Newton also derived this relationship and was disappointed to learn that Huygens had already published the same results.

Huygens's train of thought is of great importance to the history of science because here it was established—contradicting the Peripatetics and even Galileo—not only that to maintain circular motion, a force is always required (which, by the way, Descartes already knew), but also that a numerical value could be calculated for this force. In this way, Huygens smoothed the way for a precise determination of acceleration in motion along curved paths.

As we have already mentioned, Huygens was no philosopher; his strength lay—as we have seen—in the establishment of simple, reasonable, but very productive fundamental physical principles. Nonetheless, Quotation 3.45, taken from the foreword to his book dedicated to problems of optics (*Traité de la lumière*), represents one of the most cogent formulations of the basic principles of natural philosophy.

3.7 Newton and the Principia: The Newtonian Worldview

3.7.1 *The Tasks Awaiting the Advent of Newton*

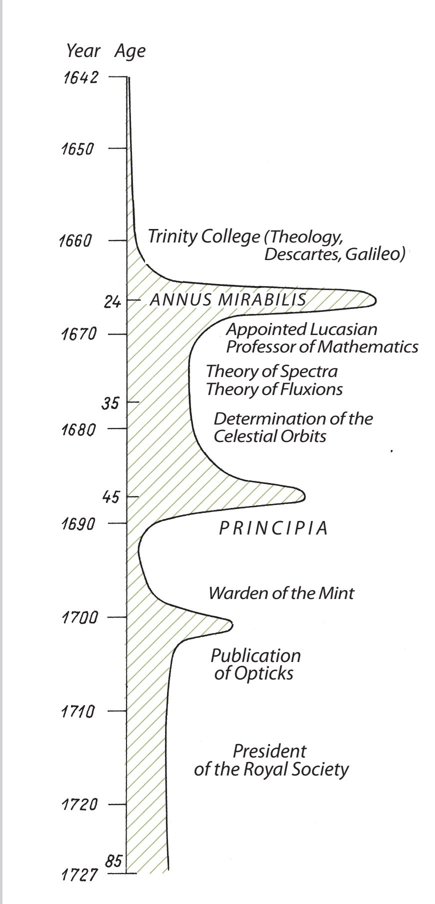
In the previous sections, we have sketched the path and followed the ideas leading to a new dynamics. Let us see—by summarizing the results of the first seven to eight decades of the seventeenth century—what was there for Newton to build on and what tasks were awaiting him.

We have spoken so far of three strands by which these ideas developed: free fall, collision, and circular motion.

- *Free fall.* The problem comprises the kinematics of bodies moving with constant acceleration, the proportionality of the distance traveled to the square of the time, and the surprising fact that every object—at least under ideal conditions—falls with the same acceleration. This fact greatly simplifies the kinematic description, but it also complicates its dynamic interpretation. Huygens's wide-ranging investigation into the problem of free fall did not bring us significantly closer to the goal, even though it was indirectly very useful because it showed that the right choice of an initial starting point—such as Huygens's principle on the center of gravity that was discussed in detail—can yield a broad multitude of concrete results.
- *Collision.* The momentum—that is, the product of the mass of the body and its velocity—as well as its change over time clearly play key roles.
- *Circular motion.* The important realization is that in order to maintain such motion, a force is necessary, contrary to the Peripatetic belief that circular motion can to some extent be seen as inertial, or naturally given, motion. More generally, it is precisely with circular motion, as the simplest form of curvilinear motion, that the vector nature of velocity and the change of velocity are the most evident and are also quantitatively accessible.

Behind all this lies a new law of inertia, recognized as final and irrevocable, according to which motion is a *state* and not a *process* and an effective cause is needed not to maintain it but to change it.

Finally, Descartes put his stamp on the worldview of physicists by requiring a unified explanation of celestial and terrestrial phenomena, and further requiring that this explanation be clearly formulated, meaning that interaction is possible only by immediately visible and perceptible contact.



▲ **Figure 3.117** Important events and creative periods in Newton's life. See also Plate XVII.

Isaac Newton (1642–1727): According to the Julian calendar then in use, born Christmas day in 1642; however, on the Continent, the new year had already begun. Newton's father had died several months before his son's birth. From 1661, Newton, with the support of his uncle, studied mathematics at Trinity College, Cambridge. During the plague epidemic in 1665, he withdrew to his estate in Woolsthorpe; this and the following year are known as Newton's "anni mirabiles" (Plate XVII). At only 24 years of age, Newton conceived the fundamental ideas for the binomial theorem, differential calculus, theory of color, centripetal force, laws of motion, and theory of gravitation. On his return to Cambridge, he dealt with problems of optics; in 1668, he completed a reflecting telescope. In 1669, he was appointed to the Lucasian Chair of Mathematics at the University of Cambridge, to succeed Isaac Barrow. In 1672, he presented his *Theory of Light and Colors* to the Royal Society; this book precipitated such dispute that he decided not to publish anything further. A summary of his work on optics was not published until

continued on next page

Figure 3.117 *continued*

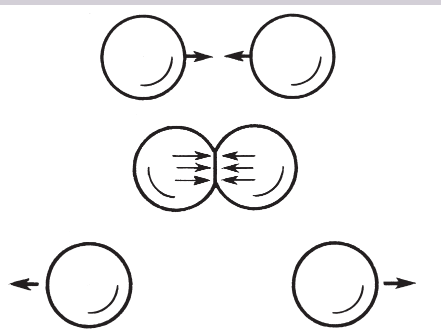
1704, in the book *Opticks*. In 1684, on the urging of Halley, he began to write the *Principia*; Halley then assumed the costs of publication. In the years 1692 and 1693, Newton suffered the consequences of a severe nervous breakdown; he recovered, retaining his full intellectual capacities, although he made no further significant scientific contributions in the remaining 35 years of his life. That he was capable of such is demonstrated by his solution in a single night of a problem posed by Bernoulli (1696), although the task was estimated to take six months, and the solution of a problem posed by Leibniz almost in the moment that he became aware of it (1716).

In 1699, Newton was named warden of the Royal Mint, and in 1705, he was knighted by the queen. From 1703 until his death in 1727, he was president of the Royal Society. He is buried in Westminster Abbey.

Quotation 3.41

It is often said in explanation of the … force which prevents the quicksilver from falling down, as it would naturally do, [that it] is internal to the vessel, arising either from the vacuum or from some exceedingly rarefied substance; but I assert that it is external and that the force comes from without. On the surface of the liquid which is in the bowl there rests the weight of a height of fifty miles of air; then what wonder is it if into the vessel, in which the quicksilver has no inclination and no repugnance, not even the slightest, to being there, it should enter and should rise in a column high enough to make equilibrium with the weight of the external air which forces it up?

—Evangelista Torricelli, Letter to Michelangelo Ricci, June 11, 1644



▲ **Figure 3.118** The phases of a symmetric elastic collision.

Toward the end of the century, a fourth strand waiting to be spun together with the above three, the problem of planetary motion, moved to the forefront. In this manner Kepler's laws could finally get the recognition they merited.

What was awaiting Newton was the task of uniting these more-or-less independent problems into a single worldview (Figure 3.117, Plate XVII).

In the end, the unified development of mechanics and indeed of the entire physical worldview rest on two fundamental realizations made by Newton. The first is the law of motion, which established a quantitative relationship between the change of a state of motion and the underlying force, that is, the recognition of the relationship

$$\text{force} = \text{mass} \times \text{acceleration}.$$

The second is the universal law of gravitation, according to which the attractive gravitational force between any two bodies is proportional to the product of their masses and is inversely proportional to the square of the distance between them.

The first law can provide the force if we know the motion, or the motion if we know the force, reducing all the previous problems of this form in the history of science to special cases; Newton himself provided posterity with an almost inexhaustible supply of new applications. The second law guarantees the unity of the celestial and terrestrial worlds because the path of a stone falling from a tower and the path traced by the Moon or any planet could now be calculated according to the same law.

In the following sections, we attempt to present as simply as possible, using today's common terminology, the train of thought that led Newton to these laws. In his *Principia*, the bible of classical physics, Newton's ideas and results already appear in their final form and in great generality wrapped, as it were, in the ceremonial vestments inherited from Euclid and Archimedes of theorems followed by proofs. We may learn about the difficult process of the birth of his ideas partly from Newton himself and partly from his contemporaries. We can especially rejoice in the fact that Newton had the habit of noting his yet unripe thoughts and calculations in a journal. This journal, which Newton called his *waste book*, is, with its collection of fragments of ideas, perhaps the most important surviving material in Newton's own hand. In the modern reappraisal of these papers, many exciting pieces of information for the history of science have come to light; however, due to their large number, we can only occasionally touch on these.

3.7.2 *A Force Is Not Required to Maintain a State of Motion but to Change It*

Let us now consider the steps that led to the formulation of the law of motion. We have frequently spoken of the importance of collision processes for the discovery of this law. Let us consider the simplest of such processes, the head-on collision of two elastic balls of equal masses moving at equal speeds. The result of the collision—both balls ricocheting off each other with reversed directions and the same speeds—is so obvious that Huygens, as we have seen, used it as an axiom in his treatment of more general collisions.

Let us consider in a bit more detail what happens physically during the course of a collision, irrespectively of how quickly it takes place. As depicted in Figure 3.118, the two balls are elastically deformed, and therefore each pushes against the other. As a result of this force, each of the balls is slowed down, and there is a moment at which the velocities of the two balls are zero. Newton's first significant insight in connection with this is that a force is required to bring a moving object to a standstill (that is, to reduce a momentum to zero), and this force is due to the pressure from the elastic deformation of the balls.

In the next phase of the collision, as a result of the forces exerted by the deformed balls, both balls recover their original velocities, but in the opposite directions. NEWTON's second observation is that the force needed to create a particular motion is the same as what is required to make that motion cease.

NEWTON's third observation is that during their interaction, each body affects the other with an equal but opposite force. A generalization of this fact later led to NEWTON's third axiom.

The collision, which lasts a very short time, is in fact a transition between two inertial states of the bodies, that is before and after the collision, they execute linear motion at constant velocity in the absence of any outside influence. The result of the collision can therefore be described quantitatively because uniform linear motion can be characterized by well-defined state parameters, namely, momentum and velocity (Figure 3.119).

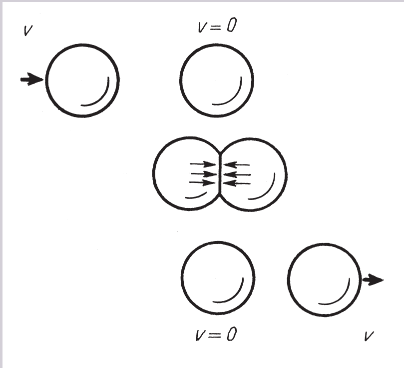
The force acting on a body in motion along a curved path can be easily understood if we reduce the effect to a series of collisions. Suppose a body is moving, as shown in Figure 3.120, along the linear path AB with constant velocity when it collides at point P with another body in such a way that, at impact, a momentum of magnitude mv_p is transmitted to the body in the direction PC. The body then continues its path in the direction PP', and suppose that at point P' it collides with another body so that its direction changes once again. The result of this sequence of collisions is that the body travels along a polygonal path.

A very important step toward generalization is if we assume that an abrupt change in the state of motion does not have to be the consequence of a collision, in other words through direct contact, but that it could also be caused by any type of force. Even gravitational force can be regarded from this point of view as a consequence of brief impulses, as had already been done by BEECKMAN (see Section 3.3).

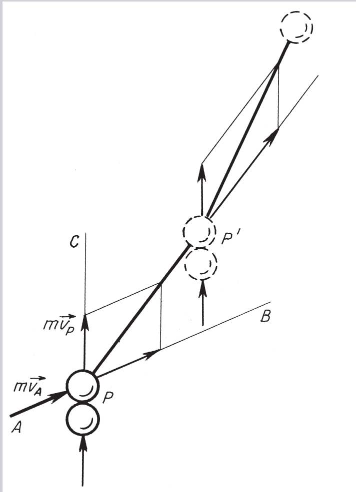
Here we call attention to a peculiarity in the terminology. In keeping with NEWTON, here we use the notion of "force" differently from the way it is used today. According to our current nomenclature, the quantity that results in a given change in momentum is not called a force; today, force means the *change* in momentum *per unit of time.* A given change in momentum is equal to the product of the force and the time over which the force acts, or more precisely, equal to the time integral over the force.

If we consider again what happens during the second phase of the elastic collision depicted in Figures 3.118 and 3.119. We see that, for example, the ball on the right is accelerated from its resting position due to the effective "force" of the pressure in the direction of that pressure, and as a result it acquires a certain momentum. This momentum is therefore proportional to the "force" and has the same direction. What happens now at point P of Figure 3.120? If no physical effects were acting here on the bodies, the ball would continue moving along the line AB. Keeping in mind the principle of "independence of motion" formulated by GALILEO, we see that the motion in the direction PC combines with the original motion, and so we may formulate the following theorem on motion resulting from a collision: *The change in momentum is proportional to and has the same direction as the effective "force."*

We stress once more that here we have been using the term "force" in NEWTON's sense; in today's language we would substitute the product of force and time.



▲ **Figure 3.119** In this particular collision, the ball on the left will come to a stop, while the equal but opposite effect will accelerate the other ball to its original velocity.

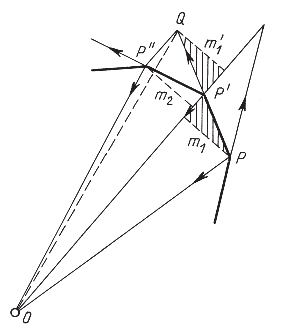


▲ **Figure 3.120** We can force a body to traverse a polygonal path through a series of collisions. In passing to the limit, we obtain motion along a curved path.

