forces whose action extends to a distance from the body as great as that at which rays are inflected. We are equally unable to admit that diffraction is caused by a shallow atmosphere which has the same thickness as the sphere of activity of these forces, and whose refractive index differs from that of the neighboring medium ; for this second hypothesis, like the first, would lead us to think that the inflection of light ought to vary with the form and the nature of the edge of the screen, and ought not to be the same, for instance, at the edge and at the back of a razor. Now, on the emission-theory it is impossible to explain in any other manner the expansion of a beam of light passing through a narrow opening, and this expansion is a well-established fact.* Consequently, *the phenomena of diffraction cannot be explained on the emission-theory.*

SECTION II

33. In the first section of this memoir I have shown that the corpuscular theory, and even the principle of interference when applied only to direct rays and to rays *reflected or inflected at the very edge of the opaque screen,* is incompetent to explain the phenomena of diffraction. I now propose to show that we may find a satisfactory explanation and a general theory in terms of waves, without recourse to any auxiliary hypothesis, by basing everything upon the principle of Huygens and upon that of interference, both of which are inferences from the fundamental hypothesis.

Admitting that light consists in vibrations of the ether similar to sound-waves, we can easily account for the inflection of rays of light at sensible distances from the diffracting body. For when any small portion of an elastic fluid under-

* The rise of a liquid in a capillary tube occurs between two surfaces separated by a finite distance, although the attraction which these surfaces exert upon the liquid extends only to an infinitely small distance. The reason of this is, that the molecules of the liquid, attracted by the surface of the tube, also in their turn attract other molecules of the liquid situated within their sphere of action, and so on, step by step ; but in the emission-theory an analogous explanation is not admissible, for the fundamental hypothesis is that the luminous particles never exert any sensible effect upon the path of neighboring particles. No interdependence of motion is here admissible, for such an assumption would be the assumption of a fluid medium.

goes condensation, for instance, it tends to expand in all directions; and if throughout the entire wave the particles are displaced only along the normal, the result would be that all points of the wave lying upon the same spherical surface would simultaneously suffer the same condensation or expansion, thus leaving the transverse pressures in equilibrium; but when a portion of the wave-front is intercepted or retarded in its path by interposing an opaque or transparent screen, it is easily seen that this transverse equilibrium is destroyed and that various points of the wave may now send out rays along new directions.

To follow by analytical mechanics all the various changes which a wave-front undergoes from the instant at which a part of it is intercepted by a screen would be an exceedingly difficult task, and we do not propose to derive the laws of diffraction in this manner, nor do we propose to inquire what happens in the immediate neighborhood of the opaque body, where the laws are doubtless very complicated and where the form of the edge of the screen must have a perceptible effect upon the position and the intensity of the fringes. We propose rather to compute the relative intensities at different points of the wave-front only after it has gone a large number of wave-lengths beyond the screen. Thus the positions at which we study the waves are always to be regarded as separated from the screen by a distance which is very considerable compared with the length of a light-wave.

34. We shall not take up the problem of vibrations in an elastic fluid from the point of view which the mathematicians have ordinarily employed—that is, considering only a single disturbance. Single vibrations are never met with in nature. Disturbances occur in groups, as is seen in the pendulum and in sounding bodies. We shall assume that vibrations of luminous particles occur in the same manner—that is, one after another and series after series. This hypothesis follows not only from analogy, but as an inference from the nature of the forces which hold the particles of a body in equilibrium. To understand how a single luminous particle may perform a large series of oscillations all of which are nearly equal, we have only to imagine that its density is much greater than that of the fluid in which it vibrates—and, indeed, this is only what has already been inferred from the uniformity of the motions of

the planets through this same fluid which fills planetary space. It is not improbable also that the optic nerve yields the sensation of sight only after having received a considerable number of successive stimuli.

However extended one may consider systems of wave-fronts to be, it is clear that they have limits, and that in considering interference we cannot predicate of their extreme portions that which is true for the region in which they are superposed. Thus, for instance, two systems of equal wave-length and of equal intensity, differing in path by half a wave, interfere destructively only at those points in the ether where they meet, and the two extreme half wave-lengths escape interference.

Nevertheless, we shall assume that the various systems of waves undergo the same change throughout their entire extent, the error introduced by this assumption being inappreciable; or, what amounts to the same thing, we shall assume in our discussion of interference that these series of light-waves represent general vibrations of the ether, and are undefined as to their limits.

