime in 1661. It was probably then that he set down the derivations outlined above and calculated a table, found in his notebooks, which lists the settings of a sliding weight for gains over 24 hours of up to 2 minutes by 5-second intervals.[51]

Huygens' first set of results made some headway in accounting for the discrepances uncovered by English experiments, but it was not until 1664that he could explain why the size of the bob made a difference. By then he had developed the mathematical techniques to apply his basic principle to various solids, most important among them the sphere. He could not locate the precise center of oscillation of a pendulum consisting of a fine wire and a round bob. It lay a significant distance from the bob's geometrical center, and for a fixed length of wire that distance depended on the bob's radius.[52]

By 1664, then, Huygens had all the elements of a theoretical treatise on the pendulum clock. But by then he also had reason to hope that the treatise could offer stunning evidence of the theory's practical value. In 1663 and 1664 the Royal Society had sent a marine version of the clock to sea to test its reliability and its accuracy as a means of determining longitude. The first results had exceed expectations.

4. Clocks at Sea

The first marine pendulum emerged from the collaboration of Huygens and the Scot Alexander Bruce during November and December 1662.[53] Huygens had already toyed with various ways of suspending the clock from a pivot so as to render it independent of the ship's motion, but Bruce's idea of a steel ball encased in a brass cylinder proved most effective in tests at The Hague. Two of the clocks went with Bruce to London early in 1663 and, after minor setbacks prompting the intervention of the Royal Society, accompanied Captain Robert Holmes on voyages first to Lisbon and then to Guinea and out into the Atlantic.[54] The first voyage yielded a log with encouraging data, including clocked longitudes that agreed well with accepted values in well known waters. The second voyage in 1664 added a dramatic touch.

Having set sail due west from the island of St. Thomas off the Guinea coast, Holmes went some 800 leagues before taking a northeast course back toward the African coast. After several days, supplies of fresh water began to run low, and Holmes's fellow masters urged that he head for the Barbados. They reckoned that the squadron was still some 100 leagues from the Cape Verde Islands. But Holmes's clock placed him only 30 leagues distant, a day's run. He continued his course and hit land the next afternoon. Huygens learned later that the ships had been becalmed for a time after heading northeast and had drifted with the current some 80 leagues eastward; traditional piloting could not detect that motion.[55]

Huygens acted on this success by publishing Holmes's account in the *Journal des S�avans* of 23 February 1665 and by placing the marine clock on public sale along with a *Kort Onderwijs*, or *Brief Instruction*, on its regulation and use in determining longitude.[56] Having secured an *octroi* from the States of Holland, he sought the same rights and recognition from France and England.[57] In the meantime, he withheld his new *Horologium*. He did not want to releasethe details of the clock, and he expected soon to have systematic data to back up the drama of the clock's seaworthiness.

Again in 1668-69 the clock met with initial success in measuring the longitudinal difference between Toulon, Crete, and several intermediate points.[58] But Huygens placed his greatest hope in long-range tests ordered by the Acad�mie des Scioences as part of an expedition to the West Indies in 1670. The experiment would benefit from the lessons of the earlier voyages and, in particular, would silence the critical murmurs coming across the Channel in Oldenburg's letters. "Personnes intelligences" in the Royal Society doubted that Huygens' clocks could be trusted to remain vertical in a rolling ship and had reason to think that the pendulum was significantly sensitive to changes in the air around it.[59]

But he did not pursue the physical implications of that mathematically dictated finding.

50. HOC.III.438.

51. HOC.IV.67; cf. The scale ruled out in the first plate of the *Horologium oscillatorium* (1673, p.4; HOC.XVIII.71) and the table at the end of Prop. XXIII of Part IV (p. 150; HOC.XVIII.347).