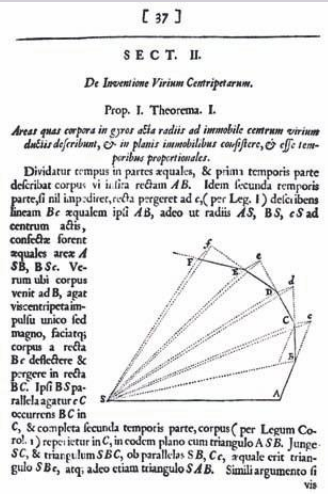
Quotation 3.42

Let all the disciples of ARISTOTLE gather together all the strength in the writings of their master and his commentators in order, if they can, to make these things reasonable by means of the horror of the vacuum. Except that they know that experiments are the true masters that must be followed in physics. And that what has been accomplished in the mountains reverses the common belief of the world that Nature abhors a vacuum; it has also established the knowledge—which will never die—that Nature has no horror of a vacuum, and that the heaviness of the mass of air is the true cause of all the effects which have previously been attributed to this imaginary cause.

—BLAISE PASCAL [Dugas 1957, p.171]



◀ **Figure 3.121** For central forces, the area law holds: The surfaces of the triangles OPP' and $OP'P''$ are equal. Since the time intervals are equal, we have $PP' = P'Q$, and therefore $m_1 = m_1'$ and $OP' \parallel QP''$, so that $m_1 = m_2$ and $OP'm_1 = OP'm_2$.



[37]

SECT. II.

De Inventione Virium Centripetarum.

Prop. I. Theorema. I.

Areas quas corpora in gyros acta radiis ad immobile centrum virium ductis describunt, & in planis immobilibus consistere, & esse temporibus proportionales.

Dividatur tempus in partes æquales, & prima temporis parte describat corpus vi insita rectam *AB*. Idem secunda temporis parte, si nil impediret, recta pergeret ad *c*, (per Leg. I) describens lineam *Bc* æqualem ipsi *AB*, adeo ut radiis *AS*, B*S*, *cS* ad centrum actis, confectæ forent æquales areæ *A SB*, B*Sc*. Verum ubi corpus venit ad B, agat vis centripeta impulsu unico sed magno, faciatq; corpus a recta B*c* deflectere & pergere in recta BC. Ipsi B*S* parallela agatur *c*C occurrens B*C* in C, & completa secunda temporis parte, corpus (per Legum Corol. 1) reperietur in *C*, in eodem plano cum triangulo A*SB*. Junge *SC*, & triangulum *SBC*, ob parallelas *SB*, *Cc*, æquale erit triangulo *SBc*, atq; adeo etiam triangulo *SAB*. Simili argumento si vis

▲ **Figure 3.122** The area law in Newton's *Principia*.

Quotation 3.43

Mr. Descartes had discovered how to make his conjectures and fictions be taken for truths. And something similar happened to those who read his *Principles of Philosophy* as to those who read the Romans who are pleasing and give the impression of being true histories. The novelty of the figures and little particles and vortices make it very charming. It seemed to me, when I read this book of *Principles* for the first time, that everything went swimmingly, and I thought, whenever I had some difficulty with it, that it was my fault for not clearly understanding his thought. I was only 15 or 16 years old. But having since then discovered things in it that are manifestly false, and others that are very unlikely, I have thoroughly returned from my former obsession, and at the present time I can find almost nothing that I can approve as true in his entire physics, or metaphysics, or meteorology.

—Christiaan Huygens, *Euvres Complètes*, vol. 10 [translated by Paul Franz]

As we have already explained, the forces acting on the vertices of the polygonal path can be of a quite general nature. For example, it can be a *central force*, directed toward one fixed point, the center, from any point in space. The importance of investigating this configuration is clear: Planetary motion involves precisely such a force. We assume now, following Figures 3.121 and 3.122, that the force directed toward a central point acts in the form of a rapid succession of impulses and that the path consequently has the form of a polygonal line. It is easy to see from the figure that the areas of the triangles that lie between the segments of the path and the center are equal; for example, the area of triangle OPP' is equal to the area of triangle $OP'P''$, since the two triangles share the common side OP', and the corresponding altitudes m_1 and m_2 are equal. This observation leads to Kepler's second law—the law of areas—where we see that this law is valid for an arbitrary central force and that no additional assumptions about the dependence of the force on distance are necessary. In the following, we show that the dependence of the force on the distance follows from Kepler's third law.

Let us return now to the polygonal path that arises under the influence of brief impulses and attempt to derive the quantitative relationships for circular motion as a limiting case, where we shall assume as known the geometric and kinematic characteristics of this path. Following Figure 3.123, we inscribe a square in the circle and assume that the body moves along the path described by this square with constant speed. At the vertices of the square, our object will presumably collide with an elastic circular ring, from which it will be reflected according to the laws of collision. Both the magnitude and direction of the force exerted by the elastic ring are easily specified. It is directed toward the center of the circle, and its magnitude can be determined by geometric considerations. From the similarity of the triangles OPP' and $BP'A$, we obtain

$$\frac{\Delta(mv)}{mv} = \frac{a}{r},$$

and therefore, for a collision at the vertex of the square, we have

$$\Delta(mv) = \frac{a}{r}mv.$$

The net effect of the force exerted during a complete circumnavigation of the square path can now be determined from

$$4\Delta(mv) = \frac{4a}{r}mv. \tag{1}$$

Rewriting this relationship in today's formalism, instead of the "effect of the force," we employ the product $F\Delta\tau$, where F is the force itself, and $\Delta\tau$ is the time over which the force acts, where we have made the simplifying assumption that the force is constant during the time of action. This assumption is valid for a collision only if we understand by "force" the average force. With these assumptions, equation (1) can be written as

$$F(4\Delta\tau) = \frac{4a}{r}mv.$$

Figure 3.124 shows a circle with an inscribed polygon of n sides. Because of the similarity of the triangles, we derive here as well the relationship

$$\frac{\Delta(mv)}{mv} = \frac{a}{r},$$

from which follows

$$\Delta(mv) = \frac{a}{r} mv,$$

and in today's usual notation, we have

$$F(n\Delta\tau) = \frac{na}{r} mv.$$

This relationship and formula (1) can be summarized thus: In a complete circumnavigation, the net effect of the force—that is, the force multiplied by the sum of its times of action—is equal to the momentum multiplied by the quotient of the total length of the path and the radius of the circle.

If we increase the number of vertices without bound, in the limit we arrive at the circle. The validity of the conclusions that we have reached for polygonal paths is independent of the number of sides of the polygon, so that they remain true for arbitrary n and therefore for the circular path as well. The length of the path, of course, coincides with the circumference of the circle, and equation (1) becomes

$$F\tau = \frac{2\pi r}{r} mv = 2\pi mv.$$

From this equation, we obtain the constant magnitude of the force directed toward the center of the circle:

$$F = \frac{2\pi}{\tau} mv = \frac{2\pi}{2\pi r / v} mv = \frac{mv^2}{r}. \qquad (2)$$

Because of its direction, Huygens called this force a *centripetal force*. It is transmitted by the circular ring to the body, and it forces the body to move along a circular path. Of course, the body traversing the circular path exerts an equal and opposite force outwardly against the ring.

Figure 3.125 shows a page from Newton's *waste book* in which the square inscribed into the circle can be recognized as the starting point for the entire train of thought sketched above.

According to his *waste book*, Newton treated the problem of circular motion in perfect analogy with Huygens, but independently of him.

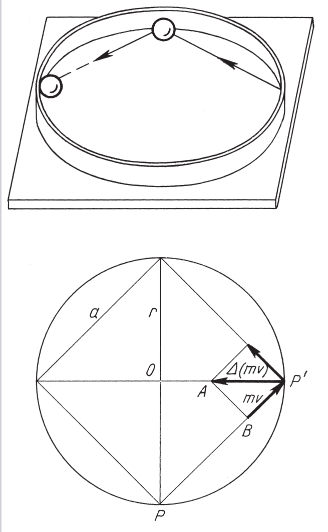
We have seen previously that circular motion can also be understood as motion with constant acceleration, given by the expression

$$a = \frac{v^2}{r},$$

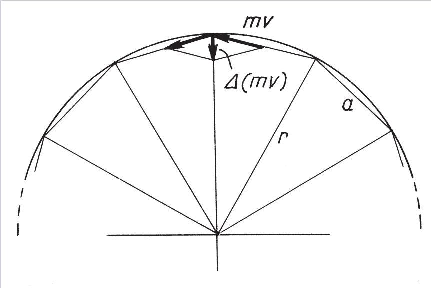
so that the force can be written as

$$F = ma = m\frac{v^2}{r}. \qquad (3)$$

This expression is identical to formula (2), but now it can be interpreted according to the relationship *force = mass × acceleration*.



◀ **Figure 3.123** For the quantitative treatment of uniform motion along a circular path, we begin with motion along a square inscribed in the circle. Because of the similarity of the triangles *ABP′* and *OPP′*, we obtain $\Delta(mv) : mv = a : r$.



▲ **Figure 3.124** For motion along an inscribed polygon with n sides, the proportion $\Delta(mv) : mv = a : r$ is still correct.

Quotation 3.44

The weight of a body is not understood, here, to be the tendency which makes it move towards the centre of the Earth, but rather to be its volume together with a certain solidity or condensation of the parts of its material which is probably the cause of its heaviness.

—Edme Mariotte, *Traité de la percussion ou choc des corps* [Dugas 1957, p. 199]

Quotation 3.45

There will be seen in it demonstrations of those kinds which do not produce as great a certitude as those of Geometry, and which even differ much therefrom, since whereas the Geometers prove their Propositions by fixed and incontestable Principles, here the Principles are verified by the conclusions to be drawn from them; the nature of these things not allowing of this being done otherwise. It is always

continued on next page

Quotation 3.45, ***continued***

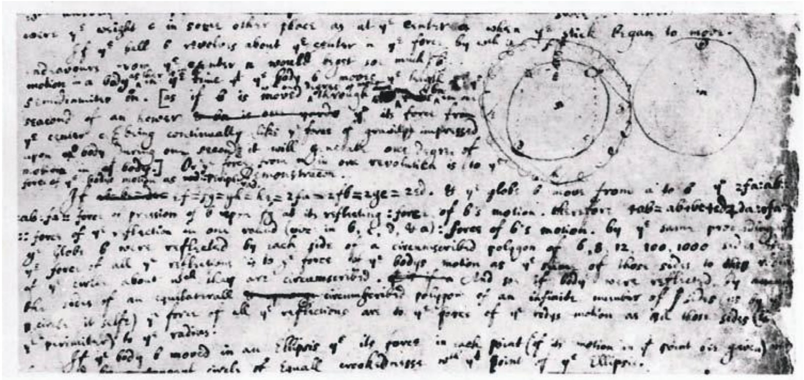
possible to attain thereby to a degree of probability which very often is scarcely less than complete proof. To wit, when things which have been demonstrated by the Principles that have been assumed correspond perfectly to the phenomena which experiment has brought under observation; especially when there are a great number of them, and further, principally, when one can imagine and foresee new phenomena which ought to follow from the hypotheses which one employs, and when one finds that therein the fact corresponds to our prevision.

—Christiaan Huygens, *Treatise on Light* [pp. vi–vii]

Quotation 3.46

After dinner, the weather being warm, we went into the garden and drank tea, under the shade of some appletrees, only he and myself. Amidst other discourse, he told me, he was just in the same situation, as when formerly, the notion of gravitation came into his mind. It was occasion'd by the fall of an apple, as he sat in a contemplative mood. Why should that apple always descend perpendicularly to the ground, thought he to himself. Why should it not go sideways or upwards, but constantly to the earth's centre? Assuredly, the reason is, that the earth draws it. There must be a drawing power in matter: and the sum of the drawing power in the matter of the earth must be in the earth's center, not in any side of the earth. Therefore does the apple fall perpendicularly, or towards the center. If matter thus draws matter, it must be in proportion to its quantity. Therefore, the apple draws the earth, as well as the earth draws the apple. That there is a power, like that we here call gravity, which extends its self thro' the universe.

—William Stukeley, *Memoirs of Sir Isaac Newton's Life*, 1936 [pp. 19–20]



▲ **Figure 3.125** Here in Newton's *waste book* these ideas appear for the first time.

Newton applied his results to circular motion—and that was the very first application of Newtonian dynamics—to investigate the relationship between the gravitational force on a body at the surface of Earth and the force that results from the rotation of Earth. Already in antiquity, the objection against Earth's rotation was raised—logically enough—that objects would be flung away from its surface. Newton showed that the force resulting from Earth's rotation is too small and that this objection is invalid. At the same time, this force is still strong enough to be measured experimentally; that is, Earth's rotation plays a role in the precise determination of the acceleration due to gravity (Figure 3.126).

3.7.3 *The Law of Universal Gravitation*

By the second half of the seventeenth century, a number of approaches toward a law of gravitational attraction had already appeared; it had even been generally formulated that all bodies mutually attract one another and that this attraction is responsible for the weight of objects on Earth's surface and therefore for motion in free fall, and also for the motion of the celestial bodies. Furthermore, the conjecture had been advanced that this force should be inversely proportional to the square of the distance separating two bodies. All this, however, was mere conjecture unsupported directly by experimental observation, and what is most important is that the supposed law of forces could not be harmonized with the elliptical planetary orbits; all such attempts had been unsuccessful.

Legend has it that the famed Newtonian apple provided the first impulse for Newton's formulation of the law of universal gravitation (Quotation 3.46).

According to Newton's own recollections and those of his friends, in deriving the law of forces, Newton proceeded as follows (Quotation 3.47): Let us assume that the planets move in circular orbits, which for most of the planets is, in fact, a good approximation. A planet moving along such a path of radius R must be acted on by the centripetal force

$$F^{\mathrm{cf}} = m\frac{v^2}{R} = m\left(\frac{2\pi R}{T}\right)^2 \frac{1}{R} = m\frac{(2\pi)^2 R}{T^2}.$$

To determine the dependence on distance of the force arising from the Sun, we must compare the orbital data of the planets in their different orbits. Such a comparison is given by Kepler's third law, according to which the squares of the orbital periods of the planets are in the same relationship as the cubes of the orbital radii:

$$T_1^2 : T_2^2 : T_3^2 : \ldots = R_1^3 : R_2^3 : R_3^3 : \ldots,$$

or

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3.$$

From this, we obtain the following relationship for the forces acting on the planets:

$$\frac{F_1}{F_2} = \frac{m_1 (2\pi)^2 R_1 / T_1^2}{m_2 (2\pi)^2 R_2 / T_2^2} = \frac{m_1}{m_2}\frac{R_1}{R_2}\frac{T_2^2}{T_1^2},$$

and since

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3,$$

we may conclude that

$$\frac{F_1}{F_2} = \frac{m_1 R_2^2}{m_2 R_1^2},$$

or

$$F_1 : F_2 : F_3 : \ldots = \frac{m_1}{R_1^2} : \frac{m_2}{R_2^2} : \frac{m_3}{R_3^2} : \ldots.$$

From this relationship, it is clear that the force is inversely proportional to the square of the distance.