The Problem of Interference

35. *Given the intensities and relative positions of any number of trains of light-waves of the same length* and travelling in the same direction, to determine the intensity of the vibrations produced by the meeting of these different trains of waves, that is, the oscillatory velocity of the ether particles.*†

* We shall not here consider light-waves of different lengths which, in general, come from different sources and which cannot, therefore, give rise to simultaneous disturbances and cannot by their interaction produce any phenomena which are uniform; and even if they were uniform, the rise and fall of intensity produced by the interference of two different kinds of waves, after the manner of beats in sound, would be far too rapid to be detected, and would produce only a sensation of constant intensity.

† It was Mr. Thomas Young who first introduced the principle of interference into optics, where he showed much ingenuity in applying it to special cases; but in the problems which he has thus solved he has considered, I think, only the limiting cases, where the difference in phase between the two trains of waves is either a maximum or a minimum, and has not computed the intensity of the light for any intermediate cases or for any number whatever of trains of waves, as I here propose to do.

Employing the general principle of the superposition of small motions, the total velocity impressed upon any particle of a fluid is equal to the sum of the velocities impressed by each train of waves acting by itself. When these waves do not coincide, these different velocities depend not only upon the intensity of each wave, but also upon its phase at the instant under consideration. We must, therefore, know the law according to which the velocity of vibration varies in any one wave, and for this purpose we must trace the wave back to the origin whence it derives all its characteristics.

36. It is natural to suppose that the particles whose vibrations produce light perform their oscillations like those of sounding bodies—that is, according to the laws which hold for the pendulum; or, what is the same thing, to suppose that the acceleration tending to make a particle return to its position of equilibrium is directly proportional to the displacement. Let us denote this displacement by x. A suitable function of this displacement can then be represented by the expression $Ax+Bx^2+Cx^3+$etc., since this will vanish when $x=0$. If, now, we suppose the excursion of the particle to be very small when compared with the radius of the sphere throughout which the forces of attraction and repulsion act, we can neglect in comparison with Ax all other terms of the series and consider the acceleration as practically proportional to the distance x. This hypothesis, to which we are led by analogy, and which is the simplest that one can make concerning the vibrations of light particles, ought to lead to accurate results, since the laws of optics remain the same for all intensities of light.

Let us represent by v the velocity of vibration of a light particle at the end of a time t. We shall then have $dv=-Axdt$; but $v=dx/dt$, or $dt=dx/v$. Substituting in the first equation, we have $vdv=-Axdx$. Integrating, we have $v^2=C-Ax^2$; and hence

$$x=-\sqrt{\frac{C-v^2}{A}}.$$

Substituting this value of x in the first equation, we have

$$dt=\frac{dv}{\sqrt{A(C-v^2}},$$

which, on integration, gives

$$t = C' + \frac{1}{\sqrt{A}} \sin^{-1} \frac{v}{\sqrt{C}}.$$

If we measure time from the instant at which the velocity is zero, the constant C' becomes zero, and we have

$$t = \frac{1}{\sqrt{A}} \sin^{-1} \frac{v}{\sqrt{C}}, \text{ or } v = \sqrt{C} \sin t\sqrt{A}.$$

If we employ as unit of time the interval occupied by the particle in one complete vibration, we have $v = \sqrt{C} \sin (2\pi t)$. Thus, in isochronous vibrations, the velocities for equal values of t are always proportional to the constant $\sqrt{C}$, which, therefore, measures the intensity of the vibration.

37. Let us now consider the wave produced in the ether by the vibrations of this particle. The energy of motion in the ether at any point on the wave depends upon the velocity of the point-source at the instant when it started a disturbance which has just reached this point. The velocity of the ether particles at any point in space after an interval of time t is proportional to that of the point-source at the instant $t - x/\lambda$, x being the distance of this point from the source of motion and λ the length of a light-wave. Let us denote by u the velocity of the ether particles. We then have

$$u = a \sin \left[2\pi\left(t - \frac{x}{\lambda}\right)\right].$$

We know that the intensity a of vibration* [*oscillatory velocity*] in a fluid is in inverse ratio to the distance of the wave from the centre of disturbance; but, considering how minute these waves are when compared with the distance which separates them from the luminous point, we may neglect the variation of a and consider it as constant throughout the extent of one or even of several waves.

38. By the aid of this expression one can compute the intensity of vibration produced by the meeting of any number of pencils of light whenever he knows the intensity of the different trains of waves and their respective positions.