52. To be precise, "Soit la sphère ABC don't le centre D, endue au filet AE, attach� en E. Il faut trouver aux lines ED, DB la troisi�me proportionelle DF, delaquelle DO faisant les 2/5. Je dis que O est le centre de vibration de cette sph�re ainsi suspendue." (HOC.V.149) That is, $DF \cdot DF = DB^2$, and $DO = \frac{2}{5}DF = \frac{2}{5}\frac{DB^2}{DE}$. Cf. *Horologium oscillatorium*, Part IV, Prop. XXII (1673, pp. 141-142; HOC.XVIII.331-333).

53. Huygens' letters to his brother Lodewijk during the summer of 1662 speak in general terms of an "horologe à petit pendule ... [dont] le cours est assez juste étant en repos pour pouvoir servir aux longitudes, et [qui] souffle sans s'arrêter le movement que je lui donne en ma chambre où elle est suspendue par des cordes de 5 pieds de long, mais je n'ai pas encore fait l'épreuve sur l'eau, pour laquelle il faudrait être dans un vaisseau de raisonnable grandeur et dans la mer même qui fût agitée, à quoi je ne sais pa quand je pourrais parvenir (9 June 1662, HOC 4,151)." Lodewijk apparently conveyed the news to Constantijn Sr., who began to talk as if his Archimede had already solved the problem; on 9 November (HOC.IV.256) Huygens asked that his father temper that enthusiasm, warning that work was far fron complete but announcing that Bruce would be undertaking sea trials. Huygens spoke of his collaboration with Bruce in letters to Moray (1 December 1662, HOC.IV.274-275; 20 December 1662, *ibid*., 280-281) and Lodewijk (14 December 1662, HOC.IV.278). In the last he noted that Bruce claimed a part in the marine clock's design and a share in any profits from it but he did not go into details. Indeed, even when Bruce later attempted to fil for the English patent (cf. Huygens to Moray, 8 December 1663, HOC.IV.45 and the correspondence of spring 1664, HOC.V.passim), Huygens avoided any explicit discussion of the clock's design and of Bruce's contribution to it Only in the *Horologium oscillatorium* does one find a hint of what the first marine clock looked like (1673, pp. 16-17; HOC.XVIII.115-117): "Instead c a weight they had a steel strip wound in a spiral, but the force of which the wheels were turned 'round, just as is commonly employed in those small watches that are wont to be carried about. So that the clocks could endure th tossing of the ship, he [sc. Bruce] suspended them from a steel ball enclosed in a brass cylinder, and extending downward the arm of the crutch that sustains the pendulum's motion (the pendulum, by the way, was a half-foot i length) he doubled it to resemble the form of an inverted letter F; namely, le the pendulum's motion wander out in a circle with the danger of stoppage."

54. For the data of the first voyage (29 April - 4 September 1663), see HOC.IV.446-451; for Huygens' response to them, cf. the surrounding correspondence between 29 October and 9 December, *ibid.*, 426-474, *passin* For Holmes's reports of the Atlantic voyage of 1664, see Moray to Huygens 23 January 1665, HOC.V.204-206, and "A Narrative concerning the success of Pendulum-Watches at Sea for the Longitudes," *Philosophical Transactior* 1(1665-66), 13-15. [online (Princeton only)]

55. Huygens at first had trouble getting additional details about the voyage. Holmes disappeared into the Tower of London for a time (Moray to Huygens, 13 February 1665, HOC.V.234), apparently as punishment for unauthorized hostile actions against foreign installations and shipping. Mora took until early March to run down another ship's officer, who confirmed the account and provided enough information for Moray to think that the origin figures needed revision (*ibid*., 269-270). Not until 27 March did Moray report the news about the currents (*ibid*., 284).