If the force exerted by Earth also satisfies this law, then we can compare the attractive forces of Earth on the Moon and on bodies near Earth's surface, or simply the acceleration measured at Earth's surface and the acceleration of the Moon in its orbit about Earth. The acceleration of the Moon in its orbit is given by the small value

$$a_{\text{Moon}} = \frac{g}{60^2} = \frac{9.8}{3600} = 2.73 \times 10^{-3} \ \text{ms}^{-2},$$

since the force decreases as the square of the distance, and the average distance of the Moon from Earth is about 60 Earth radii. From a knowledge of the orbital period of the Moon, the acceleration can be calculated as follows:

$$a_{\text{Moon}} = \frac{v_{\text{M}}^2}{R_{\text{M}}} = \frac{(2\pi)^2 R_{\text{M}}}{T_{\text{M}}^2} = \frac{(2\pi)^2 \, 3.84 \times 10^8}{(27.3 \times 24 \times 3600)^2} = 2.73 \times 10^{-3} \ \text{ms}^{-2}.$$

We can see that results of both computations agree, and according to his reminiscences, NEWTON saw this agreement as very good. However, it would be closer to the truth (Quotation 3.47) to say that NEWTON was unable to obtain a satisfactory agreement with the imprecise data available to him and therefore put off further investigation of the problem for a long time (almost 15 years).

It was only a single step from here to the formulation of the law of universal gravitation: If an attractive force from a body of mass m_A acts upon a body of mass m_B, then this force must be proportional to the mass m_A. However, in the process of interaction, the body of mass m_B attracts the body of mass m_A, and this force is proportional to the mass m_B. But because both forces are the same, it follows that

Quotation 3.47

[NEWTON's] first thoughts, which gave rise to his *Principia*, he had, when he retired from Cambridge in 1666 on account of the plague. As he sat alone in a garden, he fell into a speculation on the power of gravity: that as this power is not found sensibly diminished at the remotest distance from the center of the earth, to which we can rise, neither at the tops of the loftiest buildings, nor even on the summits of the highest mountains; it appeared to him reasonable to conclude, that this power must extend much farther than was usually thought; why not as high as the moon, said he to himself? and if so, her motion must be influenced by it; perhaps she is retained in her orbit thereby. However, though the power of gravity is not sensibly weakened in the little change of distance, at which we can place our selves from the center of the earth; yet it is very possible that so high as the moon this power may differ much in strength from what it is here. To make an estimate, what might be the degree of this diminution, he considered with himself, that if the moon be retained in her orbit by the force of gravity, no doubt the primary planets are carried round the sun by the like power. And by comparing the periods of the several planets with their distances from the sun, he found, that if any power like gravity held them in their courses, its strength must diminish in the duplicate proportion of the increase of distance. This he concluded by supposing them to move in perfect circles concentrical to the sun, from which the orbits of the greatest parts of them do not much differ. Supposing therefore the power of gravity, when extended to the moon, to decrease in the same manner, he computed whether that force would be sufficient to keep the moon in her orbit. In this computation, being absent from books, he took the common estimate in use among geographers and our seamen, before Norwood had measured the earth, that 60 English miles were contained in one degree of latitude on the surface of the earth. But as this is a very faulty supposition, each degree containing about 69½ of our miles, his computation did not answer expectation; whence he concluded, that some other cause must at least join with the action of the power of gravity on the moon. On this account he laid aside for that time any farther thoughts upon this matter.

—HENRY PEMBERTON, *A View of Sir Isaac Newton's Philosophy*, 1728

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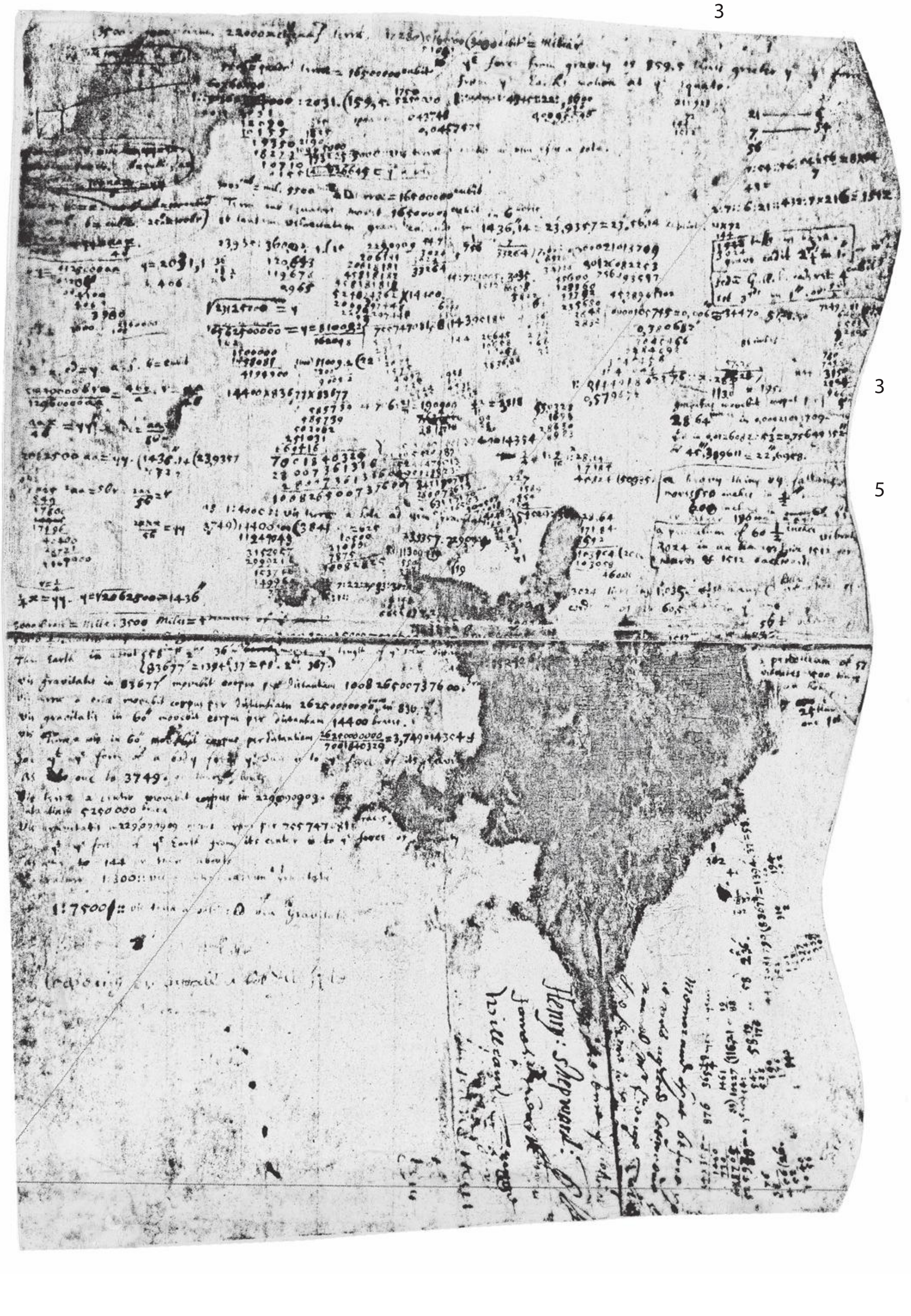

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◀ ▶ **Figure 3.126** The “vellum manuscript” (after Herivel). In the years 1665 and 1666, Newton used a sheet of parchment that had been used to draw up a rental contract as scrap paper. The numerical comparison of the gravitational and centrifugal forces and the path to this comparison can be read on this scrap.

1. Framed: *100 cubits in 5²* (100 ells in 5 seconds), data for free fall taken from the *Dialogo*.
2. $\frac{1}{4}x = yy$. The relationship for the distance traveled by a body in free fall; x is the distance and y is the time. If we write this formula as $x = \frac{1}{2}8y^2$ and compare it with today’s notation $x = \frac{1}{2}gt^2$, we then see that g = 8 ells (= cubits = braces)/s² = 8 ´ 0.685 m/s² = 5.480 m/s². This is a very poor approximation, for the correct value (g = 9.8 m/s²) is almost twice as big. The formula $x = \frac{8}{2}y^2$ corresponds exactly to the data given by Galileo (y = 5, $y^2 = 25 \rightarrow x = 100$).
3. Newton determines g more precisely. A conical pendulum of length 81 inches and half conical angle of 45° executes…
4. … in one hour 1512 oscillations (1512 *ticks in hora*). With the help of this experiment, Newton determines the value of g in two different ways: He first assumes that in the case of a conical pendulum with half conical angle of 45°, the gravitational and centrifugal forces are equal ($g = v^2 / R$ with $R = l\sin\alpha$, $l = 81$ inches, and he uses, moreover, the fact that the period of the conical pendulum agrees with that of the mathematical pendulum with length $l\cos\alpha$; that is, it is equal to the projection of the pendulum length on the vertical. Thus Newton arrives…
5. … at the conclusion: *A heavy thing in falling moves 50 inches in 1* (crossed out and corrected) *1/2², 200 inches in one ², or rather 196 inches = 5 yds.* That is already quite a good approximation ($g \approx 10$ m/s²). The last conclusion is that …
6. … *vis terrae a centro movebit corpus in* 229.09 *minutes per distantiam* 5,250,000 *braces. Vis gravitatis in* 229.09 *minutes movet corpus per* 755,747,081 *braces.*

So that the force of the earth from its centre is to the force of gravity as one to 144 or thereabout.

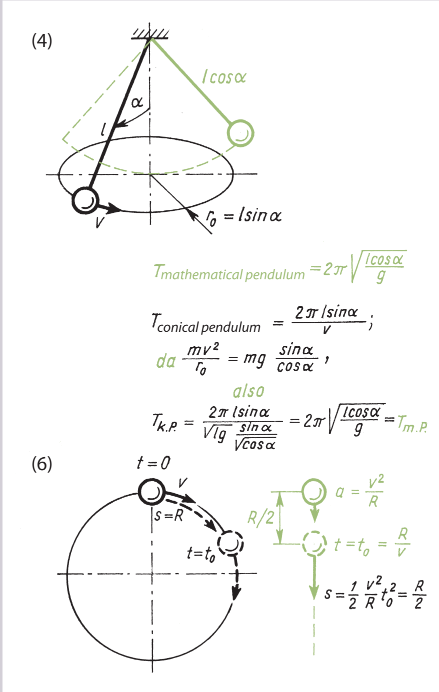
Or rather as 1 : 300 : vis a centro terrae : vim gravitatis.

Newton uses here the following properties of circular motion: If the force acting on a body moving along a circular path with radius R is used to accelerate this body along a linear path, then the body will traverse in the time that it takes to traverse a distance R on the circular path, the distance R/2. (If one sets in the formula $s = \frac{a}{2}t^2$ the values $a = v^2 / R$ and $t = R / v$, then one obtains $s = R / 2$.)

The value 229.09 minutes = 3.818 hours (= 24/2p) is the time in which a point on the Earth’s equator traverses the distance R_{Earth}. A free-falling body under the influence of centrifugal acceleration would traverse in this time the distance $R_{\text{Earth}} / 2 = 5250.000$ ells.

If for the case of actual free fall on Earth’s surface one substitutes the value for g and the time t = 229.09 minutes into the formula $s = \frac{g}{2}t^2$, then one obtains 755,747,081 (ells). The ratio of the two path lengths is equal to the ratio of the centrifugal force to the gravitational force: 1:144. (The corrected value 1:300 came from the recognition of the error made in the determination of g.)

With this, one can refute the argument against the rotation of Earth that the centrifugal force resulting from a rotating Earth would fling objects off the surface of Earth (1 cubit = brace = ell = ¾ yard = 0.685 m; 1 inch = 2.54 cm).



the attractive force must be proportional to both masses; thus, *the gravitational force is proportional to the product of the masses of the two bodies and inversely proportional to the square of the distance between them*. In mathematical form,

$$F = G\frac{m_A m_B}{R^2},$$

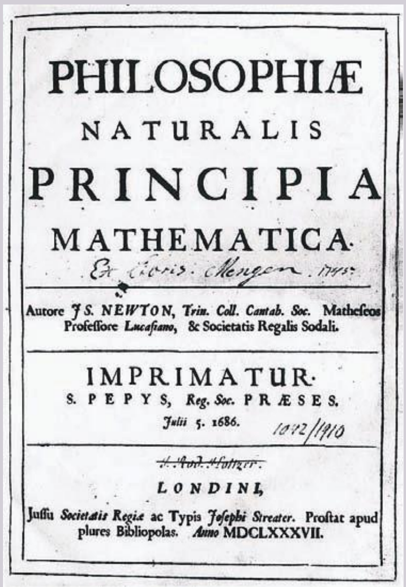
where G is a universal constant.

The proportionality factor G can be expressed in terms of the Keplerian constants and the Sun’s mass (see Figure 3.31). To this end, we write Kepler’s third law in a somewhat different form

$$\frac{R_1^3}{T_1^2} = \frac{R_2^3}{T_2^2} = \cdots = \frac{R^3}{T^2} = k_{\text{Sun}}.$$

However, the equation of motion for a planet of mass m_P is

$$G \times \frac{m_P m_S}{R^2} = m_P\frac{v^2}{R} = m_P\left(\frac{2\pi R}{T}\right)^2\frac{1}{R},$$


PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore J. S. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,
Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.