Let us first determine the velocity of a luminous particle in a vibration which results from the interference of two trains

* [*See last sentence of section* 57 *below.*]

of waves displaced, one with respect to the other, by a quarter of a wave-length [*i.e., differing in phase by* 90°], and having intensities which we shall denote by a and a'. We shall count time, t, from the moment at which the vibrations of the first train begin. Let u and u' be the velocities which the first and second trains of waves would impress upon a light particle whose distance from the source of motion is x. We then have

$$u=a \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)\right] \text{ and } u'=a' \sin\left[2\pi\left(\frac{t-x+\frac{\lambda}{4}}{\lambda}\right)\right],$$

or

$$u'=-a' \cos\left[2\pi\left(t-\frac{x}{\lambda}\right)\right].$$

Hence, the resultant velocity U will be

$$a \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)\right]-a' \cos\left[2\pi\left(t-\frac{x}{\lambda}\right)\right].$$

Putting $a=A \cos i$ and $a'=A \sin i$, this expression may always be placed in the following form:

$$A \cos i \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)\right]-A \sin i \cos\left[2\pi\left(t-\frac{x}{\lambda}\right)\right],$$

or

$$A \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)-i\right].$$

Thus the wave produced by the meeting of two others will be of the same nature, but will have a different position [*phase*] and a different intensity. From the equations $A \cos i=a$ and $A \sin i=a'$, we have for the value of A (that is, for the intensity of the resultant wave) $\sqrt{a^2+a'^2}$; but this is exactly the value of the resultant of two mutually rectangular forces, a and a'.

From the same equations it is easily seen also that the new wave exactly corresponds in angular position [*phase*] to the resultant of the two mutually rectangular forces a and a'; for the equation

$$U=A \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)-i\right]$$

shows that the linear displacement of this wave with respect to the first is $\frac{i\lambda}{2\pi}$; but i is also the angle which the force a

makes with the resultant A, because $A \cos \iota = a$. Thus we have complete analogy between the resultant of two mutually rectangular forces and the resultant of two trains of waves differing in phase by a quarter of a wave-length.

39. The solution of this particular case for waves differing by a quarter of a wave-length suffices to solve all other cases. In fact, whatever be the number of the trains of waves, and whatever be the intervals which separate them, we can always substitute for each of them its components referred to two reference points which are common to each train of waves and which are distant from each other by a quarter of a wave-length; then adding or subtracting, according to sign, the intensities of the components referred to the same point, we may reduce the whole motion to that of two trains of waves separated by the distance of a quarter of a wave-length; and the square root of the sum of the squares of their intensities will be the intensity of their resultant; but this is exactly the method employed in statics to find the resultant of any number of forces; here the wave-length corresponds to one circumference in the statical problem, and the interval of a quarter of a wave-length between the trains of waves to an angular displacement of 90° between the components.

40. It very often happens in optics that the intensities of light or the particular tint which one wishes to compute is produced by the meeting of only two trains of waves, as in the case of [*Newton's*] colored rings and the ordinary phenomena of color presented by crystalline plates. It is, therefore, well to know the general expression for the resultant of two trains of waves differing in phase by any amount whatever. The result is easily predicted from the general method which I have explained, but I think it will be wise to emphasize somewhat the theory of vibrations, and to show directly that the wave resulting from two others, separated by any interval whatever, corresponds exactly in intensity and position to the resultant of two forces whose intensities are equal to those of the two pencils of light, making an angle with each other which bears to one complete circumference the same ratio that the interval between the two trains of waves bears to one wave-length.

Let x be the distance from the origin of the first train of waves to the light particle under consideration, and t the instant for which we wish to compute its velocity. The speed impressed by the first train of waves will be

$$a \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)\right],$$

where a represents the intensity of this ray of light.

Let us call a' the intensity of the second pencil, and let us denote by c the distance between corresponding points on the two trains of waves; the [*oscillatory*] velocity due to the second train will then be

$$a' \sin\left[2\pi\left(t-\frac{x+c}{\lambda}\right)\right],$$

and hence the total velocity impressed upon the particle will be

$$a \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)\right]+a' \sin\left[2\pi\left(t-\frac{x+c}{\lambda}\right)\right],$$

or

$$\left[a+a' \cos\left(2\pi\frac{c}{\lambda}\right)\right] \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)\right]-a' \sin\left(2\pi\frac{c}{\lambda}\right) \cos\left[2\pi\left(t-\frac{x}{\lambda}\right)\right],$$