56. *Kort Onderwijs aengaende het gebruyck der horologien tot het vinden der lengthen van Oost en West, O.C. 17*, 199-235. Huygens composed the instructions even before learning the results of Holmes's second voyage. He announced them to Moray on 2 January 1665 (HOC.V.187) and sent the text to the printer on 19 February; printer's proofs went to Moray on 27 February

According to Huygens' account, which no one at the time denied, the voyage was a disaster. Shortly after sailing, one of the clocks stopped in a storm. Contrary to instructions, Jean Richer, the *el*�ve detailed to carry out the experiment, did not immediately restart it. After a delay of some 10-12 hours, the second clock also stopped. Again ignoring instructions, Richer simply abandoned the tests altogether and allowed the clocks to deteriorate in their mountings and eventually to crash to the deck. Even more distressing that Richer's negligence was the apparent lack of concern about it among colleagues at the Acad�mie. Writing from The Hague in February 1671, Huygens complained to J.B. Duhamel, "qu'on s'est fort peu applique � bien faire r�ussir cette experience," but his complaints seem to have gone unanswered.[60] A year later Huygens reported to Henry Oldenburg that, even if the newest version of his marine clock were ready for a new French expedition then departing for the West Indies, he would not send it. The ship was not properly fitted, nor wqas anyone suited to carry out the texts. "Je crois," he noted in resignation,

> que j'irais quelque jour moi-m�me en quelque petit voyage, pour voir le succ�s de cette invention, car je vois qu'il d�pend beaucoup de la diligence de ceux � qui on en commet et desquels je ne suis pas fort satisfait jusqu'� pr�sent.[61]

For the moment at least, sea trials were over. The method of longitude remained unproved, though not, Huygens insisted, by fault of the clock.

It was, therefore, in a mood more of resignation than of triumph that Huygens decided in 1671 to proceed with the publication of the *Horologium oscillatorium*, which appeared in the spring of 1673. In it he presented the 1658 clock with the addition of cycloidal cheeks, a revised escapement, and an adjustable pendulum. He also described with illustrations the newest, as yet untested marine model with a triangular suspension of the pendulum and a Cardan mounting for the clock. He condensed his *Kort Onderwijs* to a few paragraphs and recounted the results of the 1663, 1664, and 1668 trials. He then took up in separate sections his analyses of the fall of heavy bodies and their motion along a cycloid, the evolution of curves, and the center of oscillation. The last concluded with the theory of the sliding weight, the design of a 1-second pendulum as a universal standard of measure, and the measure of the vertical distance through which a body falls freely in a given time. To this central theme he added a brief account of the conical pendulum with escapement and cheek to keep it on its tautochronic paraboloidal surface. As an appendix he offered his theorems --but not their demonstration-- on the measure of centrifugal force.

5. Strings, Springs, and Other Oscillators.

In the years immediately following the appearance of the *Horologium oscillatorium* Huygens left the marine clock and the method of longitude in abeyance and turned instead to further exploration of the phenomenon of tautochronic oscillation. The new line of inquiry began with a theretofore unnoticed corollary of his analysis of motion on a cycloid: the effective motive force acting on a body at any point of the cycloid is proportional to the arc length from the vertex to that point.[62] He first used the result to analyze the empirically know tautochrony of a vibrating string, but its connection with another area of investigation soon became evident.[63] Taking up again in 1674 the study of the nature of accelerative forces sketched out in the opening of the 1659 treatise on centrifugal force, he defined as *incitation:*

> la force qui ait sur un corps pour le mouvoir quand il est en repos ou pour augmenter ou diminuer sa vitesse quand il est en movement.

"La quantity de l'incitation � chaque instant de son movement," he added,

> se measure par la force qu'il faudrait employer pour emp�cher le corps de commencer � se mouvoir, � l'endroit o� il se trouve, et dans la direction qu'il a.[64]

He offered as example a ball held in check at a point on a curved surface by a force parallel to the tangent to the surface at that point. He then turned to the spring, which he drew in the form both of a straight leaf and of a helix: the *incitation* it exerts in pushing or pulling a body is also measure at any point by the force necessary to hold the body at rest there. It took only the force relations of the cycloid to recognize that in a system where the *incitation* is proportional to the displacement from equilibrium the resulting oscillation is tautochronic, and it took only a knowledge of the behavior of springs to recognize that they form such systems and hence act ideally as tautochronic oscillators.[65]