▲ **Figure 3.127** The book *Philosophiae Naturalis Principia Mathematica* is often cited as the most significant work in the history of science, or even in the history of mankind.

Newton wrote it, as he later recounted, in a period of 17–18 months. In 1684, he began a series of lectures in Cambridge on this subject. The book appeared in 1687 with the financial support of Halley.

continued on next page

Figure 1.327 *continued*

The goal of the book is to provide answers finally to the questions of celestial mechanics.

> Dr. Vincent presented to the Society a manuscript treatise entitled *Philosophiae Naturalis Principia Mathematica*, and dedicated to the Society by Mr. Isaac Newton, wherein he gives a mathematical demonstration of the Copernican hypothesis as proposed by Kepler, and makes out all the phaenomena of the celestial motions by the only supposition of a gravitation towards the center of the sun decreasing as the squares of the distances therefrom reciprocally.
>
> —Minutes of the Royal Society, April 28, 1886 [Westfall 1980, pp. 444–445]

The work begins in the introduction with definitions and axioms; there follow three books. The first book deals with the motion of bodies, where above all, motions along a conic section and under the influence of a central force (not only in the form $1/r^2$) are considered. Essentially, the motion of a single body (mass point) is examined, although Newton also investigated the attraction between bodies of finite dimensions. The second book has as its subject motion in a viscous medium. Newton investigates here the resistance of a medium that can depend linearly, quadratically, or in a more complex manner on the velocity.

With its description of vortical motion of viscous fluids, the presentation reaches its goal of refuting Descartes's theory of vortices.

In the introduction to the third book, we meet the often quoted rules for philosophical thought. In the section *Phenomena*, we encounter tables of data for the moons of Jupiter and Saturn as well as for the five planets, with information contributed by a number of observers being compared. The section *Theses* of this book is the most important of the entire *Principia*. Here we find the law of universal gravitation and the inverse proportionality of the gravitational force to the square of the distance, an extensive description of the motion of the Moon, and an explanation of the precessional motion of Earth and an interpretation of the tides.

The *Principia* makes for quite difficult reading. It is tersely formulated, and theorems once stated are not repeated when they are applied, being merely cited by number. Thus, for example, Newton proves the assertion that the force acting on the moons of Jupiter acts in the direction of Jupiter and is inversely proportional to the square of the distance as follows:

> The first part of the theorem follows from phenomenon 1 and from Theorems 2 and 3 of the first book; the second part follows from phenomenon 1 as well as Corollary VI of the fourth theorem in the same book.

The second edition appeared in 1713 under the editorship of Roger Cotes with a large number of changes. The third edition (1726, published by Henry Pemberton) contains only a few further changes. The first English version appeared in 1729 with the title *Mathematical Principles of Natural Philosophy*.

from which follows the gravitational constant

$$G = \frac{(2\pi)^2}{m_S}\frac{R^3}{T^2} = \frac{(2\pi)^2}{m_S}k_{\text{Sun}} \Rightarrow m_S = \frac{(2\pi)^2}{G}k_{\text{Sun}},$$

where k_{Sun} is Kepler's constant for the Sun.

Available to Newton were not only data for the planets, but also for the orbits of four of Jupiter's moons and five of Saturn's. With these data, he was able to determine the masses of Jupiter and Saturn. We note that

$$G = \frac{(2\pi)^2}{m_{\text{Jupiter}}}k_{\text{Jupiter}} = \frac{(2\pi)^2}{m_{\text{Saturn}}}k_{\text{Saturn}} = \frac{(2\pi)^2}{m_{\text{Sun}}}k_{\text{Sun}} \Rightarrow \frac{m_{\text{Jupiter}}}{m_{\text{Sun}}} = \frac{k_{\text{Jupiter}}}{k_{\text{Sun}}}.$$

The gravitational constant G was determined in 1797–1798 by Cavendish with the help of a torsion balance. Its value is

$$G = 6.67 \times 10^{-11}\ \text{Nm}^2\text{kg}^{-2}.$$

With this value, it is now possible to specify the masses of the Sun and the planets numerically.

3.7.4 *Selections from the Principia*

Anyone who picks up Newton's *Principia* (Figure 3.127) today will find two things greatly surprising. First, it will be seen that the law known as Newton's fundamental law of mechanics, *force* = *mass* × *acceleration*, does not appear anywhere in the book, neither in words nor in today's usual form

$$\mathbf{F} = m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}$$

or even as

$$\mathbf{F} = \frac{\mathrm{d}}{\mathrm{d}t}(m\mathbf{v})$$

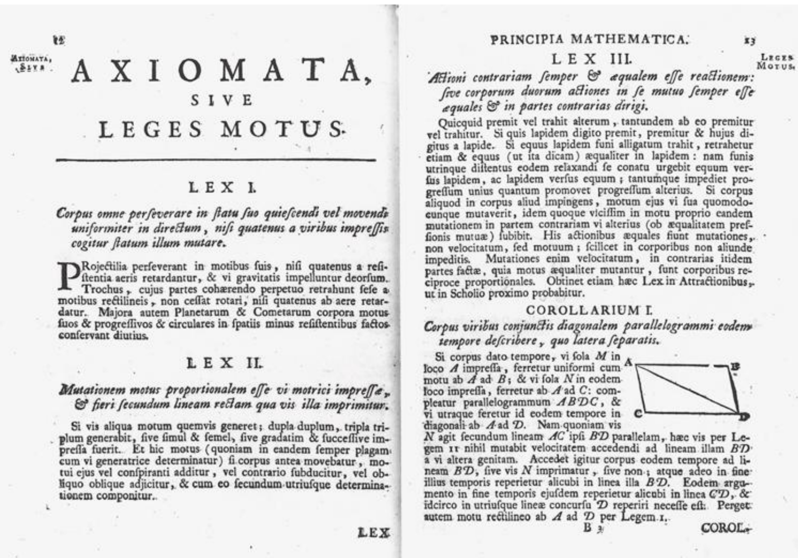
(Quotation 3.48).

The second surprising observation is that Newton, as one of the fathers of calculus, does not make use of this mathematical innovation anywhere in his book; all of his ideas are worked out on the basis of classical geometry.

We meet Newton's second axiom in the *Principia* in precisely the form already quoted: *The change in motion is proportional to the effective force and takes place along the line along which the force acts* (Figure 3.128). We have also mentioned already that by "motion," Newton means "momentum." There is no mention of changes per unit of time, nor of limits, and so the "effective force" here is not what we call force today. Accordingly, there is no mention of acceleration, mass, or force (in today's sense) in this law. In dealing with concrete cases, however, Newton uses the formulation of the law of motion familiar to us, and therefore it is apparent that he was aware of the relationship in that form as well (Quotation 3.49). The law of motion as it appears in the *Principia* can be written in today's notation as

$$\int_{t_1}^{t_2} \mathbf{F}\,\mathrm{d}t = \Delta(mv).$$

Regarding the lack of infinitesimal calculus, the general opinion is that while Newton made extensive use of the calculus of "fluxions" in his own calculations,


12

AXIOMATA, SIVE LEGES.

AXIOMATA,

SIVE

LEGES MOTUS.

LEX I.

Corpus omne perſeverare in ſtatu ſuo quieſcendi vel movendi uniformiter in directum, niſi quatenus a viribus impreſſis cogitur ſtatum illum mutare.

PRojectilia perſeverant in motibus ſuis, niſi quatenus a reſiſtentia aeris retardantur, & vi gravitatis impelluntur deorſum. Trochus, cujus partes cohærendo perpetuo retrahunt ſeſe a motibus rectilineis, non ceſſat rotari, niſi quatenus ab aere retardatur. Majora autem Planetarum & Cometarum corpora motus ſuos & progreſſivos & circulares in ſpatiis minus reſiſtentibus factos conſervant diutius.

LEX II.

Mutationem motus proportionalem eſſe vi motrici impreſſæ, & fieri ſecundum lineam rectam qua vis illa imprimitur.

Si vis aliqua motum quemvis generet; dupla duplum, tripla triplum generabit, ſive ſimul & ſemel, ſive gradatim & ſucceſſive impreſſa fuerit. Et hic motus (quoniam in eandem ſemper plagam cum vi generatrice determinatur) ſi corpus antea movebatur, motui ejus vel conſpiranti additur, vel contrario ſubducitur, vel obliquo oblique adjicitur, & cum eo ſecundum utriuſque determinationem componitur.

LEX

PRINCIPIA MATHEMATICA. 13

LEGES MOTUS.

LEX III.

Actioni contrariam ſemper & æqualem eſſe reactionem: ſive corporum duorum actiones in ſe mutuo ſemper eſſe æquales & in partes contrarias dirigi.

Quicquid premit vel trahit alterum, tantundem ab eo premitur vel trahitur. Si quis lapidem digito premit, premitur & hujus digitus a lapide. Si equus lapidem funi alligatum trahit, retrahetur etiam & equus (ut ita dicam) æqualiter in lapidem: nam funis utrinque diſtentus eodem relaxandi ſe conatu urgebit equum verſus lapidem, ac lapidem verſus equum; tantumque impediet progreſſum unius quantum promovet progreſſum alterius. Si corpus aliquod in corpus aliud impingens, motum ejus vi ſua quomodocunque mutaverit, idem quoque viciſſim in motu proprio eandem mutationem in partem contrariam vi alterius (ob æqualitatem preſſionis mutuæ) ſubibit. His actionibus æquales fiunt mutationes, non velocitatum, ſed motuum; ſcilicet in corporibus non aliunde impeditis. Mutationes enim velocitatum, in contrarias itidem partes factæ, quia motus æqualiter mutantur, ſunt corporibus reciproce proportionales. Obtinet etiam hæc Lex in Attractionibus, ut in Scholio proximo probabitur.

COROLLARIUM I.

Corpus viribus conjunctis diagonalem parallelogrammi eodem tempore deſcribere, quo latera ſeparatis.

Si corpus dato tempore, vi ſola *M* in loco *A* impreſſa, ferretur uniformi cum motu ab *A* ad *B*; & vi ſola *N* in eodem loco impreſſa, ferretur ab *A* ad *C*: compleatur parallelogrammum *ABDC*, & vi utraque feretur id eodem tempore in diagonali ab *A* ad *D*. Nam quoniam vis *N* agit ſecundum lineam *AC* ipſi *BD* parallelam, hæc vis per Legem II nihil mutabit velocitatem accedendi ad lineam illam *BD* a vi altera genitam. Accedet igitur corpus eodem tempore ad lineam *BD*, ſive vis *N* imprimatur, ſive non; atque adeo in fine illius temporis reperietur alicubi in linea illa *BD*. Eodem argumento in fine temporis ejuſdem reperietur alicubi in linea *CD*, & idcirco in utriuſque lineæ concurſu *D* reperiri neceſſe eſt. Perget autem motu rectilineo ab *A* ad *D* per Legem I.

B 3 COROL.

▲ **Figure 3.128** The two pages of the *Principia* on which the three Newtonian axioms are to be found.

he feared that the *Principia* contained enough difficult material as it was, so including the new method would simply serve to frighten readers away. Otherwise, NEWTON himself did not consider the fundamental law that bears his name to be of cardinal importance, so that in his reminiscences, in recounting his most important works, he lists the theory of color and the law of universal gravitation, but he does not even mention the fundamental law of mechanics (Plate XVII); moreover, he quite erroneously attributes the basic idea to GALILEO (Quotation 3.50).

As we have already mentioned, the *Principia*, with its definitions, axioms, propositions, and lemmas, is written according to the patterns of antiquity. With respect to the clarity of definitions and concepts as well as their arrangement in logical sequence, we note a certain carelessness of the creative genius. Thus, as we have mentioned, NEWTON used the concept of force in a sense different from that used by succeeding generations, up to the present day. Taken alone, there would be no cause for objection; but the trouble is that NEWTON himself did not use the concept of force implicit in his own law of motion in a consistent manner.

Already the very first definition in the *Principia*, that of mass, is questionable, and indeed, it is seen by many as a *circulus vitiosus*: NEWTON defines quantity of matter, or mass, as the product of volume and the mass per unit volume, that is, as the product of density and volume. This definition is certainly a vicious circle if density is defined as mass per unit volume. However, if we regard the matter as being built up of identical atoms, then density could also be defined in such a way as to avoid this difficulty. Namely, density can be understood as a quantity proportional to the number of atoms per unit volume, or, more simply, as the relative occupation of space by the material under investigation.

Clarifying the concepts used by NEWTON, primarily his notion of force, is a popular topic for many contemporary historians of science.