an expression to which may always be given the following form:

$$A \cos i \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)\right]-A \sin i \cos\left[2\pi\left(t-\frac{x}{\lambda}\right)\right],$$

or

$$A \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)-i\right],$$

where

$$a+a' \cos\left(2\pi\frac{c}{\lambda}\right)=A \cos i,$$

and

$$a' \sin\left(2\pi\frac{c}{\lambda}\right)=A \sin i.$$

Squaring and adding, we have

$$A^2=a^2+a'^2+2aa' \cos\left(2\pi\frac{c}{\lambda}\right).$$

Hence,

$$A=\pm\sqrt{a^2+a'^2+2aa' \cos\left(2\pi\frac{c}{\lambda}\right)}.$$

But this is precisely the value of the resultant of two forces, a and a', inclined to each other at an angle $2\pi\frac{c}{\lambda}$.

41. From this general expression it is seen that the resultant intensity of the light vibrations is equal to the sum of intensities of the two constituent pencils when they are in perfect agreement and to their difference when they are in exactly opposite phases, and, lastly, to the square root of the sum of their squares when their phase difference is a quarter of a wave-length, as we have already shown.

It thus follows that the phase of the wave corresponds exactly to the angular position of the resultant of two forces, a and a'. The distance from the first wave to the second is c, to the resultant wave $\frac{i\lambda}{2\pi}$, and from the resultant wave to the second is $c-\frac{i\lambda}{2\pi}$; accordingly, the corresponding angles are $2\pi.\frac{c}{\lambda}$, i, and $2\pi.\frac{c}{\lambda}-i$. Let us multiply the equation

$$a+a'\cos\left(2\pi\frac{c}{\lambda}\right)=A\cos i$$

by $\sin i$, and the following equation

$$a'\sin\left(2\pi\frac{c}{\lambda}\right)=A\sin i$$

by $\cos i$. Subtracting one from the other, we have

$$a\sin i=a'\sin\left(2\pi\frac{c}{\lambda}-i\right),$$

which, together with

$$a'\sin\left(2\pi\frac{c}{\lambda}\right)=A\sin i,$$

gives the following proportion:

$$\sin\left(2\pi\frac{c}{\lambda}-i\right):\sin i:\sin 2\pi\frac{c}{\lambda}::a:a':A.$$

42. The general expression, $A\sin\left[2\pi\left(t-\frac{x}{\lambda}\right)-i\right]$, for the velocity of the particles in a wave produced by the meeting of two others shows that this wave has the same length as its components and that the velocities at corresponding points are proportional, so that the resultant wave is always of the same nature as its components and differs only in intensity—that is to say, in the constant by which we must multiply the velocities in either of the components in order to obtain the correspond-

ing velocities in the resultant. In combining this resultant with still another new wave, one again arrives at an expression of the same form—a remarkable property of a function of this kind. Thus in the resultant of any number of trains of waves of the same length the light particles are always urged by velocities proportional to those of the components at points located at the same distance from the end of each wave. [*This is seen by multiplying each of the last three terms in the preceding proportion by* sin ωt. *For then,*

$$a \sin \omega t : a' \sin \omega t : A \sin \omega t :: a : a' : A' :: \textit{constant ratio.}]$$

Applications of Huygens's Principle to the Phenomena of Diffraction

43. Having determined the resultant of any number of trains of light-waves, I shall now show how by the aid of these interference formulæ and by the principle of Huygens alone it is possible to explain, and even to compute, all the phenomena of diffraction. This principle, which I consider as a rigorous deduction from the basal hypothesis, may be expressed thus: *The vibrations at each point in the wave-front may be considered as the sum of the elementary motions which at any one instant are sent to that point from all parts of this same wave in any one of its previous* positions, each of these parts acting independently the one of the other.* It follows from the principle of the superposition of small motions that the vibrations produced at any point in an elastic fluid by several disturbances are equal to the resultant of all the disturbances reaching this point at the same instant from different centres of vibration, whatever be their number, their respective positions, their nature, or the epoch of the different disturbances. This general principle must apply to all particular cases. I shall suppose that all of these disturbances, infinite in number, are of the same kind, that they take place simultaneously, that they

*I am here discussing only an infinite train of waves, or the most general vibration of a fluid. It is only in this sense that one can speak of two light waves annulling one another when they are half a wave-length apart. The formulæ of interference just given do not apply to the case of a single wave, not to mention the fact that such waves do not occur in nature.