Within a year Huygens had transformed that theoretical insight into a practical mechanism. The *eureka* is dated 20 January 1675 for the first sketch of a watch balance regulated by a coiled spring.[66] A series of refinements during the next week led finally to an announcement sent in anagram to the Royal Society on 30 January: "the axis of a moving circle attached to the center of a metal coil."[67] Readers of the *Journal des S�avans* learned of the invention in February.[68]

Various preoccupations over the next few years, not the least of these being bitter priority disputes with Isaac Thuret, Robert Hooke, and Jean de Hautefeuille over the invention of the spring balance, distracted Huygens from extending it beyond its initial application to pocket watches.[69] But the idea of a larger format had occurred to him from the outset, and by 1679 he was speaking openly about a spring-regulated marine clock. Work on such a seq-going version reached by 1682 a state sufficiently promising to attract formal encouragement from the Directors of the (Dutch) East India Company as a possible means of determing longitude at sea.[70]

Although the evidence is quite thin and circumstantial, it appears that Huygens' experiments with springs themselves and with spring-regulated clocks had by 1683 borne out Oldenburg's warning of 1675: springs are very sensitive to changes in temperature and humidity.[71] Toward the end of 1683, having found that cold accelerated the oscillations of a clock spring, Huygens began looking for some other form of tautochronic oscillator, one that would have "the effect of the spring without the spring." On 4 December he found it in the *pendulum cylindricum trichordon*, consisting of three equal wires hanging parallel to one another from a fixed mounting and attached at equidistant points on the inside perimeter of a heavy ring free to rotate and in doing so to rise and fall along its

so that he could get an English translation underway. That translation did no appear until 1669 in the *Philosophical Transactions* (Vol 4, no. 47, 937-953) Huygens undertook a French translation, but postponed publication until he could add data from later voyages.

57. Cf. Huygens' correspondence with Moray and Chapelain in the spring of 1665, HOC.V.*passim*. Huygens' various attempts to secure the rights to his clocks would shed light on the early history of patents and warrant careful study.

58. Huygens tried in 1667 to enlist the Dutch East India Company in making sea trials but had to reject the Company's proposal that he supervise them personally; his position in the new Acad�mie des Sciences would have made that impolitic at best. In March 1668 the Acad�mie arranged for de la Voye to accompany a squadron under the command of the Duc de Beaufort, sailing from Toulon to Candia (Crete) to attack the Turkish fleet there. The expedition lasted until the fall of 1669, although Huygens received interim reports; cf. HOC.VI.219 (11 May 1668), 379 (13 March 1669), 486 (4 September 1669). De la Voye's final report went to Colbert in October 1669; it is not extant, but Huygens' comments on it remain (HOC.VI.501-503; *18*, 633-635). For Huygens' high expectations of the Richer expedition of 1670, see his letter to Oldenburg, 22 January 1670, HOC.VII.4.

59. Oldenburg to Huygens, 31 January 1670, HOC.VII.6. Francis Vernon heard Huygens' response *viva voce*, indeed *vivissima voce*; cf. *Ibid.,* 7-13.

60. Huygens to [Duhamel?], 4 February 1671, HOC.VII.54. The editors incorrectly refer to the *M�moires de l'Acad�mie Royale des Sciences, 1666-1699,* Vol. VII, 321, for Richer's report. That source details Richer's measurement in Cayenne of the length of a one-second pendulum.

61. Huygens to Oldenburg, 13 February 1672, HOC.VII.142.

62. HOC.XVIII.489; the editors place the date of the entry in Ms. D sometime in the second half of 1673.

63. Mersenne knew from musical experience that a string of given length an tension is tautochronic: whether strongly or weakly plucked, it sounds the same note. Cf. *Harmonicorum libri* (Paris, 2nd ed., 1648; 1st ed., 1636), Pro 29, p. 24 (cited by the ed. Of HOC.XVIII.486). For Huygens' analysis by means of the tautochrony of cycloidal oscillation, see HOC.XVIII.490-494. For his early knowledge of the problem, see his letter to Moray, 8 August 1664, HOC.V.99.