It is a bit difficult, but nevertheless worth the effort, to follow NEWTON's method of determining the nature of the central force from the motion of a body along an elliptical path as set down in the *Principia*. Our goal here is of course once again to derive

Quotation 3.48

Previously the relation $F = m \times a$ was in fact generally inferred from the Definitions VII and VIII and Axiom II. However, this case resembles that of the Emperor's clothes in the fairy tale: all people saw them because they were convinced of their existence, until a child said that the Emperor had nothing on. Similarly Axiom II of the introductory chapter of NEWTON's *Principia* always used to be interpreted in the sense that a constant force produces a constant acceleration, and that their magnitudes are proportional, but if one looks at it impartially, nothing of the kind can be discovered. In order to interpret it this way, one has to assume that by change (*mutatio*) NEWTON means rate of change. Only then does it become possible to formulate the statement in a modern way as

$$F = d/dt\,(mv) \text{ [where } mv \text{ multiplies } (d/dt)]$$

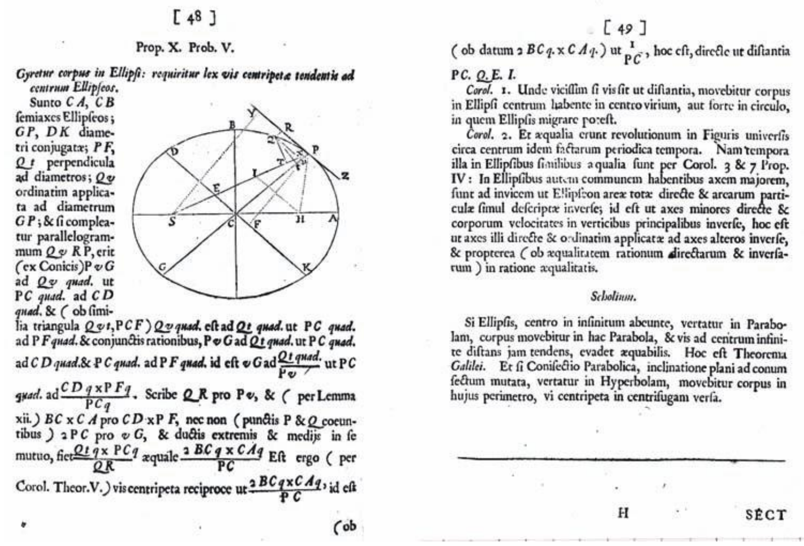
and if m is constant, this indeed amounts to

$$F = m \times a.$$

It is, however, extremely unlikely that NEWTON, who was quite capable of expressing his thoughts, should have committed the very serious mistake of confounding a magnitude and the rate of change of that magnitude in so fundamental a passage as that containing the axioms. Before making such a charge, we should consider whether it is not possible to make sense of the statement as it stands. …

That he does not explicitly argue anywhere that a constant continuous force causes a uniformly accelerated motion (which after all is really the fundamental principle of the new dynamics which radically broke with the ancient and medieval conception) can probably be explained by the fact that he considered it perfectly self-evident (just as HUYGENS had done) that if a particle is acted upon by a constant force, the velocity necessarily increases by equal amounts in equal times. Both scientists were already imbued with the new dynamic conceptions to such a degree that they did not even think it necessary to mention this most cardinal point of difference with the old notions: a striking illustration of the rapidity with which new conceptions that have at first been paradoxical become commonplace. However, from the point of view of axiomatization—and that is the main point here—the omission of the proportionality of momentum and time, i.e. of the constancy of the acceleration, amounts to a flaw: in fact, while the momentum increases as the time, the kinetic energy increases as the distance, and how is one to know either one or the other without either postulating or proving it? Moreover, according to the Aristotelian conception of axiomatization the fact that a thing is evident is precisely a reason for stating it as an axiom, not for omitting it.

—E. J. DIJKSTERHUIS, *The Mechanization of the World Picture* [pp. 471, 473]



[48]

Prop. X. Prob. V.

Gyretur corpus in Ellipſi: requiritur lex vis centripetæ tendentis ad centrum Ellipſeos.

Sunto CA, CB ſemiaxes Ellipſeos; GP, DK diametri conjugatæ; PF, Qt perpendicula ad diametros; Qv ordinatim applicata ad diametrum GP; & ſi compleatur parallelogrammum $QvRP$, erit (ex Conicis) PvG ad Qv *quad.* ut PC *quad.* ad CD *quad.* & (ob ſimilia triangula Qvt, PCF) Qv *quad.* eſt ad Qt *quad.* ut PC *quad.* ad PF *quad.* & conjunctis rationibus, PvG ad Qt *quad.* ut PC *quad.* ad CD *quad.* & PC *quad.* ad PF *quad.* id eſt vG ad $\frac{Qt\ quad.}{Pv}$ ut PC *quad.* ad $\frac{CDq \times PFq}{PCq}$. Scribe QR pro Pv, & (per Lemma xii.) $BC \times CA$ pro $CD \times PF$, nec non (punctis P & Q coeuntibus) $2PC$ pro vG, & ductis extremis & medijs in ſe mutuo, fiet $\frac{Qtq \times PCq}{QR}$ æquale $\frac{2BCq \times CAq}{PC}$. Eſt ergo (per Corol. Theor. V.) vis centripeta reciproce ut $\frac{2BCq \times CAq}{PC}$, id eſt

(ob

[49]

(ob datum $2BCq. \times CAq.$) ut $\frac{1}{PC}$, hoc eſt, directe ut diſtantia PC. Q.E.I.

Corol. 1. Unde viciſſim ſi vis ſit ut diſtantia, movebitur corpus in Ellipſi centrum habente in centro virium, aut forte in circulo, in quem Ellipſis migrare poteſt.

Corol. 2. Et æqualia erunt revolutionum in Figuris univerſis circa centrum idem factarum periodica tempora. Nam tempora illa in Ellipſibus ſimilibus æqualia ſunt per Corol. 3 & 7 Prop. IV: In Ellipſibus autem communem habentibus axem majorem, ſunt ad invicem ut Ellipſeon areæ totæ directe & arearum particulæ ſimul deſcriptæ inverſe; id eſt ut axes minores directe & corporum velocitates in verticibus principalibus inverſe, hoc eſt ut axes illi directe & ordinatim applicatæ ad axes alteros inverſe, & propterea (ob æqualitatem rationum directarum & inverſarum) in ratione æqualitatis.

Scholium.

Si Ellipſis, centro in infinitum abeunte, vertatur in Parabolam, corpus movebitur in hac Parabola, & vis ad centrum infinite diſtans jam tendens, evadet æquabilis. Hoc eſt Theorema *Galilei*. Et ſi Coniſectio Parabolica, inclinatione plani ad conum ſectum mutata, vertatur in Hyperbolam, movebitur corpus in hujus perimetro, vi centripeta in centrifugam verſa.

H SECT

▶ **Figure 3.129** The pages of the *Principia* that discuss the question of how the force law can be derived when the path is known.

the gravitational force field, which decreases as the square of the distance from the central body. However, this time we wish to arrive at it not starting from a simple circular orbit and using Kepler's third law, but on the basis of the general law of motion.

We begin with the following general theorem (*Principia, De motu corporum*, Liber I, Propositio VI, Corollarium V):

Suppose a body is moving, as depicted in Figures 3.129 and 3.130, along a curvilinear path APQ around the central point S, and let ZPR be the tangent to the path at the point P. Let us draw the segment RQ parallel to the segment SP, and the segment QT perpendicular to it. Then the centripetal force is inversely proportional to the quantity

$$\frac{SP^2 \times QT^2}{QR},$$

or, more precisely, to the limiting value of this quantity as the point Q approaches P.

The proof of this relationship is simple using today's methods. We specify the path by

$$\mathbf{r} = \mathbf{r}(t),$$

and then we expand this equation into a power series around time $t = 0$ and the place $\mathbf{r} = \mathbf{r}(0) = \mathbf{r}_p$:

$$\mathbf{r} = \mathbf{r}_P + \left.\frac{\mathrm{d}\,\mathbf{r}}{\mathrm{d}\,t}\right|_{t=0} t + \frac{1}{2}\left.\frac{\mathrm{d}^2\,\mathbf{r}}{\mathrm{d}\,t^2}\right|_{t=0} t^2 + \cdots.$$

If we now take into account the fundamental law

$$\frac{\mathrm{d}^2\,\mathbf{r}}{\mathrm{d}\,t^2} = \frac{\mathbf{F}}{m}$$

as well as the relationship $SP \times QT \propto t$, then we obtain the given expression for the centripetal force (Figure 3.131(a,b)).

We return next to the original problem, keeping to the formulation that appears in the *Principia* (Propositio IX).

Given that a body is moving along an elliptical path, let us find the law for the centripetal force directed toward the focus of the ellipse.

Suppose that at a certain moment the body is located at point P. We begin by drawing the diameter DCK conjugate to the diameter GCP (Figures 3.129 and 3.130). Newton proves that the distance of the point P from the point E, which is the intersection of the conjugate diameter with $SP \equiv \mathbf{r}_p$, is equal to the length a of the major semiaxis, that is,

$$EP = a.$$

Quotation 3.49

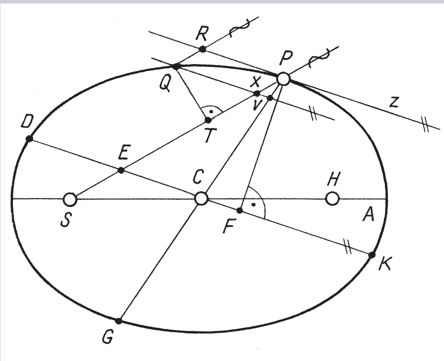
For the velocity which a given force can generate in a given matter in a given time is directly as the force and the time, and inversely as the matter. The greater the force or the time is, or the less the matter, the greater the velocity generated. This is manifest from the second Law of Motion.

—Isaac Newton, *Principia*, Book II, Section VI

Quotation 3.50

Hitherto I have laid down such principles as have been received by mathematicians, and are confirmed by abundance of experiments. By the first two Laws and the first two Corollaries, Galileo discovered that the descent of bodies varied as the square of time (*in duplicate ratione temporis*) and that the motion of projectiles was in the curve of a parabola; experience agreeing with both, unless so far as these motions are a little retarded by the resistance of the air.

—Isaac Newton, *Principia*, Axioms, or Laws of Motion, Corollary VI



▲ **Figure 3.130** The diagram from Figure 3.129 enlarged for clarity.

We now draw the line Qxv parallel to the tangent RPZ. It can then be shown by making use of the similar triangles that

$$\frac{QR}{Pv}=\frac{PE}{PC}=\frac{a}{PC}.$$

Since we also have that QT is perpendicular to SP, and PF is perpendicular to the tangent, we obtain

$$\frac{Qx}{QT}=\frac{a}{PF}.$$

From a theorem of Apollonius on conic sections, we have

$$\frac{a}{PF}=\frac{CD}{b},$$

and from that, we obtain the relationship

$$QT=\frac{PF\times Qx}{a}=\frac{bQx}{CD},$$

as well as

$$QR=\frac{aPv}{PC}.$$

We now let the point Q approach the point P. Then the quotient Q_v/Q_x approaches 1, so that we obtain

$$\lim QT=\frac{bQv}{CD}.$$

We now calculate the expression appearing in the theorem given above as

$$\lim\frac{SP^2\times QT^2}{QR}=\lim SP^2\frac{Qv^2\times b^2\times PC}{CD^2\times a\times Pv}. \quad (4)$$

The equation of the ellipse in the coordinate system determined by the conjugate semiaxes CD and CP is

$$\frac{Qv^2}{CD^2}+\frac{Cv^2}{CP^2}-1=0,$$

and with this, we can eliminate the quotient $Q_v^{\,2}/CD^2$ appearing in equation (4):

$$\frac{Qv^2}{CD^2}=\frac{CP^2-Cv^2}{CP^2}=\frac{(CP+Cv)Pv}{CP^2},$$

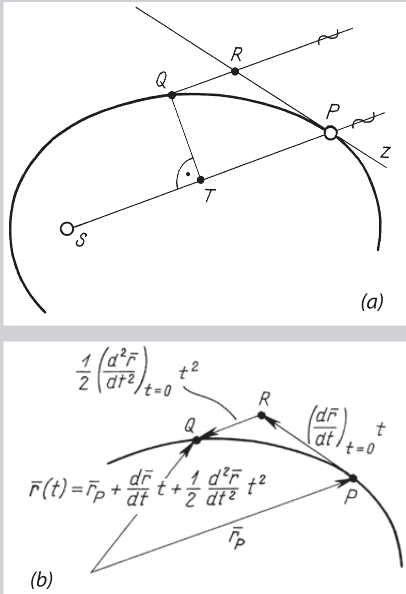
and thus we have

$$\lim\frac{SP^2\times QT^2}{QR}=\lim SP^2\frac{(CP+Cv)Pv\times b^2\times PC}{CP^2\times a\times Pv}=\frac{2PC^2\times b^2}{a\times PC^2}\times SP^2=\frac{2b^2}{a}SP^2$$
$$=L\times SP^2.$$

We can state the conclusion of the above derivation in the original words of the *Principia*: The centripetal force is reciprocal to the quantity $L\times SP^2$ and therefore inversely proportional to the square of the distance SP.

In this way, Newton, beginning with the general elliptical path, arrived at the law of the force being proportional to the inverse square of the distance.

Later, Newton raised the inverse question: given a central force that diminishes by the square of the distance, what is the path of a body in the general case? He



▲ **Figure 3.131** Since

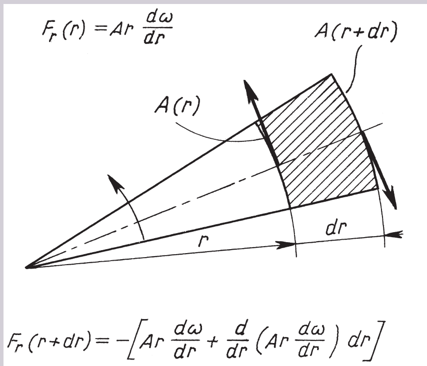
$$\mathbf{QR}=\frac{1}{2}\frac{d^2\mathbf{r}}{dt^2}\bigg|_{t=0}t^2,\quad \frac{d^2\mathbf{r}}{dt^2}=\frac{\mathbf{F}}{m},$$

and the area $SP\cdot QT$ is proportional to the time t (in a neighborhood of $\mathbf{r}_P$, that is, for small QT), it follows that

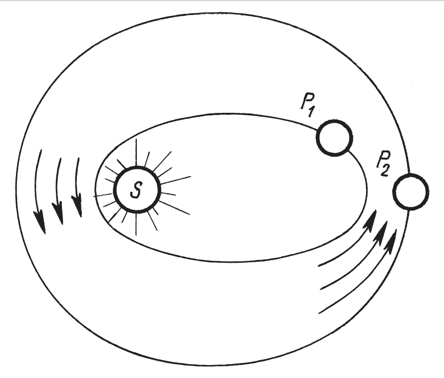
$$QR\propto F\cdot SP^2\cdot QT^2,$$

and finally, the proportion

$$\frac{1}{F}=\frac{SP^2\cdot QT}{QR}.$$



▲ **Figure 3.132** Cylindrically symmetric vortical flow of a viscous fluid.



▲ **Figure 3.133** For the planetary orbits shown, DESCARTES's vortex theory would give higher velocities when the planets are further from the Sun, in contradiction to KEPLER's law.