64. HOC.XVIII.496-497.

65. It is not clear when or where Huygens gained his knowledge of what is known as Hooke's Law. Hooke himself did not announce the law -ut tension sic vis- until 1678 in his *De potentia restitutive* (cf. HOC.VII.525, no. 24). Writing to Leibniz in 1691 (HOC.X.58; HOC.XVIII.484) Huygens based th law on *exp�rience* without mentioning Hooke.

66. HOC.VII.408. There follows immediately (409-416) on that sketch, found in Ms. E, an almost day-by-day account of Huygens' dealing with Thuret from 21 January to 25 February. Although that account, evidently composed in retrospect, may shae some circumstances in Huygens' favor, or datum is fixed by external reference, namely, the anagram sent to Oldenburg on 30 January; cf. *Ibid*., 409.

67. Huygens to Oldenburg, 20 February 1675, HOC.VII.422.

68. *Journal des S�avans*, 25 February 1675; extracted in the *Philosophical Transactions*, No. 112 (25 March 1675 [O.S.]) and reprinted in HOC.VII.424-425. The French *privil�ge* is dated 15 February 1675 (*ibid*., 419-420)..

69. The various contentions dominate Huygens' correspondence of 1675 unt his final settlement with Thuret in September (HOC.VII.497). To pursue the claims and their justification would go beyond the scope of the present pape although in the case of Thuret it seems clear that he deserved credit for designing an effective escapement; cf. J.H. Leopold, "Christiaan Huygens and his Instrument Makers", *Studies on Christiaan Huygens* (Lisse, 1980), 227-228. Moreover, it should be noted -as it does not seem to be in HOC- that Huygens had heard from Moray in 1665 about an idea of Hooke's to use a spring instead of a pendulum to regulate a balance wheel. Huygens all but dismissed the notion at the time; cf. His replies of 18 September, HOC.V.48 and 24 December 1665, *ibid*., 549.

70. Both the French *privilege* (above, no. 68) and the resolution of the States of Holland to award an *octroi* (25 September 1675, HOC.XVIII.523) - though not the *octroi* itself (*ibid*., 524) - speak of a large version of the spring-balance watch to be used for determining longitude. Huygens talked about it in 1679 in a "M�moire concernant l'Acad�mie des Sciences" written for Paul Pellisson's use in composing his *Histoire du Roy* (HOC.VIII.197) and referred to recent experiments in a letter of 1 October 1682 to the Abb� Gallois (*ibid*., 394). In December 1682 he asked Gallois to arrange for an extension of his leave of absence from the Acad�mie so that he could supervise sea trials in Holland; cf. HOC.VIII.405-406, for Gallois' reply of 7 January 1683 to Huygens' no longer extant request. The Directors' first resolution of support came on 31 December 1682 (HOC.XVIII.533).

71. Oldenburg to Huygens, 21 March 1675, HOC.VII.427.

72. HOC.XVIII.527-533.

perpendicular axis (see Figure 6).[72] His analysis of the mechanism began on 7 December with the condition of tautochrony. Considering each cord as a pendulum with one third of the ring's weight as bob, he invoked the displacement principle to show that the motion would be tautochronic if the pendulum followed a certain parabola, which he then demonstrated to be very close to the actual space-curve generated by the endpoints of the cords. Noting that the period (but not the approximation to tautochrony) depended on the square root of the length of the cords and on the size (but not the weight for that size) of the ring, he foresaw calibrating the pendulum by small weights movable radially on the ring; one of his drawings even shows small cheeks about the suspension of one of the cords, apparently to guide the tricorn pendulum into the proper parabolic paths.

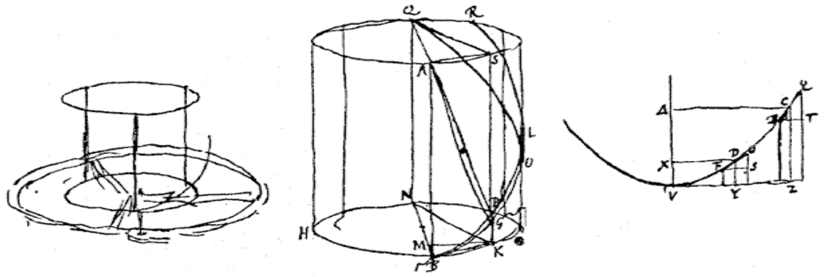

Figure 6. (HOC.XVIII.527)

A highly imaginative device posing significant difficulties for mathematical analysis, the tricorn pendulum nonetheless proved practicable within a short time. On 17 December drawings and a rough model went to the clockmaker Johannes van Ceulen, who soon produced two clocks with the new regulator. After some adjustments in design during the spring, Huygens had a clock capable of convincing the East India Company in July 1684 to pay van Ceulen and to place a boat at Huygens' disposal for trials on the Zuyder Zee.[73]

At about the same time Huygens began to sketch the first versions of what emerged in early 1693 as his *balancier marin parfait*.[74] This new tautochronic regulator basically took the form of a vertical balance wheel over which a cord passed linking two systems of small weights, each of which by its rise and fall in following the swinging balance increased the force exerted alternately on the cord by the weights. Two of the systems had originally been designed in 1659 to counterbalance the centrifugal force of a conical pendulum and to pull it back into a paraboloidal envelope: the first consisted of two piles of chain resting on a common support; the second, of two weights partially immersed in mercury.[75] As one side rose and thus pulled more heavily on the cord, the other side rested more heavily on the support or was immersed more deeply in the mercury. In each case, the torque acting on the wheel varied as its displacement from equilibrium and hence produced tautochronic oscillation.

73. See the Directors' resolutions, HOC.XVIII.533-535. The tests went on well into 1685; cf. HOC.IX.24-25, 30-31.

74. HOC.XVIII.536-538 (1683/84); 546-561 (1693).

75. Cf. HOC.XVII.88-89.

6. Final Test and New Problems

As noted above, the spring balance and its successors prompted Huygens as early as 1679 to propose new sea trials of his method of determining longitude by means of clocks. The time the Dutch were the first to respond, starting in 1682.[76] Formal encouragement became financial support in 1684, and when Huygens' personal experiments on the Zuyder Zee proved successful, the East India Company made provision for two clocks and two attendants to accompany a fleet to the Cape of Good Hope in 1686.[77]

The finely detail instructions prepared by Huygens for one of the attendants, Thomas Helder, on 23 April 1686 incorporated not only all the procedures of the *Kort Onderwijs* but, with the memory of Richer's behavior perhaps still fresh set out explicit rules for the mounting, regulatin, and maintenance of the clocks.[78] Huygens' instructions make clear by words and drawings that those clocks were spring-driven remontoirs with triangularly suspended cycloidal pendulums.[79] That is, for reasons not stated, he abandoned the experimental designs that had stimulated the new trials and placed his reliance on the marine clock as it stood in 1671.

The ship *Alkmaar* carrying the clocks returned to Texel on 15 August 1687. Helder did not return with it; shortly after leaving the Cape he had died at sea. Moreover, on the outgoing boyage he had had trouble with the clocks, which had responded badly to heavy seas. The other attendant, Johannes de Graaf, picked up the experiment and brought home enough measurements for Huygens to plot a course that could be compared with that of the fleet's pilots.[80] According to the clocks, the *Alkmaar* had sailed right through Ireland and Scotland rather than around them.