Quotation 3.51

I have endeavored in this Proposition to investigate the properties of vortices, that I might find whether the celestial phenomena can be explained by them; for the phenomenon is this, that the periodic times of the planets revolving about Jupiter are as the 3/2th power of their distances from Jupiter's centre; and the same rule obtains also among the planets that revolve about the sun. And these rules obtain also with the greatest accuracy, as far as has been yet discovered by astronomical observation. Therefore if those planets are carried round in vortices revolving about Jupiter and the sun, the vortices must revolve according to that law. But here we found the periodic times of the parts of the vortex to be as the square of the distances from the centre of motion; and this ratio cannot be diminished and reduced to the 3/2th power, unless either the matter of the vortex be more fluid the farther it is from the centre, or the resistance arising from the want of lubricity in the parts of the fluid should, as the velocity with which the parts of the fluid are separated goes on increasing, be augmented with it in a greater ratio than that in which the velocity increases. But neither of these suppositions seems reasonable. The more gross and less fluid parts will tend to be the circumference, unless they are heavy towards the centre. And though, for the sake of demonstration, I proposed, at the beginning of this Section, an Hypothesis that the resistance is proportional to the velocity, nevertheless, it is in truth probable that the resistance is in a less ratio than that of the velocity; which granted, the periodic times of the parts of the vortex will be in a greater ratio than the square of the distances from its centre. If, as some think, the vortices move more

continued on next page

showed that such a path will be a conic section and gave conditions under which an elliptical, parabolic, or hyperbolic path would be realized.

We select three topics from the second book of the *Principia*, of which the first two are of particular historical interest. NEWTON worked intensively on vortices in fluids, obviously with the goal of proving that the theory of vortices, which plays such a great role in Cartesian cosmology, was absurd.

In the case of a cylindrically symmetric vortex, NEWTON begins with the correct assumption that the frictional force deriving from the internal friction of the fluid "is proportional to the velocity with which the fluid elements move against one another."

We describe, following HUND [1972], a simplified description of NEWTON's train of thought.

Since the velocity at a particular point is given by the product of the radius and angular velocity, the relative velocity of the fluid elements to one another with respect to the relative velocities of the particles of a rotating rigid body are given by the first term of the total differential of the velocity,

$$\mathrm{d}(\omega r) = r\,\mathrm{d}\omega + \omega\,\mathrm{d}r.$$

The second term describes the portion that is the result of rigid rotation. It follows then that the frictional force is proportional to the surface area $A(r)$ and to the relative velocity gradient $r\frac{\mathrm{d}\omega}{\mathrm{d}r}$:

$$A(r)r\frac{\mathrm{d}\omega}{\mathrm{d}r}.$$

According to Figure 3.132, the equilibrium condition is

$$F_r(r) + F_r(r + \mathrm{d}r) = 0,$$

or

$$\frac{\mathrm{d}}{\mathrm{d}r}\left[A(r)r\frac{\mathrm{d}\omega}{\mathrm{d}r}\right] = 0,$$

from which follows

$$A(r)r\frac{\mathrm{d}\omega}{\mathrm{d}r} = \text{constant}.$$

If we further consider that the area A is proportional to the radius, the angular velocity must satisfy the relationship

$$\frac{\mathrm{d}\omega}{\mathrm{d}r} = \frac{\text{constant}}{r^2}, \quad \omega \propto \frac{1}{r}.$$

However, NEWTON's argumentation for the derivation of the relationship $\omega \propto 1/r^2$ between angular velocity and radius for the vortical flow around a sphere is not so convincing. But if the planets are carried along in their orbits by such vortices, then the relationship

$$T \propto r^2$$

for the orbital periods of the planets and their orbital radii would have to be satisfied, in contradiction to the relationship

$$T \propto r^{3/2}$$

required by Kepler's third law. From this contradiction, NEWTON concludes that the motion of the planets cannot be explained by vortices.

More convincing is the qualitative argument that the theory of vortices is incapable of leading to Kepler's second law. To see this, consider two planets as in Figure 3.133, one of which moves in a nearly circular orbit, while the orbit of the other planet is an ellipse of greater eccentricity; that is, the second planet moves along an elongated ellipsis. The streaming substance carrying the planets along must have a greater velocity farther from the Sun than closer in, due to the smaller cross section of the stream's channel; but then the same must hold for the planets, in contradiction to Kepler's second law. NEWTON concluded from this that the vortex theory could not be brought into accord with astronomical observation. The theory explained nothing and just caused confusion (Quotation 3.51).

In the second problem that we have selected, NEWTON deals with the question of how deeply a moving solid cylindrical body would penetrate into a fluid or another solid body. He has this to say about the problem (Liber II, Propositio XXXVII): The resistance generated by the cross section of a cylinder that moves uniformly along its axis in a dense, unbounded, and inelastic liquid is in the same proportion to the force that could cause or stop its movement during the time during which it could traverse four times its length, as the density of the medium is to the density of the cylinder, approximately.

The meaning of this difficult proposition can be restated in a simplified form: As the cylinder moves in the fluid, its front face makes way by pushing the material ahead of it sideways. Today we would say that it continuously transfers its kinetic energy to the surrounding material. This continuous transfer of impulse is the breaking force, whose magnitude is specified by NEWTON's statement. Let us investigate, says NEWTON, what force would be required to accelerate the cylinder of the given mass to its present speed within a well-defined time, namely, the time in which the cylinder traverses a path that corresponds to four times its length, that is,

$$\frac{4l}{v_0}.$$

This force should be equal to the breaking force. This assertion is noteworthy in that from it, one can derive the surprising conclusion that the length of the path traversed by the cylinder in the fluid (until it comes to rest) depends not on the initial velocity but only on the length of the cylinder. Namely, to a good approximation, the ratio between the length of the path traversed by the cylinder to the cylinder's own length is the same as that of the density of the cylinder's material to that of the fluid. In connection with this, GAMOW relates an interesting story: During the Second World War, it was surprising how little the depth to which bombs penetrated the soil depended on the height from which they were dropped. A theorem from Newton's *Principia* provided a solution to the mystery.

To investigate this problem in somewhat greater detail, we cast NEWTON's assertion in the form

$$R : F \propto \rho_{\text{fluid}} : \rho_{\text{metal}}.$$

The magnitude of the force is given by

$$F = m\frac{v_0}{4l/v_0} = \frac{mv_0^2}{4l},$$

from which we obtain the resistive force

$$R = F\frac{\rho_{\text{fluid}}}{\rho_{\text{metal}}} = \frac{mv_0^2}{4l}\frac{\rho_{\text{fluid}}}{\rho_{\text{metal}}}.$$

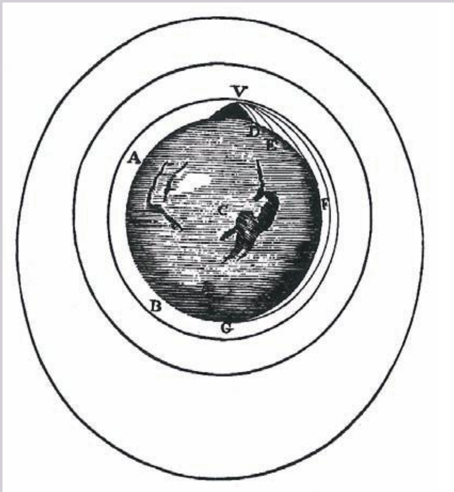
If now assume, by way of a crude approximation, that this force remains constant until the cylinder comes to rest, then the distance traveled by the cylinder is given by

$$s = \frac{v_0^2}{2a},$$

Quotation 3.51, ***continued***

swiftly near the centre, then slower to a certain limit, then again swifter near the circumference, certainly neither the 3/2th power, nor any other certain and determinate power, can obtain in them. Let philosophers then see how that phenomenon of the 3/2th power can be accounted for by vortices.

—ISAAC NEWTON, *Principia*, Book II, Section IX



▲ **Figure 3.134** We meet here for the first time in the history of physics the idea of an artificial satellite and the associated theory that is still valid today. (*De mundi systemate liber Isaaci Newtoni*, 1728.)

A body moves in a circular path about the Earth if the attractive force GMm/R^2 yields precisely the necessary acceleration V^2/R. Therefore, for this (horizontal) initial

continued on next page

Figure 3.134 *continued*

velocity—called the first cosmic velocity—one has the equation

$$\frac{GMm}{R^2} = \frac{mv_1^2}{R} = mg \rightarrow v_1 = \sqrt{\frac{GM}{R}} = \sqrt{Rg} = 7900 \text{ m/s}$$

(R = 6380 km, g = 9.8066 m/s^2).

If the body's kinetic energy $\frac{1}{2}mv^2$ is greater at the surface of Earth than its potential energy GMm/R, then the body will leave Earth's gravitational field along a parabolic path. The equation $\frac{1}{2}mv^2 = GMm/R$ determines the second cosmic velocity, the escape velocity

$$v_2 = \sqrt{\frac{2GM}{R}} = \sqrt{2Rg} = \sqrt{2}v_1 = 11190 \text{ m/s}.$$

Newton's theory could, of course, also be applied to the "mysterious" comets. Newton himself gave a graphical method for an approximate determination of comets' paths and of the times of their reappearances, all from a small number of observations. Edmond Halley (1656–1742), publisher and supporter of the book *Principia*, applied this method to the comet appearing in 1681/1682 and was able to identify it with other historical comets. The correct prediction of the comet's return in 1758 was a triumph for Newton's theory of great scientific and indeed psychological consequence.

Although Halley's predictions were made more precise by the French mathematician Alexis Clairaut (1713–1765), the comet is known today as Halley's comet.

The first historical record of this comet comes from the year 240 BCE Since then, it has been regularly observed at periods of 74 to 79 years (with a single exception). The most recent years of return were 1531, 1607, 1682, 1758, 1835, 1910, 1986. Historical associations are linked with the years 451 (battle against Attila), 1066 (Battle of Hastings), and 1456 (Siege of Belgrade by the Turks).

Quotation 3.52

And hence appears a method both of comparing bodies one with another, as to the quantity of matter in each; and of comparing the weights of the same body in different places, to know the variation of its gravity. And by experiments made with the greatest accuracy, I have always found the quantity of matter in bodies to be proportional to their weight.

—Isaac Newton, *Principia*, Book II, Section VI

and using the relationship given above, we conclude that

$$s = \frac{v_0^2}{2(R/m)} = \frac{4l\rho_{\text{metal}}Rm}{m\rho_{\text{fluid}}2R} = 2\frac{\rho_{\text{metal}}}{\rho_{\text{fluid}}}l,$$

that is,

$$\frac{s}{l} = 2\frac{\rho_{\text{metal}}}{\rho_{\text{fluid}}}.$$

We see that the ratio of the two lengths indeed depends only on that of the two densities.

Our crude approximation can be justified in that when a bomb penetrates a solid medium, only the impulses transferred at high speeds play a significant role, since with reduced velocity, the bomb simply "gets stuck."

In Theorem XXIV of the second book, we encounter a statement of major theoretical significance: The concepts of weight (W) and mass (m) are separated here—in an experimentally measurable manner. If we do not use the relationship $W = mg$ in the equation of the pendulum, then for the period, we obtain

$$T = 2\pi\sqrt{\frac{ml}{W}},$$

which yields the relationship

$$m = \frac{T^2}{(2\pi)^2} \times \frac{W}{l}$$

for the mass. At this point (Quotation 3.52), Newton points out the proportionality between the weight W and mass m.

The third part of the *Principia* treats planetary motion on the basis of the general law of attraction between masses. In addition to the assertion that the same laws hold for both terrestrial and celestial phenomena, Newton also specifies the precise conditions under which a terrestrial body can become a celestial one. With reference to Figure 3.134, let us consider the paths of projectiles fired horizontally with varying velocities from the peak of a high mountain. Of course, with increasing velocity, the distance between the launching and landing sites of the projectile increases, and if the launching velocity is sufficiently great, then the projectile could—in principle—orbit Earth, landing at the spot from which it was launched. Let us suppose that the motion takes place in a vacuum. Then in this last case, the launching and landing velocities will be the same, so that the projectile can orbit Earth repeatedly like an artificial satellite. For today's reader, this is perhaps the most interesting part of the *Principia*, since we are so familiar with satellites and Moon landings as memorable events of our modern era and therefore are astonished that not only the requisite theoretical tools, but also the explanatory qualitative illustration, can already be found in the *Principia*.

For Newton's contemporaries it was of greater significance that besides the regular motion of the planets he also qualitatively explained a host of other phenomena whose interpretations had been attempted in vain by outstanding scholars before him. These include the tides, which, as we have mentioned, were investigated intensively by both Galileo and Descartes without either of them arriving at a satisfactory explanation. A similar once baffling problem was the precession of Earth's axis, which is manifested—as we have already mentioned in connection with ancient natural science (Section 1.4)—in the shifting of the equinoxes along the ecliptic.

3.7.5 *Newton as Philosopher*

Newton's significance as a philosopher is threefold: he formulated and therefore determined the method of research for the natural sciences, for a long time to come; he set the basic goals of scientific research; and finally, he established a coherent, unified cosmology.

These contributions remained unchallenged in the three centuries that have elapsed since then—the Newtonian conception was at most refined in certain points.

Newton's method of scientific investigation was already established in the foreword to the *Principia* (Quotation 3.53). The following sentence there deserves special emphasis: "[F]or the whole burden of philosophy seems to consist in this—from the phenomena of motions to investigate the forces of nature, and then from these forces to demonstrate the other phenomena."

Our first glance into the third book of the *Principia* is sufficient to convince us of how much Newton respected facts and shows that his considerations started from facts and led back to facts. The subject of the book is the structure of the universe, and it starts with a most precise and detailed collection of observations. The sheer volume of tables and data in such a theoretical work is astounding.

Today, we are so accustomed to the scientific method as laid down by Newton that we consider it to be self-evident. However, we should recall that even Galileo set out from considerations of a completely different sort. He argued, for example, that motion with uniform acceleration must be realized in nature because it is the simplest form of change in velocity, or even that circular motion is the natural form of motion for celestial bodies. Or recall that Descartes forced his entire cosmology into a rationally clear and easily understandable starting point. In this sense, Newton realized Bacon's program, but in contrast to Bacon, he gave mathematics its rightful place in scientific investigation.