Having examined the logs kept by both assistants, Huygens knew he could not fault their work. Instead, he concluded that he had to take seriously a perturbing effect of which he had first heard in about 1685 but which he had then considered unproved. On the 1672-73 expedition to Cayenne -- on which Huygens had refused to send his new clocks -- Richer had carried out instructions to determine the length of a 1-second pendulum. In his *Observations astornomique et physiques faites en l'Isle de Caienne* published in Paris in 1679,[81] he reported that careful and repeated measurements agreed on a length shorter by 1-3/4 *lignes* than that found for Paris.[82] Richer drew no horological consequences of that result, but on reading it Huygens did: a clock regulated to mean solar time in Paris and then carried to Cayenne would fall behind by just less than 2*m* 10*s* as a

76. See above, n.10.

77. Huygens agreed to the tests in a letter to Johan Hudde, 26 October 1685, of which only Huygens' minute remains (HOC.IX.37); cf. Huygens' busy preparation, reported to brother Constantijn, 6 April 1686, *ibid*., 51.

78. HOC.IX.55-76; according to the minute of 26 October 1685 (see previous note), the instructions were a Dutch translation of those given to de la Voye in 1668.

79. In remontoir clocks the escapement is driven directly by a small spring c weight, which is rewound -hence the name- at very short intervals by the mainspring or main weight. Ideally the clocks have the advantage of maintaining a small, steady force on the escapement (and hence on the pendulum or spring balance). Huygens was experimenting with both types c remontoir as early as 1665 (cf. his exchange with Chapelain concerning Thuret's mechanism, HOC.V.511, 525), but at the time he found the design too sensitive to disturbance; see his letter to Lodewijk, 4 December 1667, HOC.VI.167.

80. See Huygens' "Rapport aengaende de Lengdevindingh door mijne Horologien op de Reys van de Caep de B. Esperance tot Texel A^{o} 1687," 24 April 1688, HOC.IX.272-291. Huygens' three-color map is reproduced at th end of HOC 9; cf. the discussion in HOC.XVIII.636-642.

81. Republished in *M�moires de l'Acad�mie Royale des Sciences* (1666-1699), VII, 233-326; the measurement is reported on p. 320.

82. Although the foot differed in length from region to region, all systems subdivided it into 12 inches (*pouce*, *duym*) and 144 lines (*lignes*, *linie*).

result of the apparent lengthening of its pendulum. The deviation was of the same order as the equation of time and hence had to affect the method of longitude.

Richer was not alone in reporting variations in the length of the 1-second pendulum. A Gabriel Mouton had published a value for Lyon that significantly differed from that for Paris, and most recently a group sent by the Acad�mie to the Cape Verde Islands and the West Indies had announced differing lengths for various locations. Yet, astronomers and geographers of the stature of Jean Picard doubted these findings and insisted that the length was constant.[83] Hugyens felt as late as May 1687 that the matter was undecided; he could give theoretical reasons both for a variation in length and for a universaly constant length.[84] The trials of 1686-87 apparently decided him. To account for the seeming failure of his clocks on the *Alkmaar*, he reasoned that the earth's rotation produced a centrifugal force that diminished the weights of bodies by a factor dependent on their latitude. A body at either pole suffered no diminution; at the equator it underwent a maximum decreased of roughly $\frac{1}{289}$ of its weight.[85] Translatged into horological terms, the diminution of weight by centrifugal force meant that a clock regulated to mean time at the pole would fall behind at the equator by a bit more than 2-1/2 minutes a day. In navigational terms, a clock carried along the same meridian from higher latitudes toward the equator would appear to indicate a longitudinal shift to the east.

Huygens' theoretical calculations of the centrifugal effect agreed on the one hand with his treatise on the cause of weight, which he had composed in 1669 but which only now found reason to publish, and on the other with most of the reported lengths of 1-second pendulums. More importantly they brought de Graaf's raw data into general agreement with the course plotted by the fleet's pilots. The agreement was not complete; Huygens knew it could not be so. The pilots had based their calculations on an incorrect longitude for the Cape, which had meanwhile been determined quite accurately by means of the satellites of Jupiter.[86] Moreover, the charts used were demonstrably incorrect at several important points of reference.