Due to Newton's influence, the criterion for truth in the natural sciences is no longer the logical derivability from some set of simple axioms, but rather agreement with the conclusions of fundamental laws that were read from nature, with experiment having the final say. If theoretical conclusions and experimental observations cannot be brought into agreement, then the underlying theory needs to be revised; Newton expresses this categorically in the *Philosophical Rules* that preface the third book of the *Principia* (Figure 3.135). We quote here the following of the rules:

Rule I: We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances. …

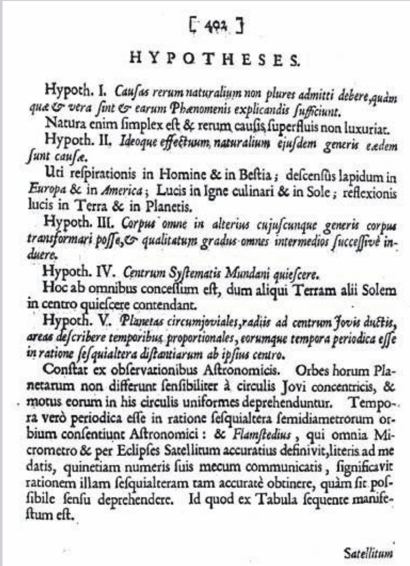
Rule II: Therefore to the same natural effects we must, as far as possible, assign the same causes.

> As to respiration in a man and in a beast; the descent of stones in Europe and in America; the light of our culinary fire and of the Sun; the reflection of light in Earth, and in the planets. ...

Rule IV: In experimental philosophy, we are to look upon propositions inferred by general induction from phenomena as accurately or very nearly true, notwithstanding any contrary hypotheses that may be imagined, till such time as other phenomena occur, by which they may either be made more accurate, or liable to exceptions.

> This rule we must follow, so that the argument of induction may not be evaded by hypotheses.

Even in his book *Optics*, Newton dealt with questions of natural philosophy. "In the beginning, God created the universe and the atoms." These atoms were considered by Newton to be fixed and "indestructible," and he reformulated the



[402]

HYPOTHESES.

Hypoth. I. *Cauſas rerum naturalium non plures admitti debere, quàm quæ & vera ſint & earum Phænomenis explicandis ſufficiunt.*

Natura enim ſimplex eſt & rerum cauſis ſuperfluis non luxuriat.

Hypoth. II. *Ideoque effectuum naturalium ejuſdem generis eædem ſunt cauſæ.*

Uti reſpirationis in Homine & in Beſtia; deſcenſûs lapidum in *Europa* & in *America*; Lucis in Igne culinari & in Sole; reflexionis lucis in Terra & in Planetis.

Hypoth. III. *Corpus omne in alterius cujuſcunque generis corpus transformari poſſe, & qualitatum gradus omnes intermedios ſucceſſivè induere.*

Hypoth. IV. *Centrum Syſtematis Mundani quieſcere.*

Hoc ab omnibus conceſſum eſt, dum aliqui Terram alii Solem in centro quieſcere contendant.

Hypoth. V. *Planetas circumjoviales, radiis ad centrum Jovis ductis, areas deſcribere temporibus proportionales, eorumque tempora periodica eſſe in ratione ſeſquialtera diſtantiarum ab ipſius centro.*

Conſtat ex obſervationibus Aſtronomicis. Orbes horum Planetarum non differunt ſenſibiliter à circulis Jovi concentricis, & motus eorum in his circulis uniformes deprehenduntur. Tempora verò periodica eſſe in ratione ſeſquialtera ſemidiametrorum orbium conſentiunt Aſtronomici: & *Flamſtedius*, qui omnia Micrometro & per Eclipſes Satellitum accuratius definivit, literis ad me datis, quinetiam numeris ſuis mecum communicatis, ſignificavit rationem illam ſeſquialteram tam accuratè obtinere, quàm ſit poſſibile ſenſu deprehendere. Id quod ex Tabula ſequente manifeſtum eſt.

Satellitum

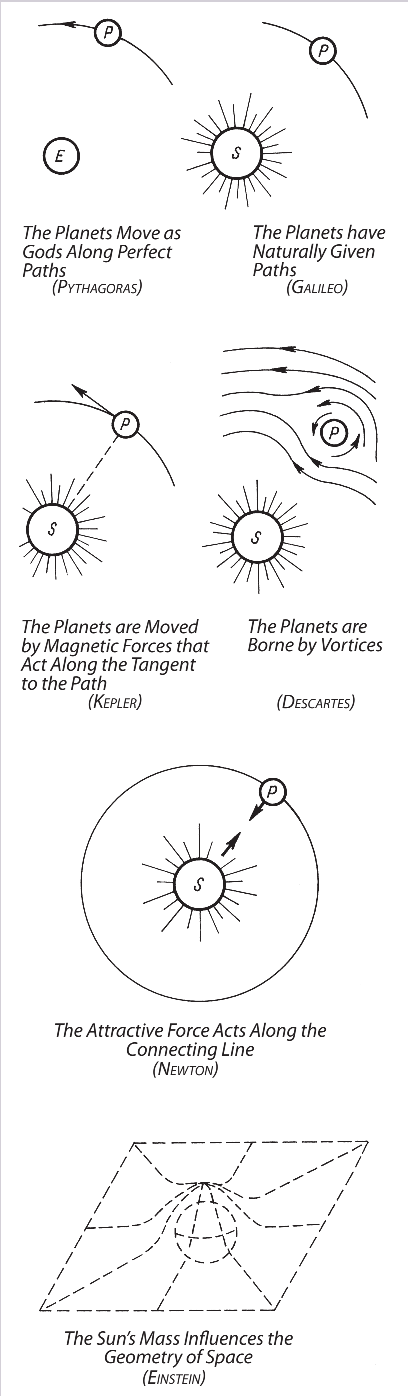
▲ **Figure 3.135** The initial hypotheses.

Quotation 3.53

... the whole burden of philosophy seems to consist in this—from the phenomena of motions to investigate the forces of nature, and then from these forces to demonstrate the other phenomena ....

I wish we could derive the rest of the phenomena of Nature by the same kind of reasoning from mechanical principles, for I am induced by many reasons to suspect that they may all depend upon certain forces by which the particles of bodies, by some causes hitherto unknown, are either mutually impelled towards one another, and cohere in regular figures, or are repelled and recede from one another. [These forces being unknown, philosophers have hitherto attempted the search of Nature in vain; but I hope the principles here laid down will afford some light either to this or some truer method of philosophy.]

—Isaac Newton, *Principia*, preface to the first edition, 1638



▲ **Figure 3.136** Explanations for the motion of planets at different times in history.

old principle of Democritus that stated that changes in macroscopic bodies are a result of the association, disassociation, and motion of atoms.

Although Newton used almost the same words as those used by the atomists of antiquity, their meaning was now deeper, in the sense that atomic theory had now become a program that could be quantitatively formulated. Specifically, if we can find the forces acting between particles, then the phenomena can also be described quantitatively. In the Newtonian world, we are dealing with centers of force and mechanistic motion that takes place under the influence of forces. Today, we say that Newton wanted to reduce all processes to mechanics, and the Newtonian worldview represents the completeness of the mechanistic worldview. However, we must note that Newton's contemporaries—and here we are thinking of the most astute and critical contemporaries, such as Huygens and Leibniz—understood something quite different by a mechanical explanation because, in their view, an interaction could be caused only by direct contact. They saw Newton's introduction of attractive forces acting at a distance as a step backward that would smuggle back into physics such ancient occult qualities as affinity, desire, and affection. Newton recognized the legitimacy of Huygens's objections, but he justified himself, correctly, with the argument that with the help of the force acting at a distance that he had postulated, "the phenomena of the heavens and the oceans" could be correctly described. Newton also felt that there was something there that needed explaining; he himself had long thought about this and, being unable to come up with the cause of gravitation, declared, "*Hypotheses non fingo*" (I frame no hypotheses). By "hypothesis," Newton meant an assumption that was unsupported by observation and could not be derived from it (Quotation 3.54). According to Newton, the Cartesian vortices are such a hypothetical concept, whereas the law of gravitation should be seen not as a hypothesis, but as fact.

Ultimately, the results of Newton's investigations and the creation of the Newtonian worldview are not only scientific but also philosophical achievements. We must therefore note with a measure of surprise and disappointment that despite his achievements, Newton is accorded no mention or at best a minor place in the history of philosophy. Many books on the history of philosophy can be found in which Newton's name does not appear in the index at all. This is all the more surprising when one considers that Aristotle and the Aristotelian cosmology is one of the favored topics in every such book. But it was exactly that cosmology that was replaced by the Newtonian worldview in the consciousness of mankind.

The old problem of a finite, closed universe was essentially restricted to the solar system: the regular motion of the sphere of fixed stars raised no particular questions, but the apparently irregular wanderings of the planets gave cause for much speculation. In Figure 3.136 we can trace the visions of our solar system up to Newton, and then beyond Newton to the picture given by Einstein's theory of general relativity. We should add that the corrections made to Newton's theory by the most modern theories are almost imperceptible even today, at least on the scale of the solar system. But the Newtonian cosmology goes beyond the solar system and attempts to describe a homogeneous and infinite universe through the connection of the laws of force and motion. By homogeneous we should understand a universe that is everywhere composed of the same matter and that is subject to the same laws, whether the matter is on Earth's surface, even is a planet, or is the Sun itself.

The carrier of the phenomena of the Newtonian world and the arbiter of their rhythm are Newtonian absolute space and absolute time. Newton defined these two notions at the beginning of the *Principia* (Figure 3.137):

Absolute space, in its own nature, without relation to anything external, remains always similar and immovable.

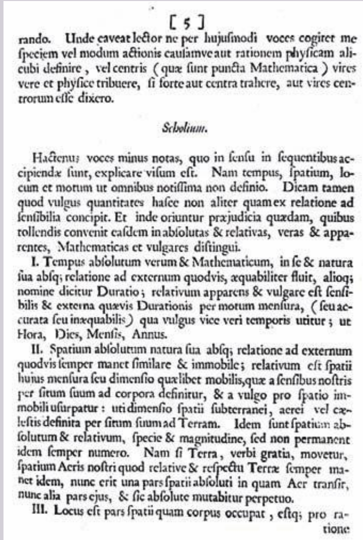
Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external.

Absolute motion is the translation of a body from one location in absolute space to another.

Newton also defined a relative space and a relative, perceptible, time, which we are able to measure in some manner with our instruments. Newton's notion of absolute space and time stands in sharp contrast to that of Descartes, who spoke only of relative position and relative motion. When Descartes discussed motion, he always meant a change of position of a body relative to the other bodies surrounding it. In this way, he could maintain that Earth does not move because its position does not change in relation to the surrounding vortical material (Section 3.4). However, Newton required the concepts of absolute space and time to relate the interaction of bodies separated by large reaches of space with the motion of those bodies. Newton clearly recognized the problems raised by his postulate of an absolute space. Leibniz sharply attacked this concept from the philosophical point of view and argued that we have no possibility of verifying any uniform motion of absolute space because it is not permitted to relate absolute space to any other object. Newton attempted to support his assumption of absolute motion—at least with respect to circular motion—with his well-known bucket analogy, which we can also demonstrate experimentally (Figure 3.138). In sharp contradiction to his own principle not to fabricate hypotheses, he even surmised that in distant reaches of the universe there are large masses that fix absolute space.

Although Newton strove to establish even his very general principles on a scientific and rational basis, he was deeply religious, and at many places in the *Principia*—especially in the later editions—he made reference to the necessity of divine influence, which, however, he limited to the setting of the universe into motion. Without this divine impulse, a host of observations made no sense to him, for example, the fact that all the planets orbit the Sun in the same direction and that the orbital planes are almost all coplanar. The Creator is even assigned the task in the Newtonian cosmology of intervening before a phenomenon could deviate from the laws. Leibniz commented sarcastically on this that God is apparently a poor mechanic, since he constructed a faulty machine that has to be repaired from time to time, and Dijksterhuis correctly remarked that Newton's god is an engineer who designed the world and set it in motion, and now God may still be an engineer, but He is in retirement.

Two great scientific revolutions of our era, the theory of relativity and quantum theory, call the Newtonian cosmology into question from two different points of view. With its concepts of quantities that are the same from any point of reference, relativity theory clears up the ideas of space and time, whereas quantum mechanics provides new equations of motion for particles of the micro world that replace those given by Newton. We have already mentioned that there is a great difference between the "wounding" of the Newtonian theory and attacks on the older, say the Aristotelian, theories in that Newtonian cosmology completely displaces the Aristotelian, taking on none of its results, whereas the Newtonian theory is a valid approximation of the theory of relativity and quantum theory in the limiting case of small velocity and large mass and therefore *remains a solid component of the natural sciences*.



[5]

rando. Unde caveat lector ne per hujuſmodi voces cogitet me ſpeciem vel modum actionis cauſamve aut rationem phyſicam alicubi definire, vel centris (quæ ſunt puncta Mathematica) vires vere et phyſice tribuere, ſi forte aut centra trahere, aut vires centrorum eſſe dixero.

Scholium.

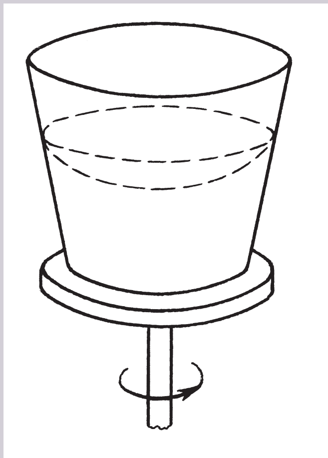
Hactenus voces minus notas, quo in ſenſu in ſequentibus accipiendæ ſunt, explicare viſum eſt. Nam tempus, ſpatium, locum et motum ut omnibus notiſſima non definio. Dicam tamen quod vulgus quantitates haſce non aliter quam ex relatione ad ſenſibilia concipit. Et inde oriuntur præjudicia quædam, quibus tollendis convenit eaſdem in abſolutas & relativas, veras & apparentes, Mathematicas et vulgares diſtingui.

I. Tempus abſolutum verum & Mathematicum, in ſe & natura ſua abſq; relatione ad externum quodvis, æquabiliter fluit, alioq; nomine dicitur Duratio; relativum apparens & vulgare eſt ſenſibilis & externa quævis Durationis per motum menſura, (ſeu accurata ſeu inæquabilis) qua vulgus vice veri temporis utitur; ut Hora, Dies, Menſis, Annus.

II. Spatium abſolutum natura ſua abſq; relatione ad externum quodvis ſemper manet ſimilare & immobile; relativum eſt ſpatii hujus menſura ſeu dimenſio quælibet mobilis, quæ a ſenſibus noſtris per ſitum ſuum ad corpora definitur, & a vulgo pro ſpatio immobili uſurpatur: uti dimenſio ſpatii ſubterranei, aerei vel cæleſtis definita per ſitum ſuum ad Terram. Idem ſunt ſpatium abſolutum & relativum, ſpecie & magnitudine, ſed non permanent idem ſemper numero. Nam ſi Terra, verbi gratia, movetur, ſpatium Aeris noſtri quod relative & reſpectu Terræ ſemper manet idem, nunc erit una pars ſpatii abſoluti in quam Aer tranſit, nunc alia pars ejus, & ſic abſolute mutabitur perpetuo.

III. Locus eſt pars ſpatii quam corpus occupat, eſtq; pro ratione

▲ **Figure 3.137** The page of *Principia* with the Newtonian definition of absolute space and absolute time.



▲ **Figure 3.138** If a bucket of water is rotated, then the surface of the water takes the form of a paraboloid of revolution, which does not depend on the relative motion of the bucket and the water. Newton argued that the rotation is to be understood relative to absolute space.

Berkeley already pointed out that rotation can be imagined only with respect to something else, such as the system of fixed stars, for only the relative motion of the two makes sense. The correct comparison is therefore the following: rotating bucket, resting universe ↔ resting bucket, rotating universe. The evident physical and epistemological problem that arises here was investigated by Mach (1872) and later Einstein (1916, Quotation 5.2.7).

continued on next page

Figure 3.138 *continued*

The effects which distinguish absolute from relative motion are the forces of receding from the axis of circular motion. For there are no such forces in a circular motion purely relative, but in a true and absolute circular motion, they are greater or less, according to the quantity of the motion. If a vessel, hung by a long cord, is so often turned about that the cord is strongly twisted, then filled with water, and held at rest together with the water; thereupon, by the sudden action of another force, it is whirled about the contrary way, and while the cord is untwisting itself, the vessel continues for some time in this motion; the surface of the water will at first be plain, as before the vessel began to move; but after that, the vessel, by gradually communicating its motion to the water, will make it begin sensibly to revolve, and recede by little and little from the middle, and ascend to the sides of the vessel, forming itself into a concave figure (as I have experienced), and the swifter the motion becomes, the higher will the water rise, till at last, performing its revolutions in the same times with the vessel, it becomes relatively at rest in it. This ascent of the water shows its endeavor to recede from the axis of its motion; and the true and absolute circular motion of the water, which is here directly contrary to the relative, becomes known, and may be measured by this endeavor. At first, when the relative motion of the water in the vessel was greatest, it produced no endeavor to recede from the axis; the water showed no tendency to the circumference, nor any ascent towards the sides of the vessel, but remained of a plain surface, and therefore its true circular motion had not yet begun. But afterwards, when the relative motion of the water had decreased, the ascent thereof towards the sides of the vessel proved its endeavor to recede from the axis; and this endeavor showed the real circular motion of the water continually increasing, till it had acquired its greatest quantity, when the water rested relatively in the vessel. And therefore this endeavor does not depend upon any translation of the water in respect of the ambient bodies, nor can true circular motion be defined by such translation.

—Isaac Newton, *Principia,* Book I, Definitions, Scholium

Newton's experiment with the rotating vessel of water simply informs us, that the relative rotation of the water with respect to the sides of the vessel produces no noticeable centrifugal forces, but that such forces are produced by its relative rotation with respect to the mass of the earth and the other celestial bodies. No one is competent to say how the experiment would turn out if the sides of the vessel increased in thickness and mass till they were ultimately several leagues thick.

—Ernst Mach, *The Science of Mechanics: A Critical and Historical Account of Its Development* [p. 284]

Newton's personal achievements did more than fill his educated contemporaries with admiration, many of whom devoted their lives to working out only a small portion of his contributions. Even those such as Huygens and Leibniz, who while understanding Newton's work, criticized it, respected the greatness of his accomplishments. Newton's apotheosis began already in his lifetime. Maupertuis, whom we shall meet again later, asked people who had had direct contact with Newton, "Does he walk, eat, drink like other mortals?" Poets composed verses in his honor, of which the following couplet by Pope is the most famous:

> All Nature and its laws lay hid in Night
> God said, let Newton be, and all was light.

The following inscription appears beneath the bust of Newton at Trinity Chapel, Cambridge: *Newton qui genus humanum ingenio superavit* (Newton, who in genius towered above the human race).

In France, no less a personality than Voltaire propagated Newton's teachings. Ten years after Newton's death, in 1737, a book appeared in Italian with the title *Neutonianismo per le donne*, which was then translated into English with the title *Sir Isaac Newton's Philosophy explain'd for the use of the Ladies*. The author was one Algarotti, on whose grave, for him the highest honor, appears the epitaph, "A student of Newton" (Figure 3.139).

The seventeenth century has been called the century of geniuses, and indeed, no other century seems to have brought forth so many great thinkers. In Table 3.4 we provide a listing of the most important of these, and with this table we take leave of this remarkable century. All who are mentioned here knew of one another, frequently with mutual regard, seldom with love, often criticizing and even despising. To show this, we have provided in the table a characteristic quotation from each about the others and, when possible, how they regarded themselves.

We conclude with the observations of a towering figure of the eighteenth century on two giants from the seventeenth:

> The famous Newton, this destroyer of the Cartesian system, died in March, anno 1727. His countrymen honoured him in his lifetime, and interred him as though he had been a king who had made his people happy.
>
> The English read with the highest satisfaction, and translated into their tongue, the Elogium of Newton, which Fontenelle spoke in the Academy of Sciences. Fontenelle presides as judge over philosophers; and the English expected this decision, as a solemn declaration of the superiority of the English philosophy over that of the French. But when it was found that this gentleman had compared Descartes to Newton, the whole Royal Society in London rose up in arms. So far from acquiescing with Fontenelle's judgment, they criticised his discourse. And even several (who, however, were not the ablest philosophers in that body) were offended at the comparison, and for no other reason but because Descartes was a Frenchman.
>
> It must be confessed that these two great men differed very much in conduct, in fortune, and in philosophy.
>
> Nature had indulged Descartes with a shining and strong imagination, whence he became a very singular person both in private life and in his manner of reasoning. This imagination could not conceal itself even in his philosophical works, which are everywhere adorned with very shining, ingenious metaphors and figures. Nature had almost made him a poet; and indeed he wrote

◀ **Table 3.4** How the great figures of the seventeenth century judged one another.

Quotation 3.54

Hitherto we have explained the phenomena of the heavens and of our sea by the power of gravity, but have not yet assigned the cause of this power. This is certain, that it must proceed from a cause that penetrates to the very centres of the sun and planets, without suffering the least diminution of its force; that operates not according to the quantity of the surfaces of the particles upon which it acts (as mechanical causes used to do), but according to the quantity of the solid matter which they contain, and propagates its virtue on all sides to immense distances, decreasing always as the inverse square of the distances. Gravitation towards the sun is made up out of the gravitations towards the several particles of which the body of the sun is composed; and in receding from the sun decreases accurately as the inverse square of the distances as far as the orbit of Saturn, as evidently appears from the quiescence of the aphelion of the planets; nay, and even to the remotest aphelion of the comets, if those aphelions are also quiescent.

But hitherto I have not been able to discover the cause of those properties of gravity from phenomena, and I frame no hypotheses, for whatever is not deduced from the phenomena is to be called an hypothesis; and hypotheses, whether metaphysical or physical, whether of occult qualities or mechanical, have no place in experimental philosophy. In this philosophy particular propositions are inferred from the phenomena, and afterwards rendered general by induction. Thus it was that the impenetrability, the mobility, and the impulsive force of bodies, and the laws of motion and of gravitation, were discovered. And to us it is enough that gravity does really exist, and act according to the laws which we have explained, and abundantly serves to account for all the motions of the celestial bodies, and of our sea.

—Isaac Newton, *Principia*, preface to the first edition, 1638

a piece of poetry for the entertainment of Christina, Queen of Sweden, which however was suppressed in honour to his memory.

He embraced a military life for some time, and afterwards becoming a complete philosopher, he did not think the passion of love derogatory to his character. He had by his mistress a daughter called Froncine, who died young, and was very much regretted by him. Thus he experienced every passion incident to mankind.

He was for a long time of opinion that it would be necessary for him to fly from the society of his fellow creatures, and especially from his native country, in order to enjoy the happiness of cultivating his philosophical studies in full liberty.

Descartes was very right, for his contemporaries were not knowing enough to improve and enlighten his understanding, and were capable of little else than of giving him uneasiness.

He left France purely to go in search of truth, which was then persecuted by the wretched philosophy of the schools. However, he found that reason was as much disguised and depraved in the universities of Holland, into which he withdrew, as in his own country, for at the time that the French condemned the only propositions of his philosophy which were true, he was persecuted by the pretended philosophers of Holland, who understood him no better; and who, having a nearer view of his glory, hated his person the more, so that he was obliged to leave Utrecht. Descartes was injuriously accused of being an atheist, the last refuge of religious scandal: and he who had employed all the sagacity and penetration of his genius, in searching for new proofs of the existence of a God, was suspected to believe there was no such Being. ...

At last Descartes was snatched from the world in the flower of his age at Stockholm. His death was owing to a bad regimen, and he expired in the midst of some literati who were his enemies, and under the hands of a physician to whom he was odious.

The progress of Newton's life was quite different. He lived happy, and very much honoured in his native country, to the age of fourscore and five years.

It was his particular felicity, not only to be born in a country of liberty, but in an age when all scholastic impertinences were banished from the world. Reason alone was cultivated, and mankind could only be his pupil, not his enemy.

One very singular difference in the lives of these two great men is that Newton, during the long course of years he enjoyed, was never sensible to any passion, was not subject to the common frailties of mankind, nor ever had any commerce with women—a circumstance which was assured me by the physician and surgeon who attended him in his last moments.

We may admire Newton on this occasion, but then we must not censure Descartes.

The opinion that generally prevails in England with regard to these new philosophers is that the latter was a dreamer, the former a sage.

Very few people in England read Descartes, whose works indeed are now useless. On the other side, but a small number peruse those of Newton, because to do this the student must be deeply skilled in the mathematics, otherwise those works will be unintelligible to him. But notwithstanding this, these great men are the subject of everyone's discourse. Newton is allowed every advantage, whilst Descartes is not indulged a single one. According to some, it is to the former that we owe the discovery of a vacuum, that the air is a heavy body, and the invention of telescopes. In a word, Newton is here as the Hercules of fabulous story, to whom the ignorant ascribed all the feats of ancient heroes.

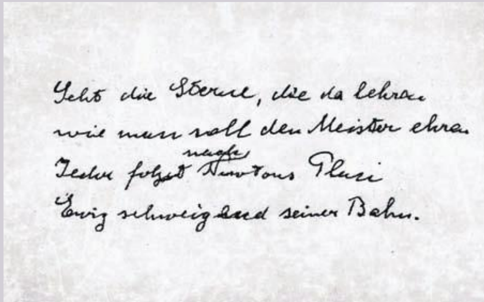
In a critique that was made in London on Fontenelle's discourse, the writer presumed to assert that Descartes was not a great geometrician. Those who make such a declaration may justly be reproached with flying in their master's face. Descartes extended the limits of geometry as far beyond the place where he found them, as Newton did after him. The former first taught the method of expressing curves by equations. This geometry which, thanks to him for it, is now grown common, was so abstruse in his time, that not so much as one professor could undertake to explain it; and Schotten in Holland, and Fermat in France, were the only men who understood it. ...

Geometry was a guide he himself had in some measure fashioned, which would have conducted him safely through the several paths of natural philosophy. Nevertheless, he at last abandoned this guide, and gave entirely into the humour of forming hypotheses; and then philosophy was no more than an ingenious romance, fit only to amuse the ignorant. He was mistaken in the nature of the soul, in the proofs of the existence of a God, in matter, in the laws of motion, and in the nature of light. He admitted innate ideas, he invented new elements, he created a world; he made man according to his own fancy; and it is justly said, that the man of Descartes is, in fact, that of Descartes only, very different from the real one.

He pushed his metaphysical errors so far, as to declare that two and two make four for no other reason but because God would have it so. However, it will not be making him too great a compliment if we affirm that he was valuable even in his mistakes. He deceived himself, but then it was at least in a methodical way. He destroyed all the absurd chimeras with which youth had been infatuated for two thousand years. He taught his contemporaries how to reason, and enabled them to employ his own weapons against himself. If Descartes did not pay in good money, he however did great service in crying down that of a base alloy.

I indeed believe that very few will presume to compare his philosophy in any respect with that of Newton. The former is an essay, the latter a masterpiece. But then the man who first brought us to the path of truth was perhaps as great a genius as he who afterwards conducted us through it.

—Voltaire, "Letters on England" ("Lettres Anglaises") XIV



▲ **Figure 3.139** It was not only his contemporaries who were in awe of Newton. This verse appears on the reverse side of a manuscript page by Einstein filled with complex formulas. Einstein wrote a number of ironic verses for his own and his friends' amusement, but this poem shows his great respect toward Newton:

Seht die Sterne, die da lehren
Wie man soll den Meister ehren
Jeder folgt nach Newtons Plan
Ewig schweigend seiner Bahn

(Look at the stars, which instruct us
How to honor the master
Each follows according to Newton's plan
Eternally silent its course)