Reviewed and generally approved by Burchard de Volder, Huygens' results encouraged the East India Company to undertake another trial, again under the supervision of de Graaf, in 1690-92. The outcome disappointed everyone. Although Huygens salvaged the measurements for the longitudinal distance from St. Iago (inthe Cape Verde Islands) to the Cape of Good Hope, and although he could cite errors in de Graaf's handling of the clocks, he had to admit that his marine pendulums did not perform well at sea.[87] Clearly, the method of longitude still hinged on a reliable clock. "I have found the business much more difficult than I thought at the outset," he had written to Bernad Fullenius in December 1683, thinking perhaps of the new tricord pendulum, "and it is still not accomplished; yet, there has been reason for no small hope." He still had reason to hope in 1693: what the pendulum lacked the "perfect marine balance" might still provide. He continued to look confidently toward yet further trials.[88] Death came first.

To the end, Huygens' work on time and longitude at sea continued the interplay between theory and practice that had characterized his earliest efforts. Each sea trial pitted the calculable world of theoretical mechanics against the arbitrary reality of wind and water. Each trial yielded new events and data to be incorporated as perturbations of the mathematical model. Yet, even while that combination of theory and practice led to ever more refined mechanics and timepieces, it also encountered the limits of Huygens' theoretical world and of the realm in which he was willing to practice. Faced in 1687 with the fact of geographical variations in the length of a one-second pendulum, Huygens turned to his mechanical theory of weight and to the measure of the perturbing centrifugal force. He did not turn back to the spring balance, which is immune to that perturbation.[89] The reason seems clear, and it points to the core of Huygens' science. The factors that perturbed the spring resided in its chemistry, in the matter within it and in the effects of temperature and humidity on that matter. Between these factors and the science of motion stood the tedious, dirty, empirical search of men like Mariotte, Boyle, and Hooke, a search in which Huygens had never taken part. As John Harrison showed, metallurgy, not mechanics, held the answer to longitude, and that answer lay behind Huygens' reach.

83. See, for example, his "observations astronomiques faites en divers endroits du Royaume," *M�m. de l'Acad.* (1666-1699, VII, 346-47, where h responded to Richer's findings and to those of Mouton in his *Observationes diametrorum solis et lunae apparentium ... Cum tabula declinationum solis Huic adjecta est Brevis dissertatio de dierum naturalium inaequalittate et d temporis aequatione* (Paris, 1670), but not to those of Varin, des Hayes, and de Glos, reported on pp. 435ff. of the same volume of the *M�moires* and made in 1682.

84. Hugyens to Philippe de la Hire, 1 May 1687, HOC9, 130-131. Huygens was suspicious of Richer's failure to record the details of his experiment; cf. HOC.XVIII.635.

85. "Rapport," 275. Huygens offered no derivation here, since he had alread do so in 1669 inhis treatise *De la cause de la pesanteur*, which he was not revising for publication (Paris, 1690).

86. Rapport," 274. Huygens referred to Guy Tachard, S.J., *Voyage de Siam des Peres J�suites envoiez par le Roy aux Indes et � la Chine* (Paris, 168 and Amsterdam, 1687). The expedition used Cassini's most recent tables of the eclipses of Jupiter's moons as a means of determining longitudinal distances from Paris.

87. Huygens, "Verklaeringh en aenmerckingen op het Journael van Jo. de Graef en 't geen ontrent de Horologien is voorgevallen in de laetste proeve der Lengdevindingh A^{o} 1690, 1691 en 1692," 24 March 1693, HOC.XVIII.643-657; cf. Huygens ' correspondence with de Graaf and de Volder during the spring of 1693, HOC.X.*passim*.

88. See, for example, Huygens' note in the *Acta eruditorum* (October 1693), 475ff. (HOC.X.512-515), where he spoke of applying one of the tractrices arising outof his solution to Debeaune's problem to the design of a marine clock and hence to the problem of longitude (*ibid*., 514-515).

89. J.H. Leopold brought this point to my attention.